The Radio Sky

Lecture 11

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PHY-236.3



Concept test

Question

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Answer: the column density of the emitting species

1. Emission adds linearly along the sightline: $\int T_{\rm b}\,{\rm d}v \propto N_u$.
2. Known excitation (one $T_{\rm ex}$, partition function) converts the upper-level column into the total column.

Opening question

Retrieval (from Lecture 10)

The H I 21-cm transition has a spontaneous emission coefficient $A_{10} \approx 2.9 \times 10^{-15}\,\mathrm{s}^{-1}$: an isolated atom waits $\sim 10^7$ yr to emit. Why is this line nevertheless one of the brightest features of the Galactic radio sky?

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- Huge columns: Galactic sightlines contain $N_{\mathrm{HI}} \sim 10^{20}$–$10^{22}\,\mathrm{cm}^{-2}$.
- Almost every atom contributes: $h\nu/k_{\mathrm{B}} = 0.068\,\mathrm{K}$ is tiny, so collisions keep $\approx 3/4$ of atoms in the upper level.
- The Galaxy is nearly transparent in the line, so emission from the whole path adds up; the line is narrow, so it stands out against the continuum.

Central question

How do H I emission and absorption reveal atomic gas and Galactic structure, and where do they fail?

The hyperfine transition

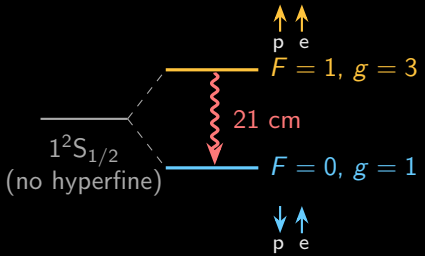
- Ground state $1^2S_{1/2}$ of hydrogen is split by the interaction of electron and proton magnetic moments.
- Total spin $F = 1$ (parallel, $g_1 = 3$) lies above $F = 0$ (antiparallel, $g_0 = 1$).
- Magnetic dipole transition:

$$\nu_{10} = 1420.405\,751\,8\ \mathrm{MHz}$$

$$\lambda = 21.106\ \mathrm{cm}$$

$$T_* \equiv h\nu_{10}/k_\mathrm{B} = 0.0682\ \mathrm{K}$$

- Predicted by van de Hulst (1944); detected by Ewen & Purcell (1951; doi:10.1038/168356a0).



Schematic, not to scale.

Why the line is weak, and why that does not matter

- Magnetic dipole, low frequency: $A = \dfrac{64\pi^4\nu^3}{3hc^3}\,|\mu_{10}|^2$, where $\mu_{10} = \langle F{=}1|\boldsymbol{\mu}|F{=}0\rangle \approx \mu_{\rm B}$ is the transition (spin-flip) matrix element, not the atom's net moment (cgs). This gives

$$A_{10} = 2.88\times10^{-15}\,\mathrm{s}^{-1}, \qquad A_{10}^{-1} \approx 1.1\times10^{7}\,\mathrm{yr}.$$

- Collisional de-excitation in the cold neutral medium takes $\sim 10^2$–10^3 yr, far shorter than A_{10}^{-1} (order-of-magnitude, depends on n and T).
- Hence level populations are set by collisions, and by Lyα pumping (Wouthuysen–Field effect), not by spontaneous decay. Field (1958; doi:10.1109/JRPROC.1958.286741).

Consequence

The emissivity per atom is small but *fixed* (independent of temperature) whenever $T_{\rm s} \gg T_* \equiv h\nu_{10}/k_{\rm B} = 0.068\,\mathrm{K}$, since then 3/4 of the atoms are in the upper level. Emission therefore counts atoms.

Spin temperature

Define the spin temperature T_s (the excitation temperature of this transition; Lecture 10) by

$$\frac{n_1}{n_0} = \frac{g_1}{g_0}\,\mathrm{e}^{-T_*/T_\mathrm{s}} = 3\,\mathrm{e}^{-0.0682\,\mathrm{K}/T_\mathrm{s}}$$

