

The Radio Sky

Lecture 8

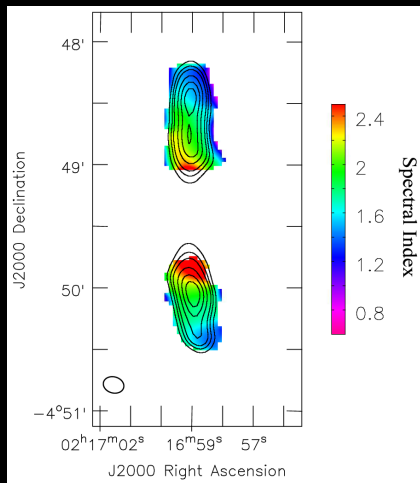
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PHY-236.3



Spectral index map: ageing across the lobes



- Index between 0.325 and 1.4 GHz, pixels above 3σ at both; contours: 325 MHz.
- Colour bar uses $S_\nu \propto \nu^{-\alpha}$: plotted values are $-\alpha$.
- α runs from -1.4 at the outer ends of the lobes to -2.5 at the inner ends, nearest the host.
- The 325 MHz lobes extend beyond the 1.4 GHz emission: the high-frequency electrons have already faded there.

Credit: Tamhane et al. 2015, MNRAS 453, 2438 ([arXiv:1508.00030](https://arxiv.org/abs/1508.00030); [doi:10.1093/mnras/stv1768](https://doi.org/10.1093/mnras/stv1768)), Fig. 3 (right).

Reading the map

- Electrons were last accelerated at the outer ends, where the hot spots were while the jet was on.
- **Backflow**: plasma flows back from there towards the host, so the inner lobe holds the oldest electrons: steepest spectra.
- In an active source the gradient, with an assumed B , gives a spectral age gradient, hence the hot-spot advance speed.

Why spectral ages are model-dependent

- B is rarely measured; usually it is the equipartition value (this lecture), and $t \propto B^{-3/2}$ unless IC losses dominate.
- B may vary in space and time; electrons diffuse and mix along the line of sight.
- Re-acceleration resets the clock; adiabatic expansion shifts the spectrum.
- Spectral and dynamical ages often disagree; broad-band fitting helps (Harwood et al. 2013, [doi:10.1093/mnras/stt1526](https://doi.org/10.1093/mnras/stt1526)).

The age of the relic

Inputs: $\nu_{\text{b,rest}} = 1.32 \text{ GHz}$ (fit), $B \approx 3.3 \mu\text{G}$ (from inverse-Compton X-rays, Lecture 8), $B_{\text{IC}} = 17.5 \mu\text{G}$.

$$t \approx 1.3 \times 10^3 \text{ Myr} \frac{(3.3)^{1/2}}{3.3^2 + 17.5^2} (1.32)^{-1/2} \approx 6.5 \text{ Myr}$$

- Lobes stay visible for only $\sim 10^6\text{--}10^7$ yr after the jet stops: relics are rare, and best found at low frequencies.

Concept test

Question

In a source where radiative ageing dominates and no fresh particles are injected, which part of the synchrotron spectrum steepens first?

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Answer: the high-frequency part

1. High-energy electrons radiate fastest: $t_{\text{cool}} = 1/(b\gamma)$.
2. They produce the high-frequency emission, since $\nu \approx \gamma^2 \nu_G$.

Synchrotron self-absorption: the source function

Deep inside an optically thick source $I_\nu = S_\nu \equiv j_\nu / \kappa_\nu$. For a thermal gas this is the Planck function (Kirchhoff, Lecture 3).

- The electrons that emit also absorb (absorption is the reverse of emission, including stimulated emission). For $N(E) = KE^{-p}$, with $P(\nu, E)$ the power one electron radiates per unit frequency [R&L Ch. 6, eqs. 6.50]:

$$j_\nu = \frac{1}{4\pi} \int P(\nu, E) KE^{-p} dE, \quad \kappa_\nu = \frac{c^2}{8\pi\nu^2} (p+2) \int P(\nu, E) KE^{-p-1} dE$$

- $P(\nu, E)$ is sharply peaked at the energy $E_* = \gamma m_e c^2$ with $\nu \approx \gamma^2 \nu_G$, so $\int P E^{-p} dE \approx E_* \int P E^{-p-1} dE$ and

$$S_\nu = \frac{j_\nu}{\kappa_\nu} = \frac{2\nu^2}{c^2} \frac{E_*}{p+2} \equiv \frac{2k_B T_{\text{eff}} \nu^2}{c^2}, \quad k_B T_{\text{eff}} = \frac{\gamma m_e c^2}{p+2}$$

T_{eff} is the energy of the electrons radiating at ν , not a thermal temperature: it *rises* with ν .

