The Radio Sky

Lecture 10

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Retrieval: a shift of 100 kHz

Calculation

The H I line has rest frequency $\nu_0 = 1420.405752\,\mathrm{MHz}$. A line is observed 100 kHz below ν_0. What radial velocity does this correspond to, and in which direction?

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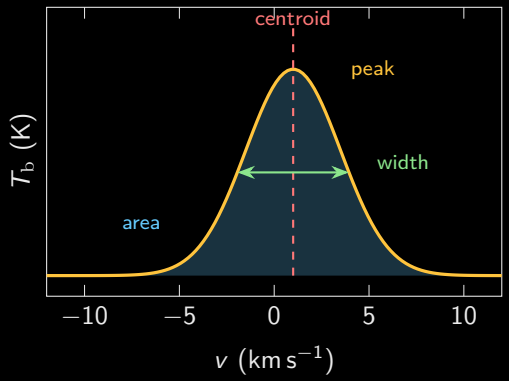
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$$v \approx c\,\frac{\nu_0 - \nu}{\nu_0} = 3.00 \times 10^5\,\mathrm{km\,s^{-1}} \times \frac{0.100}{1420.4} \approx 21.1\,\mathrm{km\,s^{-1}}$$

- Lower observed frequency: the source is receding.
- Rule of thumb: near 1.4 GHz, 1 kHz $\approx 0.21\,\mathrm{km\,s^{-1}}$.
- The same 100 kHz at 22.2 GHz (H_2O) is only $1.35\,\mathrm{km\,s^{-1}}$: velocity resolution depends on $\Delta\nu/\nu$, not on $\Delta\nu$.

Central question: from line profile to physics

How does a line profile become a measurement of gas physics and kinematics?



Schematic line profile.

- Area $\int T_b \, dv$: column density (if thin).
- Peak T_b: excitation temperature (if thick).
- Centroid: line-of-sight velocity.
- Width: temperature, turbulence, bulk motion.
- Shape and ratios: opacity, excitation, structure.

Every arrow from observable to physics passes through an assumption. Today: which ones, and when they fail.

Recap (Lectures 3–4) and Einstein coefficients

Levels u, l with statistical weights g_u, g_l, energy difference $h\nu_0$, column densities N_u, N_l (SI throughout; velocities in $\mathrm{km\,s^{-1}}$).

From Lectures 3–4

$$\frac{N_u}{N_l} = \frac{g_u}{g_l} \mathrm{e}^{-h\nu_0/k_\mathrm{B}T_\mathrm{ex}} \quad \text{(defines } T_\mathrm{ex}\text{; } T_\mathrm{ex} = T_\mathrm{k} \text{ only in LTE; masers have } T_\mathrm{ex} < 0\text{)}$$

$$\Delta T_\mathrm{b}(\nu) = \left[T_\mathrm{ex} - T_\mathrm{bg}\right]\left(1 - \mathrm{e}^{-\tau(\nu)}\right) \quad \text{(thick: peak} \to T_\mathrm{ex} - T_\mathrm{bg}\text{; thin: } \approx (T_\mathrm{ex} - T_\mathrm{bg})\tau\text{;}$$
$T \to \Theta_\nu(T)$ at mm wavelengths)

- A_{ul}, B_{ul}, B_{lu}: the Einstein coefficients (next two slides).
- In H I, T_ex is called the spin temperature T_s.

Observational caveat

A source smaller than the beam is diluted: measured T_b is reduced by the beam filling factor. A low peak does not prove low T_ex.

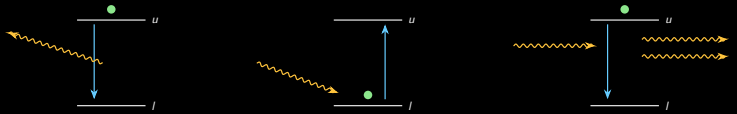
An aside: a warning from the statistical-mechanics literature

> *"Ludwig Boltzmann, who spent much of his life studying statistical mechanics, died in 1906, by his own hand. Paul Ehrenfest, carrying on the work, died similarly in 1933. Now it is our turn to study statistical mechanics. Perhaps it will be wise to approach the subject rather cautiously."*

David L. Goodstein, *States of Matter* (1975), opening paragraph. The Boltzmann factor $\mathrm{e}^{-h\nu/k_{\mathrm{B}}T}$ used on the previous slide is his.