- For any astrophysical $T_\mathrm{s} \gtrsim 3\,\mathrm{K}$: $n_1/n_0 \approx 3$, so $n_0 \approx n_\mathrm{H}/4$, $n_1 \approx 3n_\mathrm{H}/4$.
- Field (1958): $T_\mathrm{s}^{-1} = \dfrac{T_\mathrm{R}^{-1} + x_c\,T_\mathrm{k}^{-1} + x_\alpha\,T_\mathrm{c}^{-1}}{1 + x_c + x_\alpha}$, with T_R the 21-cm radiation temperature and x_c, x_α the collisional and Lyα couplings.
- Lyα colour temperature T_c: the slope of the Lyα photon spectrum near line centre, $\mathrm{d}\ln n_\nu/\mathrm{d}\nu = -h/k_\mathrm{B}T_\mathrm{c}$. A Ly$\alpha$ absorption and re-emission can leave the atom in the other hyperfine level, so the photons drive n_1/n_0 toward T_c. Many scatterings thermalise them: $T_\mathrm{c} \approx T_\mathrm{k}$.
- Dense cold gas: $T_\mathrm{s} \approx T_\mathrm{k}$. Low-density warm gas: collisions may be too slow, so $T_\mathrm{s} < T_\mathrm{k}$ is possible.

Derivation: 21-cm optical depth (cgs units)

Assumptions: two levels, line profile $\phi(\nu)$ with $\int \phi \, \mathrm{d}\nu = 1$, uniform T_s along the path, $T_* \ll T_\mathrm{s}$. Absorption minus stimulated emission, then $1 - \mathrm{e}^{-x} \approx x$ and $n_0 = n_\mathrm{H}/4$:

$$\kappa_\nu = \frac{c^2}{8\pi\nu^2} \frac{g_1}{g_0} n_0 A_{10} \left[1 - \mathrm{e}^{-T_*/T_\mathrm{s}}\right] \phi(\nu) \approx \frac{3c^2 h A_{10}}{32\pi k_\mathrm{B} \nu} \frac{n_\mathrm{H}}{T_\mathrm{s}} \phi(\nu)$$

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Integrate along the line of sight and convert to velocity, $\mathrm{d}\nu = (\nu/c)\,\mathrm{d}v$:

$$\int \tau \, \mathrm{d}v = \frac{3c^3 h A_{10}}{32\pi k_\mathrm{B} \nu^2} \frac{N_\mathrm{HI}}{T_\mathrm{s}}$$

$$\boxed{N_\mathrm{HI} = 1.82 \times 10^{18} \, \mathrm{cm}^{-2} \int \left(\frac{T_\mathrm{s}}{\mathrm{K}}\right) \tau(v) \, \frac{\mathrm{d}v}{\mathrm{km\,s^{-1}}}}$$

$\tau \propto N_\mathrm{HI}/T_\mathrm{s}$: cold gas absorbs far more strongly than warm gas of the same column. (Often quoted as 1.823×10^{18}; the last digit depends on the adopted A_{10}.)

Transfer: emission in the optically thin limit

Uniform slab of spin temperature T_s in front of a background T_bg (Lectures 3 and 10):

$$\Delta T_\mathrm{b}(\nu) = (T_\mathrm{s} - T_\mathrm{bg})\left[1 - \mathrm{e}^{-\tau(\nu)}\right]$$

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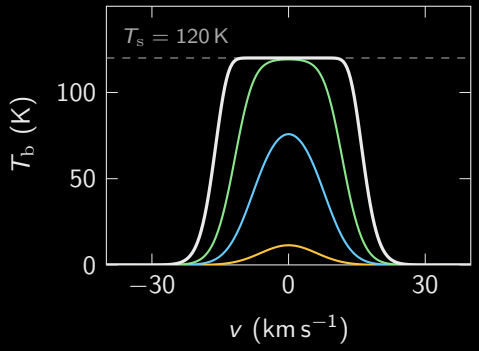
$$\Delta T_\mathrm{b}(v) = (T_\mathrm{s} - T_\mathrm{bg})\left[1 - \mathrm{e}^{-\tau(v)}\right]$$

If $\tau \ll 1$ and $T_\mathrm{s} \gg T_\mathrm{bg}$: $\Delta T_\mathrm{b} \approx T_\mathrm{s}\tau$, so

$$N_\mathrm{HI} \approx 1.82 \times 10^{18}\,\mathrm{cm^{-2}} \int \frac{T_\mathrm{b}\,\mathrm{d}v}{\mathrm{K\,km\,s^{-1}}}$$

- T_s cancels: optically thin emission measures column density without knowing the temperature.
- If $\tau \gtrsim 0.3$, this underestimates N_HI by the factor $\tau/(1 - \mathrm{e}^{-\tau})$.