The optically thick spectrum

With $\gamma \approx (\nu/\nu_G)^{1/2}$ and $\nu_G \propto B$:

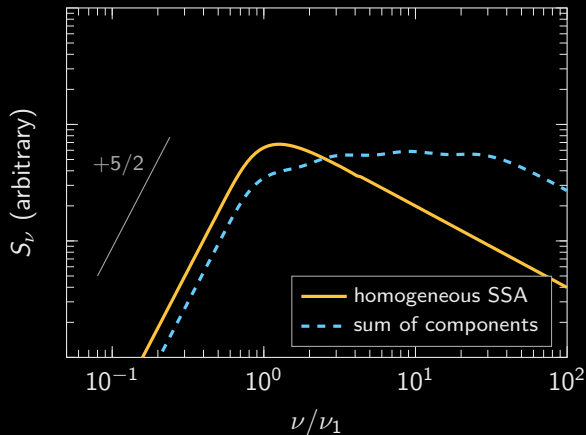
$$I_\nu^{\text{thick}} = \frac{2k_B T_{\text{eff}} \nu^2}{c^2} \propto \nu^2 \left(\frac{\nu}{\nu_G} \right)^{1/2} \propto B^{-1/2} \nu^{5/2}$$

A source of solid angle $\Omega \propto \theta^2$ has $S_\nu = I_\nu \Omega$:

$$S_\nu \propto \theta^2 B^{-1/2} \nu^{5/2} \quad (\text{thick}), \quad S_\nu \propto \nu^\alpha \quad (\text{thin})$$

- The thick slope $+5/2$ is independent of p . It is steeper than the Rayleigh–Jeans $+2$ because $T_{\text{eff}} \propto \nu^{1/2}$ rises with frequency.
- Absorption coefficient (scaling from κ_ν above): $\kappa_\nu \propto KB^{(p+2)/2} \nu^{-(p+4)/2}$, so τ_ν falls steeply with ν : thick below some frequency, thin above.

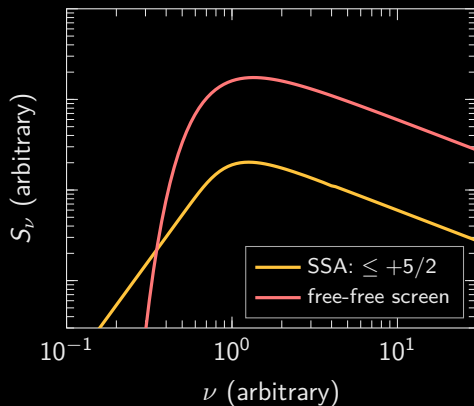
Self-absorbed spectra: homogeneous and inhomogeneous



Schematic: $\alpha = -0.7$, $\tau = (\nu/\nu_1)^{\alpha-5/2}$.

- Homogeneous source:
 $S_\nu \propto \nu^{5/2} [1 - e^{-\tau_\nu}]$, peak near $\tau_\nu \sim 1$.
- A jet with components of decreasing size along it: each peaks at a different ν ; the sum is **flat**, $\alpha \approx 0$.
- Flat-spectrum compact cores of AGN are interpreted this way (Lecture 18).
- A slope steeper than $+5/2$ cannot come from SSA in a single homogeneous source.

Slopes steeper than $+5/2$: not self-absorption



Schematic, arbitrary normalization.

- **Free-free screen:** $S_\nu \propto \nu^\alpha e^{-\tau}$, $\tau \propto \nu^{-2.1}$. Local slope $\alpha + 2.1\tau$: no upper limit as τ grows at low ν . It exceeds $+5/2$ once $\tau \gtrsim 1.5$ ($\alpha = -0.7$).
- **Razin–Tsytovich:** in a dense plasma the emission is suppressed below $\nu_R \approx 20 n_e / B$ Hz (n_e in cm^{-3} , B in G).
- Slope $> +5/2$: an ionized absorber or dense plasma, not SSA alone.