Einstein coefficients: three radiative processes

Two levels, upper u and lower l, with $h\nu_0 = E_u - E_l$. A line profile $\phi(\nu)$, $\int \phi \, \mathrm{d}\nu = 1$, is centred on ν_0. The atoms see radiation from all directions: the mean intensity $J_\nu = \dfrac{1}{4\pi} \oint I_\nu \, \mathrm{d}\Omega$ [R&L eq. 1.8], averaged over the line profile, $\bar{J} = \int J_\nu \, \phi(\nu) \, \mathrm{d}\nu$.



- Spontaneous: independent of the radiation field, in a random direction. A_{ul} is a rate in s^{-1} [R&L eq. 1.64].
- Absorption $B_{lu}\bar{J}$ [R&L eq. 1.66] and stimulated emission $B_{ul}\bar{J}$ [R&L eq. 1.68] both scale with the intensity. The stimulated photon copies the frequency, direction and phase of the one that triggered it, so it counts as negative absorption.

R&L §1.6 label the levels 1 (lower), 2 (upper): $A_{21} = A_{ul}$, $B_{12} = B_{lu}$, $B_{21} = B_{ul}$.

Einstein coefficients: the relations and what they give

- In equilibrium at T, $\bar{J} = B_\nu(T)$ and upward and downward transitions balance [R&L eqs. 1.69–1.71]: $n_l B_{lu} \bar{J} = n_u A_{ul} + n_u B_{ul} \bar{J}$.
- Matching the Planck function at every T gives the Einstein relations [R&L eq. 1.72]. The coefficients are atomic properties, so they hold out of equilibrium too: one fixes the others.

$$g_l B_{lu} = g_u B_{ul}, \qquad A_{ul} = \frac{2h\nu^3}{c^2} B_{ul}$$

Emission and absorption coefficients [R&L eqs. 1.73, 1.75, 1.78; R&L write α_ν for κ_ν]:

$$j_\nu = \frac{h\nu}{4\pi} n_u A_{ul} \phi(\nu), \qquad \kappa_\nu = \frac{h\nu}{4\pi} \phi(\nu) \left(n_l B_{lu} - n_u B_{ul}\right) = \frac{h\nu}{4\pi} n_l B_{lu} \left(1 - \frac{g_l n_u}{g_u n_l}\right) \phi(\nu)$$

- $j_\nu / \kappa_\nu = B_\nu(T_{\rm ex})$ [R&L eq. 1.79]; $\kappa_\nu < 0$ if $n_u/g_u > n_l/g_l$ (a maser). Eliminating B_{lu} with the relations gives the line opacity on the next slide.

Line opacity: why radio lines are special

Normalized line profile $\phi(\nu)$, $\int \phi \, \mathrm{d}\nu = 1$. Absorption corrected for stimulated emission:

$$\tau_\nu = \frac{c^2 A_{ul}}{8\pi\nu^2} N_u \left[\exp\left(\frac{h\nu}{k_\mathrm{B} T_\mathrm{ex}} \right) - 1 \right] \phi(\nu)$$

$$h\nu \ll k_\mathrm{B} T_\mathrm{ex} : \qquad \tau_\nu \approx \frac{c^2 A_{ul}}{8\pi\nu^2} \frac{h\nu}{k_\mathrm{B} T_\mathrm{ex}} N_u \, \phi(\nu)$$

- $h\nu/k_\mathrm{B} = 0.068\,\mathrm{K}$ (H I), $1.1\,\mathrm{K}$ (NH_3, $23.7\,\mathrm{GHz}$), $5.5\,\mathrm{K}$ (CO 1–0, $115\,\mathrm{GHz}$).
- Stimulated emission cancels all but a fraction $h\nu/k_\mathrm{B} T_\mathrm{ex} \sim 10^{-3}$ of the absorption, so $\tau_\nu \propto N/T_\mathrm{ex}$ (why, on the next slide).
- If $n_u/g_u > n_l/g_l$, then $T_\mathrm{ex} < 0$ and $\tau_\nu < 0$: the medium amplifies, a maser (Lecture 12).