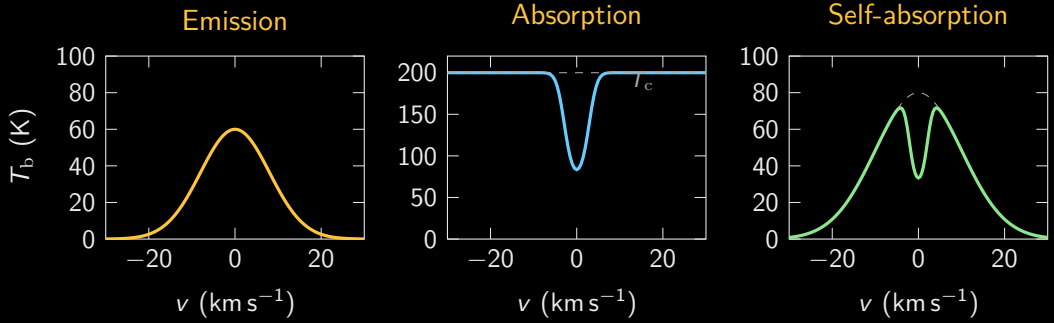
A saturated H I line: the Galactic plane



Peak $\tau_0 = 0.1$, 1, 5, 30 (bottom to top), same line shape; $T_b = T_s(1 - e^{-\tau})$. Schematic.

- Flat top at $T_b \approx T_s$: more gas makes the line *wider*, not taller.
- Emission cannot exceed T_s, so the plateau is a lower limit on T_s.
- $\int T_b \, dv$ stops tracking N_{HI}: the thin formula underestimates it by $\tau/(1 - e^{-\tau})$.
- Toward the inner plane, many clouds overlap along the line of sight, and the observed spectrum is a broad, flat-topped blend.

Three ways to see H I in one spectrum



- Emission: $T_s \gg T_{bg}$; measures $\int T_s \tau \, dv$, hence N_{HI} if thin.
- Absorption against a continuum source ($T_c = 200$ K, $T_s = 50$ K): measures $\tau(v)$ directly.
- H I self-absorption (HISA): cold foreground H I ($T_s = 20$ K) against warmer background H I emission.

Schematic profiles; illustrative parameters.

Concept test

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An H I cloud produces an *absorption* line against a continuum source. Must the cloud's spin temperature be above or below the background brightness temperature at line centre?

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Answer: below the background

1. The continuum-subtracted line is $\Delta T_\mathrm{b} = (T_\mathrm{s} - T_\mathrm{bg})(1 - \mathrm{e}^{-\tau})$; absorption means $\Delta T_\mathrm{b} < 0$.
2. Since $1 - \mathrm{e}^{-\tau} > 0$, this requires $T_\mathrm{s} < T_\mathrm{bg}$.

Cold and warm neutral media

	Cold neutral medium (CNM)	Warm neutral medium (WNM)
T_k	$\sim$40–200 K	$\sim$5000–8000 K
n_H	$\sim$20–50 cm^{-3}	$\sim$0.2–0.5 cm^{-3}
Thermal FWHM[†]	$\approx 2\,\mathrm{km\,s^{-1}}$ at 100 K	$\approx 19\,\mathrm{km\,s^{-1}}$ at 8000 K
Seen in	absorption *and* emission	emission; absorption very weak, $\tau \sim 10^{-3}$

- Two thermally stable phases in rough pressure equilibrium ($P/k_\mathrm{B} \sim 10^3$–$10^4\,\mathrm{K\,cm^{-3}}$) (Wolfire et al. 2003; doi:10.1086/368016).
- Weak is not zero: for $N_\mathrm{HI} = 10^{20}\,\mathrm{cm^{-2}}$ and thermal widths only, peak $\tau \approx 3 \times 10^{-4}$ at 8000 K but $\approx 2 \times 10^{-2}$ at 500 K, since $\tau \propto N/(T_\mathrm{s}\,\Delta v)$.
- Sensitive absorption surveys (Heiles & Troland 2003, doi:10.1086/367828; Murray et al. 2018, doi:10.3847/1538-4365/aad81a) detect it, and find a substantial fraction of the "warm" gas at intermediate, thermally unstable temperatures ($\sim$500–5000 K), where absorption is detectable.

[†]FWHM $= 0.214\sqrt{T_\mathrm{k}/\mathrm{K}}\,\mathrm{km\,s^{-1}}$ for H atoms. Ranges are representative textbook values, not sharp boundaries.