The turnover measures compactness and field

Textbook estimate (ERA eq. 5.91; Longair, *HEA*, eq. 9.118): set $T_b(\nu_p) \approx \gamma m_e c^2 / (3k_B)$ and $\nu_p \approx \gamma^2 \nu_G$, where ν_p is the **synchrotron turnover frequency** (the spectral peak):

$$B \approx \frac{\nu_p}{2.8 \text{ MHz}} \left(\frac{m_e c^2}{3k_B T_b} \right)^2 \text{ G} \approx 1.4 \times 10^{12} \frac{\nu_p}{\text{Hz}} \left(\frac{T_b}{\text{K}} \right)^{-2} \text{ G}.$$

Exact homogeneous source ($p = 2.4$; R&L eqs. 6.36, 6.53): the peak is at $\tau_\nu \approx 0.47$, so T_b there is only ≈ 0.24 of the estimate above and B is ≈ 18 times smaller. With T_b from a Gaussian FWHM θ :

$$B \approx 5 \times 10^{-5} \left(\frac{\theta}{\text{mas}} \right)^4 \left(\frac{\nu_p}{\text{GHz}} \right)^5 \left(\frac{S_p}{\text{Jy}} \right)^{-2} \frac{\delta}{1+z} \text{ G}$$

- Same form as Kellermann & Pauliny-Toth 1969 ([doi:10.1086/180305](https://doi.org/10.1086/180305)); the coefficient depends on geometry and p . δ : Doppler factor (Lecture 18). Radio supernovae: Chevalier 1998 ([doi:10.1086/305676](https://doi.org/10.1086/305676)).
- θ^4 : a factor 2 error in size is a factor 16 in B . Size must be **measured** (VLBI) or bounded by variability.

Worked example: a compact peaked source

Peak $S_p = 1$ Jy at $\nu_p = 5$ GHz; Gaussian FWHM θ (illustrative values, $z \approx 0$, no beaming).

θ (mas)	T_b (K)	B (G), exact	B (G), textbook	$\gamma = (\nu_p/\nu_G)^{1/2}$
0.5	2.0×10^{11}	0.010	0.18	420
1.0	4.9×10^{10}	0.16	2.9	105
2.0	1.2×10^{10}	2.6	47	26

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- T_b from Lecture 2: $1.22 \times 10^3 \text{ K } (S/\text{mJy})(\nu/\text{GHz})^{-2}(\theta^2/\text{arcsec}^2)^{-1}$.
- The textbook estimate is an **upper estimate**: good for scalings ($B \propto \theta^4 \nu_p^5 S_p^{-2}$), not for the coefficient.
- Radiative lifetime at 5 GHz: ≈ 560 yr for $B = 0.010$ G, ≈ 9 yr for $B = 0.16$ G. Compact sources are young or continuously re-accelerated.

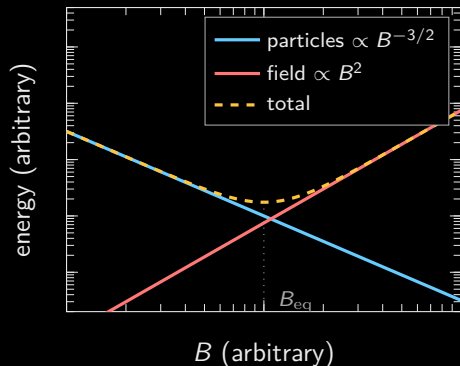
Brightness-temperature ceilings for incoherent synchrotron

- The same electrons inverse-Compton scatter their own synchrotron photons:
 $P_{\text{IC}}/P_{\text{syn}} = U_{\text{rad}}/U_B$.
- U_{rad} rises steeply with T_b . Above $T_b \sim 10^{12}$ K second-order scattering dominates and the electrons cool catastrophically: the **inverse-Compton limit** (Kellermann & Pauliny-Toth 1969).
- If the source is near equipartition (next section), $T_b \sim 10^{11}$ K: the **equipartition brightness temperature** (Readhead 1994, [doi:10.1086/174038](https://doi.org/10.1086/174038)).

Interpretation of an apparent $T_b > 10^{12}$ K

1. Relativistic beaming: intrinsic T_b lower by a power of δ .
2. Short-lived non-equilibrium state (the catastrophe takes time).
3. Coherent emission (Lecture 6), if polarization and time structure agree.
4. Size underestimated, e.g. variability caused by scintillation.