Why stimulated emission nearly cancels the absorption

- Absorption removes a photon from the beam. Stimulated emission adds a *clone*: same direction and frequency.
- Net absorption is the *difference*:

$$1 - \mathrm{e}^{-h\nu/k_\mathrm{B}T_\mathrm{ex}} \approx \frac{h\nu}{k_\mathrm{B}T_\mathrm{ex}}$$

- H I at $T_\mathrm{s} = 100\,\mathrm{K}$: 7×10^{-4}, so of 1500 photons absorbed, 1499 are returned.
- Cause: $h\nu \ll k_\mathrm{B}T_\mathrm{ex}$ makes $n_u/g_u \approx n_l/g_l$.

What this changes at radio wavelengths

- Optical, $h\nu \gg k_\mathrm{B}T$: the upper level is nearly empty, the bracket is ≈ 1, and stimulated emission can be dropped. Radio: the bracket is $\sim 10^{-3}$ and *must* be kept.
- Thin emission has no cancellation ($T_\mathrm{b} = \tau T_\mathrm{s} \propto N$): H I *emission* counts all gas, *absorption* favours cold gas.
- $A_{ul} \propto \nu^3$ is tiny, so collisions set T_ex.

Column density from an optically thin line

For $\tau \ll 1$ and $T_{\rm ex} \gg T_{\rm bg}$, every spontaneous photon escapes:

$$N_u = \frac{8\pi k_{\rm B} \nu^2}{h c^3 A_{ul}} \int T_{\rm b}\, {\rm d}v$$

- To get the total column, one needs the fraction in level u: $N_{\rm tot} = N_u\, Q(T_{\rm ex})\, {\rm e}^{E_u/k_{\rm B}T_{\rm ex}}/g_u$, with partition function Q. This step assumes a single $T_{\rm ex}$ for all levels.
- H I (exact for thin emission, $T_{\rm s} \gg T_{\rm bg}$):

$$N_{\rm HI} = 1.82 \times 10^{18}\,{\rm cm^{-2}} \int \frac{T_{\rm b}}{\rm K}\, \frac{{\rm d}v}{\rm km\,s^{-1}}$$

- Molecules: Mangum & Shirley 2015 (doi:10.1086/680323); multi-line population diagrams: Goldsmith & Langer 1999 (doi:10.1086/307195).

When does $T_{\rm ex} = T_{\rm k}$? Critical density

Collisional de-excitation rate $n\,\gamma_{ul}$ (γ_{ul}: rate coefficient, $\mathrm{m^3\,s^{-1}}$) competes with A_{ul}:

$$n_{\rm crit} = \frac{A_{ul}}{\gamma_{ul}}$$

- $n \gg n_{\rm crit}$: collisions set the populations, $T_{\rm ex} \to T_{\rm k}$ (LTE).
- $n \lesssim n_{\rm crit}$: subthermal, $T_{\rm bg} < T_{\rm ex} < T_{\rm k}$.
- $A_{ul} \propto \nu^3 \mu^2$ (dipole moment μ): high-frequency lines and polar molecules need dense gas.
- Optically thick lines trap photons; the effective excitation density is lower, roughly $n_{\rm crit}/\tau$ (Shirley 2015, doi:10.1086/680342).

Pitfall

Detecting a line does not mean $n \geq n_{\rm crit}$. Subthermal excitation and trapping both allow emission from lower-density gas.

What sets the line width

- Natural width $\sim A_{ul}/2\pi$: negligible in the radio ($A \sim 10^{-15}$–$10^{-4}\,\mathrm{s}^{-1}$).
- Thermal: Maxwellian velocities of particles of mass m:

$$\Delta v_{\mathrm{th}} = \sqrt{\frac{8 \ln 2\, k_{\mathrm{B}}\, T_{\mathrm{k}}}{m}} = 0.214\,\mathrm{km\,s^{-1}} \left(\frac{T_{\mathrm{k}}/\mathrm{K}}{m/\mathrm{u}}\right)^{1/2} \quad \text{(FWHM)}$$

- Non-thermal: turbulence and unresolved bulk motions. If Gaussian and independent, they add in quadrature: $\Delta v^2 = \Delta v_{\mathrm{th}}^2 + \Delta v_{\mathrm{nt}}^2$.
- Opacity broadening: for $\tau \gg 1$ the profile saturates and flattens at the top.