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The warm neutral medium has $T_\mathrm{k} \approx 8000\,\mathrm{K}$, where $k_\mathrm{B} T_\mathrm{k} \approx 0.7\,\mathrm{eV}$ is far above the thermal energy of a cold cloud. Shouldn't hydrogen at that temperature be ionised?

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Answer: no: it is $\sim 99.99\%$ neutral by collisions alone

1. The ionisation energy is 13.6 eV, so only a fraction $\mathrm{e}^{-13.6/0.69} \approx 3 \times 10^{-9}$ of collisions can ionise.
2. Temperature does not set the ionisation fraction; a *balance* of ionisation and recombination rates does (next slide).

Why gas at 8000 K stays neutral

Saha (LTE): predicts ionised

$$\frac{n_e n_p}{n_{\rm H}} = \left(\frac{2\pi m_{\rm e} k_{\rm B} T}{h^2}\right)^{3/2} {\rm e}^{-13.6\,{\rm eV}/k_{\rm B}T}$$

At 8000 K the right side is $\sim 5 \times 10^{12}\,{\rm cm}^{-3}$, so for $n_{\rm H} \sim 0.5\,{\rm cm}^{-3}$ Saha gives $x \approx 1$.

- Saha assumes matter *and radiation* in equilibrium at T. The ISM radiation field is far too dilute for that.

Rate balance: what actually happens

Collisional ionisation = radiative recombination:

$$C\, n_e n_{\rm H} = \alpha_{\rm B}\, n_e n_p \;\Rightarrow\; \frac{x}{1-x} \approx \frac{C}{\alpha_{\rm B}}$$

With $C \approx 10^{-17}$ and $\alpha_{\rm B} \approx 3 \times 10^{-13}$ cm^3 s^{-1} at 8000 K: $x \sim 3 \times 10^{-5}$, independent of density.

- Observed WNM is a few per cent ionised, supplied by cosmic rays, X-rays and EUV photons, not by its temperature.

Worked example: why absorption selects the cold gas

Same column $N_{\rm HI} = 10^{20}\,{\rm cm^{-2}}$, Gaussian line (illustrative values). Peak $\tau = \int \tau\,{\rm d}v / (1.064\,\Delta v_{\rm FWHM})$.

Phase	$T_{\rm s}$	$\Delta v_{\rm FWHM}$	$\int \tau\,{\rm d}v$	$\tau_{\rm peak}$
CNM	50 K	$2\,{\rm km\,s^{-1}}$	$1.1\,{\rm km\,s^{-1}}$	0.52
WNM	5000 K	$20\,{\rm km\,s^{-1}}$	$0.011\,{\rm km\,s^{-1}}$	5×10^{-4}

- Both contribute equally to emission ($\propto N_{\rm HI}$).
- Absorption is $\sim 10^3$ times stronger for the CNM: an absorption spectrum is a census of cold gas only.
- Detecting WNM absorption needs $\tau \sim 10^{-3}$ sensitivity against very bright continuum sources.

Measuring $T_{\rm s}$: the on–off method

Off source (beside a compact continuum source):

$$T_{\rm off}(v) = T_{\rm s}\left[1 - {\rm e}^{-\tau}\right]$$

On source (continuum $T_{\rm c}$ subtracted):

$$\Delta T_{\rm on}(v) = (T_{\rm s} - T_{\rm c})\left[1 - {\rm e}^{-\tau}\right]$$

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Subtract:

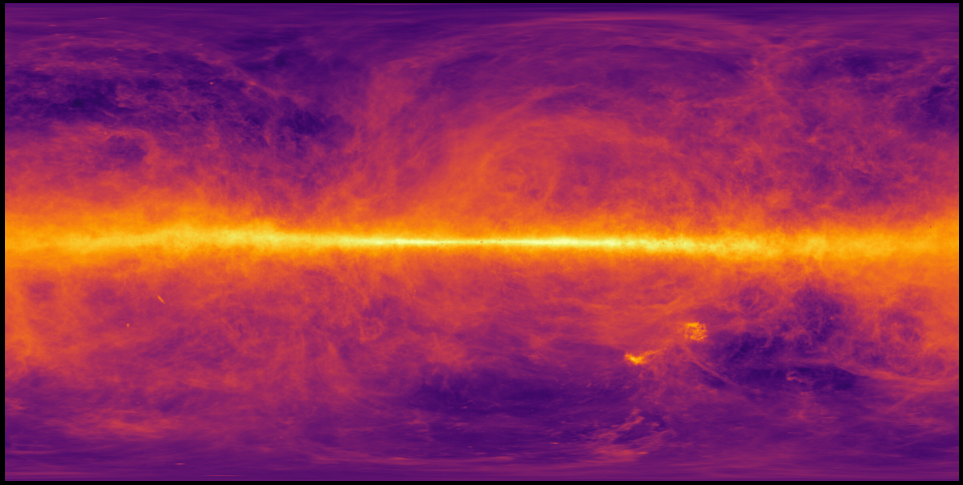
$$1 - {\rm e}^{-\tau} = \frac{T_{\rm off} - \Delta T_{\rm on}}{T_{\rm c}}, \quad T_{\rm s} = \frac{T_{\rm off}}{1 - {\rm e}^{-\tau}}$$