Equipartition: a way to estimate B



Schematic, at fixed observed luminosity.

Motivation. The radio flux fixes only $KB^{(p+1)/2}$: a weak field with many electrons fits as well as a strong field with few. We need a second equation.

Idea. Pick the field that needs the least total energy.

1. At fixed luminosity, a stronger B needs fewer electrons: particle energy $\propto B^{-3/2}$.
2. Field energy $\propto B^2$.
3. The total is smallest when $U_B = \frac{3}{4} U_{\text{part}}$: field and particles hold about equal energy, hence “equipartition”.

Solve for B_{eq} from the measured luminosity and the source volume. It is an *assumption*, not a measurement.

The equipartition field from luminosity and volume

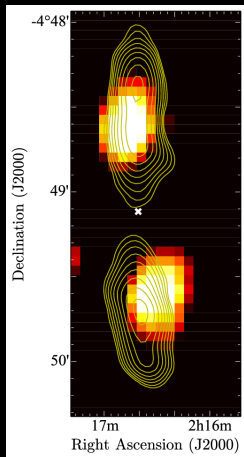
Minimizing $U_{\text{part}} + U_B$ with $U_B = \frac{3}{4} U_{\text{part}}$ gives (cgs units, Pacholczyk 1970):

$$B_{\text{eq}} = \left[\frac{6\pi (1+k) c_{12} L}{\phi V} \right]^{2/7} \text{ G}, \quad U_{\text{min}} = \frac{7}{24\pi} B_{\text{eq}}^2 \phi V$$

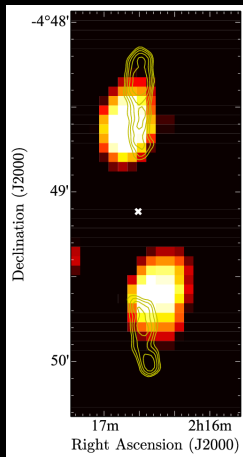
- L : radio luminosity integrated from ν_1 to ν_2 (usually 10 MHz–10 GHz), in erg s^{-1} .
- V : source volume in cm^3 ; ϕ : filling factor of the emitting plasma.
- k : energy in heavy particles per unit electron energy.
- $c_{12}(\alpha, \nu_1, \nu_2)$: a tabulated constant that depends on the spectral index and the frequency range.

Pacholczyk, *Radio Astrophysics*, Freeman 1970 (no DOI). Note $B_{\text{eq}} \propto (L/V)^{2/7}$: a weak dependence.

The relic in X-rays: inverse-Compton scattering of the CMB



325 MHz contours



1.4 GHz contours

- Colour: XMM-Newton 0.3–2 keV image of J021659–044920. The X-ray lobes are nearly co-spatial with the 325 MHz lobes.
- X-ray spectrum: a power law, photon index $\Gamma = 1.86^{+0.49}_{-0.41}$. A thermal-gas fit gives unphysical or unconstrained temperatures.
- Synchrotron X-rays would need $\gamma \sim 3 \times 10^8$, with lifetimes of a few hundred years: excluded in a dead source.
- Inverse-Compton scattering of CMB photons by the lobe electrons.

Credit: Tamhane et al. 2015, MNRAS 453, 2438 ([arXiv:1508.00030](https://arxiv.org/abs/1508.00030); [doi:10.1093/mnras/stv1768](https://doi.org/10.1093/mnras/stv1768)), Fig. 1 (middle and right).

Which electrons make the X-rays?

A photon of energy ϵ scattered by an electron of Lorentz factor γ (Thomson regime) emerges with mean energy $\langle E \rangle \approx \frac{4}{3}\gamma^2\epsilon$.

- CMB at $z = 1.33$: $T = 2.725(1 + z) = 6.3$ K, mean photon energy $2.7k_B T \approx 1.5$ meV.
- $\gamma = 10^3$: $\langle E \rangle \approx 2$ keV (rest frame), 0.8 keV observed. X-ray band electrons: $\gamma \sim 10^3$.
- Electrons radiating at rest-frame 0.76–3.25 GHz in $3.3 \mu\text{G}$:
 $\gamma = (\nu/\nu_G)^{1/2} \approx (0.9\text{--}1.9) \times 10^4$.
- Cooling time $\propto 1/\gamma$: ≈ 8 Myr at $\gamma = 10^4$ but ≈ 80 Myr at $\gamma = 10^3$. The X-rays trace older, lower-energy electrons that outlive the GHz emitters.