	H I at 100 K	NH_3 at 10 K	CH_3OH at 10 K
Δv_{th} ($\mathrm{km\,s^{-1}}$)	2.1	0.16	0.12

Doppler conventions: three definitions

$$v_{\rm radio} = c\,\frac{\nu_0 - \nu}{\nu_0}$$

$$v_{\rm opt} = c\,\frac{\nu_0 - \nu}{\nu} = cz$$

$$\frac{\nu}{\nu_0} = \sqrt{\frac{1-\beta}{1+\beta}} \quad \text{(relativistic)}$$

v (relativistic)	$v_{\rm radio}$	$v_{\rm opt}$
100	100.0	100.0
1000	998.3	1001.7
10000	9839	10173
30000	28639	31664

All in $\mathrm{km\,s^{-1}}$ (computed).

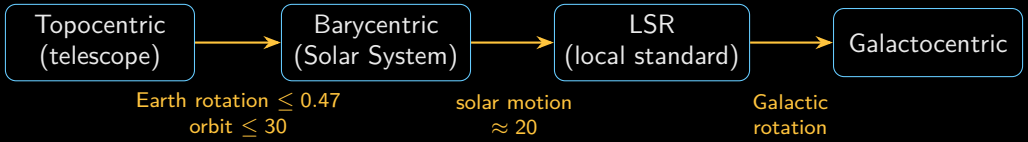
- All agree to first order in v/c.
- $v_{\rm radio}$ is linear in ν: equal frequency channels give equal velocity channels.

Pitfall

At $1000\,\mathrm{km\,s^{-1}}$ the conventions differ by $3\,\mathrm{km\,s^{-1}}$: more than a channel width. Always state the convention.

Galaxies at cosmological distances: quote z, not a velocity.

Velocity reference frames



Corrections in $\mathrm{km\,s^{-1}}$.

- LSRK (kinematic, the radio convention): Sun moves at $20\,\mathrm{km\,s^{-1}}$ towards RA 18^{h}, Dec $+30^{\circ}$ (B1900). This is a convention, not a measurement (Kerr & Lynden-Bell 1986, doi:10.1093/mnras/221.4.1023).
- Modern measured solar motion $(U, V, W) = (11.1, 12.2, 7.3)\,\mathrm{km\,s^{-1}}$ (Schönrich, Binney & Dehnen 2010, doi:10.1111/j.1365-2966.2010.16253.x).
- Uncorrected Earth motion changes by up to $\pm 30\,\mathrm{km\,s^{-1}}$ ($\sim 60\,\mathrm{km\,s^{-1}}$ between opposite sides of the orbit): spectra taken months apart will not align.
- Extragalactic work uses barycentric (heliocentric) velocities.

Channel width and spectral resolution

$$\Delta v_{\mathrm{chan}} = c\,\frac{\Delta\nu_{\mathrm{chan}}}{\nu}$$

Line	ν	$\Delta\nu_{\mathrm{chan}}$	Δv_{chan}
H I	1.420 GHz	10 kHz	$2.1\,\mathrm{km\,s^{-1}}$
H I	1.420 GHz	1 MHz	$211\,\mathrm{km\,s^{-1}}$
H_2O maser	22.235 GHz	10 kHz	$0.13\,\mathrm{km\,s^{-1}}$
NH_3 (1,1)	23.694 GHz	100 kHz	$1.27\,\mathrm{km\,s^{-1}}$
NH_3 (1,1)	23.694 GHz	1 MHz	$12.7\,\mathrm{km\,s^{-1}}$

- Resolution is not channel spacing: the correlator window sets it, typically 1–2 channels (Hanning smoothing gives 2).
- Measured profile = true profile convolved with the spectral response. A width needs $\gtrsim 3$ resolution elements across the FWHM.
- A $0.2\,\mathrm{km\,s^{-1}}$ NH_3 line is unresolved in $1.27\,\mathrm{km\,s^{-1}}$ channels.