Assumptions to state

- On and off sightlines sample the same gas (beam mismatch, small-scale structure).
- One isothermal component per velocity channel.
- Real spectra: several CNM components and WNM overlap; fit Gaussians with ordering along the sightline.

Measured: $\tau(v)$. Inferred: $T_{\rm s}$, $N_{\rm HI}$.

The H I sky



Credit: HI4PI all-sky H I column density, Galactic coordinates, centred on $l = 0$. Parkes/Effelsberg/CSIRO/MPIfR, image by NOIRLab, CC BY 4.0, Wikimedia Commons. Survey: HI4PI Collaboration 2016, doi:10.1051/0004-6361/201629178

Reading the H I sky

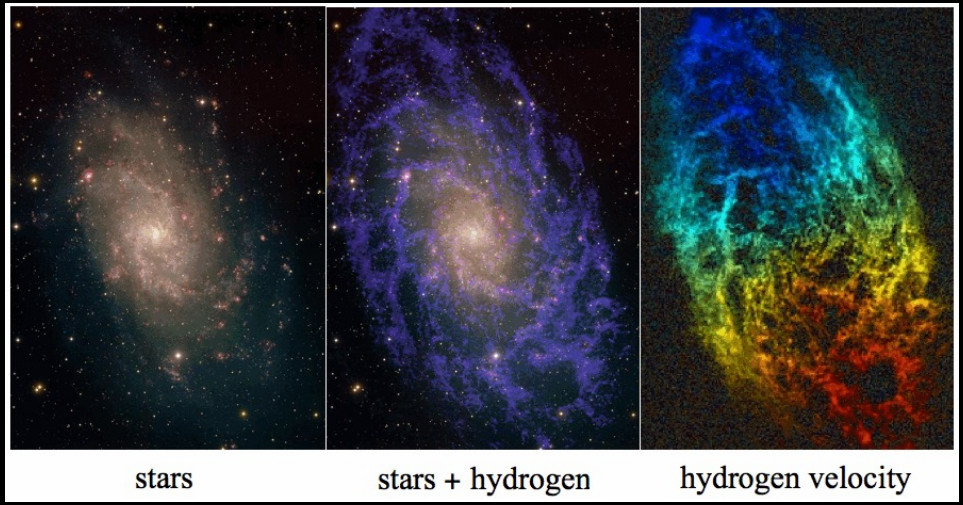
- The disc dominates: $N_{\rm HI} \gtrsim 10^{21}\,{\rm cm}^{-2}$ (velocity-integrated, thin-limit) in the plane.
- High latitudes: $N_{\rm HI} \sim 10^{20}\,{\rm cm}^{-2}$, with filaments, shells and chimneys.
- Lower right: the Magellanic Clouds, connected by H I bridges and stream.
- Each pixel is a full spectrum: the map is the zeroth moment of a cube $T_{\rm b}(l, b, v)$.

What the map does *not* give

- Distance to any feature.
- Correction for $\tau > 0$ in the plane, so plane columns are lower limits.
- Separation of CNM from WNM.

Review: Kalberla & Kerp 2009, doi:10.1146/annurev-astro-082708-101823.