Consequences seen in the data

X-ray index $\alpha_X = 1 - \Gamma \approx -0.86$ is much flatter than the radio -1.83 : different parts of the electron spectrum. The 1.4 GHz lobes are smaller than the X-ray lobes.

The second equation: B from X-rays and radio

The same electrons radiate in two fields, the magnetic field and the CMB:

$$\frac{L_{\text{IC}}}{L_{\text{syn}}} = \frac{U_{\text{CMB}}}{U_B} = \left(\frac{B_{\text{IC}}}{B} \right)^2$$

- U_{CMB} is known exactly at a given z , so the flux ratio gives B **without assuming equipartition**. In practice the X-ray and radio electrons differ in γ , so the electron spectrum is extrapolated between them (Tucker 1977 formula).
- J021659–044920: $B \approx 3.3 \mu\text{G}$ from IC/synchrotron; $B_{\text{eq}} \approx 3.5 \mu\text{G}$ from equipartition.
- Energies: electrons $\gtrsim 4.2 \times 10^{59}$ erg (from the X-ray luminosity, $\gamma \sim 10^3$); field $\approx 6.4 \times 10^{59}$ erg (cylinders of 180 kpc diameter, 1 Mpc in total). Within $\approx 30\%$: **equipartition holds here**.
- Assumptions remain: the lobe volume and geometry, the electron spectrum between $\gamma \sim 10^3$ and 10^4 , and co-spatial electrons and field.

Two ways to make a low-frequency turnover

Synchrotron self-absorption (internal)

$$S_\nu \propto \nu^{5/2} [1 - e^{-\tau_\nu}], \quad \tau_\nu \propto \nu^{\alpha-5/2}$$

- Thick slope $\leq +5/2$.
- Needs a compact, high- T_b source.

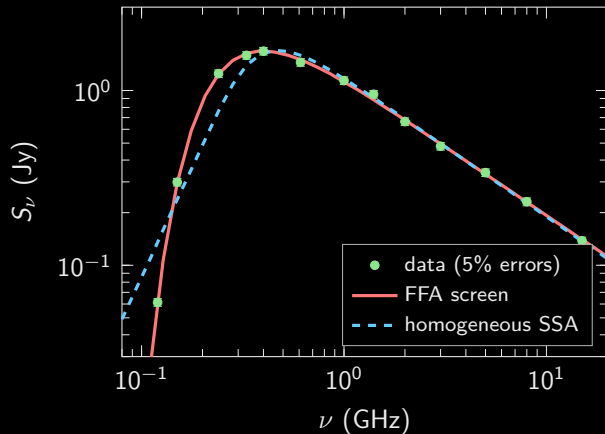
Free-free absorption (external screen)

$$S_\nu \propto \nu^\alpha e^{-\tau_\nu^{\text{ff}}}$$

- Exponential cutoff: slope can exceed $+5/2$.
- Needs ionized gas, not compactness.
- Internal mixing: $(1 - e^{-\tau})/\tau$, thick slope $\alpha + 2.1$.

- Lecture 4: $\tau^{\text{ff}} \approx 3.28 \times 10^{-7} (T_e/10^4 \text{ K})^{-1.35} (\nu/\text{GHz})^{-2.1} (\text{EM}/\text{pc cm}^{-6})$. For $\tau = 1$ at 0.25 GHz ($T_e = 10^4 \text{ K}$): $\text{EM} \approx 1.7 \times 10^5 \text{ pc cm}^{-6}$, dense gas near an AGN, not the diffuse ISM.