Concept test

Question

Two H I spectra of the same cloud, taken six months apart, are plotted in *topocentric* velocity. The peak has shifted by about $20\,\mathrm{km\,s^{-1}}$. Has the cloud changed?

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Answer: not necessarily: the telescope's velocity has changed

1. Topocentric velocities include Earth's orbital motion, whose projection on the line of sight changes by up to $\sim 60\,\mathrm{km\,s^{-1}}$ over a year ($\pm 30\,\mathrm{km\,s^{-1}}$).
2. Convert both spectra to barycentric or LSRK velocities before comparing; only a shift that survives this correction belongs to the cloud.

Radio spectral lines: H I is our focus

Several lines matter at centimetre wavelengths; in this course we work mainly with H I. The others are listed for reference.

Line	Rest frequency	Origin	Traces
H I 21 cm	1420.405752 MHz	hyperfine	atomic gas (WNM, CNM)
OH 18 cm main lines	1665.402, 1667.359 MHz	Λ-doubling	molecular gas, masers
CH_3OH masers	6.6685, 44.0694 GHz	torsion–rot.	high-mass star formation
H_2O 6_{16}–5_{23}	22.23508 GHz	rotational	masers in shocks and discs
NH_3 (1,1), (2,2)	23.6945, 23.7226 GHz	inversion	dense-gas temperature
RRL, e.g. H109α	5008.923 MHz	$n \to n-1$	ionized gas, T_e

- Millimetre and sub-millimetre lines are quite important: CO (115 GHz and up) and [C II] 158 μm are the standard tracers of molecular gas and photodissociation regions. They belong to mm/sub-mm/far-IR astronomy and will *not* be covered in this course.

When the standard assumptions fail

Assumption	Fails when	Symptom
LTE ($T_{\rm ex} = T_{\rm k}$)	$n < n_{\rm crit}$; strong radiation field	ladder ratios inconsistent with one T
Optically thin	bright lines, high column	flat-topped profile; NH_3 main/satellite ratio $\to 1$
Single $T_{\rm ex}$	temperature gradient on the sightline	self-absorbed dip; blue/red asymmetry
Full beam filling	clumpy or compact source	low peak $T_{\rm b}$, high column from ratios
$T_{\rm ex} > 0$	population inversion	$T_{\rm b} \gg T_{\rm k}$, narrow spikes: a maser

- Infall in a centrally heated core gives a blue-peaked self-absorbed profile: a single-$T_{\rm ex}$ fit would be wrong *and* miss the dynamics.
- Masers (Lecture 12): line analogue of the coherent continuum emission of Lecture 9.

Summary

- Excitation. $T_{\rm ex}$ describes level populations and equals $T_{\rm k}$ only for $n \gg n_{\rm crit} = A_{ul}/\gamma_{ul}$; below it the line is subthermal but still emitted.
- Einstein coefficients. $g_l B_{lu} = g_u B_{ul}$ and $A_{ul} = (2h\nu^3/c^2)B_{ul}$ tie emission to absorption, so $j_\nu/\kappa_\nu = B_\nu(T_{\rm ex})$; $\kappa_\nu < 0$ means a maser.
- Radio lines. Because $h\nu \ll k_{\rm B}T_{\rm ex}$, stimulated emission cancels all but a fraction $h\nu/k_{\rm B}T_{\rm ex}$ of the absorption: $\tau_\nu \propto N/T_{\rm ex}$.
- Transfer and column. $\Delta T_{\rm b} = (T_{\rm ex} - T_{\rm bg})(1 - {\rm e}^{-\tau})$; for thin gas $\int T_{\rm b}\,{\rm d}v \propto N_u$, and a partition function converts N_u to $N_{\rm tot}$.
- Widths and velocities. Line width is thermal $\oplus$ non-thermal; state the Doppler convention, the velocity frame and the channel width.
- Caution. Check τ, a single $T_{\rm ex}$ and the background before reading a line.

Further reading

- Rybicki & Lightman §1.6: Einstein coefficients.
- ERA Ch. 7, *Spectral Lines* ("Line Radiative Transfer", "Excitation Temperature"); TRA Ch. 12, 13, 15 and 16.
- Mangum & Shirley 2015, how to calculate molecular column density (doi:10.1086/680323).