H I beyond the Milky Way: M33



Credit: Spiral galaxy M33: stars; stars plus H I; H I velocity field. Image index source: "Spitzer | HST | SDSS" [attribution to be confirmed]

Derivation: H I mass from integrated flux

Assumptions: optically thin, $T_{\rm s} \gg T_{\rm bg}$, $z \ll 1$, distance D. $m_{\rm H} = 1.67 \times 10^{-24}$ g is the mass of one hydrogen atom.

$$
\begin{aligned}
M_{\rm HI} &= m_{\rm H} \int N_{\rm HI}\, {\rm d}A = m_{\rm H} D^2 \int N_{\rm HI}\, {\rm d}\Omega && (\text{area } {\rm d}A = D^2 {\rm d}\Omega) \\
&= m_{\rm H} D^2 \, (1.82 \times 10^{18}) \iint T_{\rm b}\, {\rm d}v\, {\rm d}\Omega && (\text{thin limit}) \\
&= m_{\rm H} D^2 \, (1.82 \times 10^{18}) \, \frac{c^2}{2 k_{\rm B} \nu^2} \int S_\nu \, {\rm d}v && \left(S_\nu = \tfrac{2 k_{\rm B} \nu^2}{c^2} \int T_{\rm b}\, {\rm d}\Omega\right)
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$$
\boxed{\frac{M_{\rm HI}}{M_\odot} = 2.36 \times 10^5 \left(\frac{D}{\rm Mpc}\right)^2 \left(\frac{\int S_\nu \, {\rm d}v}{\rm Jy\,km\,s^{-1}}\right)}
$$

$\int S_\nu \, {\rm d}v$ is measured; $M_{\rm HI}$ needs D. Excludes helium; multiply by ≈ 1.36 for total atomic gas mass.

Worked example: H I mass and its systematics

A spiral galaxy has $\int S_\nu \, \mathrm{d}v = 20\,\mathrm{Jy\,km\,s^{-1}}$ at a Hubble-flow distance $D = 10\,\mathrm{Mpc}$ (illustrative values).

$$M_{\mathrm{HI}} = 2.36 \times 10^5 \times 10^2 \times 20\, M_\odot = 4.7 \times 10^8\, M_\odot$$

Distance

- $M \propto D^2$: 20% in D gives 40% in M.
- Peculiar velocities of a few hundred $\mathrm{km\,s^{-1}}$ dominate for nearby galaxies.

Opacity and measurement

- Edge-on and dense discs: τ not $\ll 1$, mass is a lower limit.
- Interferometers can miss smooth extended flux; single dishes suffer confusion.

Concept test

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An H I source has the same integrated flux but is placed at twice the distance. What happens to its inferred H I mass?

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Answer: four times larger

1. $M_{\mathrm{HI}} \propto D^2 \int S_\nu \, \mathrm{d}v$, and $\int S_\nu \, \mathrm{d}v$ is unchanged.
2. Doubling D multiplies the inferred mass by $2^2 = 4$.

Summary

- **The line.** A magnetic-dipole hyperfine transition at 1420.4 MHz, $A_{10} = 2.88 \times 10^{-15}\,\mathrm{s}^{-1}$. Collisions and Ly$\alpha$ pumping, not spontaneous decay, set the populations.
- **Spin temperature.** $T_* = 0.068\,\mathrm{K} \ll T_\mathrm{s}$, so 3/4 of the atoms are in the upper level and emission counts atoms. T_s is set by Field's mix of T_R, T_k and the Lyα colour temperature.
- **Transfer.** $\Delta T_\mathrm{b} = (T_\mathrm{s} - T_\mathrm{bg})(1 - \mathrm{e}^{-\tau})$. Thin: $N_\mathrm{HI} = 1.82 \times 10^{18} \int T_\mathrm{b}\,\mathrm{d}v$. Saturated: a flat top at T_s and an underestimated N_HI. Absorption has $\tau \propto N/T_\mathrm{s}$ and selects cold gas; the on–off method gives T_s.
- **Phases.** CNM ($\sim$100 K) and WNM ($\sim$8000 K). The WNM stays neutral because ionisation and recombination balance, not because of its temperature; absorption surveys also find gas at unstable temperatures.
- **Galaxies.** $M_\mathrm{HI} = 2.36 \times 10^5 D^2 \int S_\nu\,\mathrm{d}v\,M_\odot$. At $z \approx 1$, stacking gives a mean M_HI for a sample, not for any one galaxy.

Further reading

- **The line:** Ewen & Purcell 1951 (doi:10.1038/168356a0); Field 1958 (doi:10.1109/JRPROC.1958.286741).
- **Phases:** Wolfire et al. 2003 (doi:10.1086/368016); Heiles & Troland 2003 (doi:10.1086/367828); Murray et al. 2018 (doi:10.3847/1538-4365/aad81a).
- **The H I sky:** HI4PI survey (doi:10.1051/0004-6361/201629178); Kalberla & Kerp 2009 (review); IRA and Kraus on atomic hydrogen.
- **Stacking at $z \approx 1$:** Kaur et al. 2026, arXiv:2609.13382.