Worked example: fitting a curved spectrum

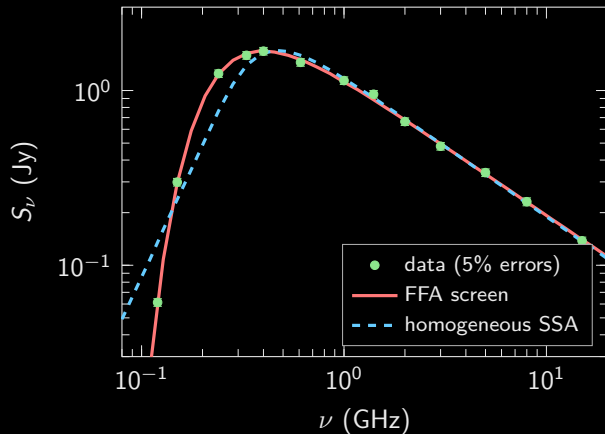


Three-parameter fits to 13 points
(10 degrees of freedom):

- FFA: $\alpha = -0.79$,
 $\nu_{\text{ff}} = 0.25$ GHz, $\chi^2 = 4$.
- SSA: $\alpha = -0.80$, $\chi^2 \approx 380$.

Synthetic data (illustrative), generated from an FFA model.

Worked example: fitting a curved spectrum



Synthetic data (illustrative), generated from an FFA model.

Three-parameter fits to 13 points (10 degrees of freedom):

- FFA: $\alpha = -0.79$,
 $\nu_{\text{ff}} = 0.25$ GHz, $\chi^2 = 4$.
- SSA: $\alpha = -0.80$, $\chi^2 \approx 380$.
- Above 0.3 GHz the two models are indistinguishable.
- The decision rests on the two lowest points: slopes +7 and +3, both steeper than +5/2.
- Next measurements: VLBI size (does T_b reach the value SSA needs?) and evidence for ionized gas.

Discriminating SSA from FFA

Test	Favours SSA	Favours FFA
Optically thick slope T_b at peak (from VLBI size)	$\leq +5/2$ $\sim 10^{10}\text{--}10^{11}$ K	$> +5/2$ (exponential) well below this
B implied by peak Time evolution	plausible (mG–G) peak drifts down as source expands	absurdly low or high tracks screen, not source
Ionized-gas evidence	none needed	HI/RRL, X-ray absorption, depolarization

- Peaked-spectrum (GPS/CSS) radio sources: both models have advocates (O'Dea 1998, [doi:10.1086/316162](https://doi.org/10.1086/316162); Bicknell, Dopita & O'Dea 1997, [doi:10.1086/304400](https://doi.org/10.1086/304400); Tingay & de Kool 2003, [doi:10.1086/376600](https://doi.org/10.1086/376600)).
- Physical question behind them: **young** (small because recently born) or **frustrated** (confined by dense gas)?

Concept test

Question

A compact synchrotron source has a low-frequency turnover and a very high brightness temperature. Which explanation is favoured first: synchrotron self-absorption or foreground free-free absorption?

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Answer: synchrotron self-absorption

1. SSA requires a compact, bright synchrotron source with T_b near the electron effective temperature.
2. Free-free absorption does not itself imply a high intrinsic T_b ; the high T_b is explained without extra gas.

Summary

- Radiative losses $d\gamma/dt = -b\gamma^2$ remove the highest-energy electrons first: a break $\nu_b \propto B^{-3}t^{-2}$ (synchrotron-dominated) appears and moves down in frequency.
- CMB inverse-Compton losses act like a field $B_{IC} = 3.24(1+z)^2 \mu\text{G}$ and cap the lifetime.
- Self-absorption gives $S_\nu \propto \nu^{5/2}$; the peak gives $B \propto \theta^4 \nu_p^5 S_p^{-2}$ if the size is measured.
- Incoherent synchrotron $T_b \lesssim 10^{11}\text{--}10^{12} \text{ K}$; above that, beaming, non-equilibrium, coherence or wrong size.
- Minimum energy gives $E_B = \frac{3}{4}E_{\text{part}}$ and $B_{\text{min}} \propto [(1+k)L/\phi V]^{2/7}$: an estimate under stated assumptions, not a measurement.

Next lecture

Mildly relativistic electrons (gyrosynchrotron) and coherent electron-cyclotron maser emission: how magnetized stars and planets produce bright, narrow-band, circularly polarized bursts.

End of topic question and reading

End of topic question

Name one observation that can discriminate a young compact source from an absorbed old source, and state what result you would expect in each case.

- **Review:** ERA Ch. 5, “Synchrotron Spectra”, “Synchrotron Sources” and “Inverse-Compton Scattering” (<https://www.cv.nrao.edu/~sransom/web/xxx.html>).
- **Deepen:** TRA Ch. 10, synchrotron spectra and absorption; TRA Ch. 11, compact and extragalactic non-thermal sources.
- **Optional:** IRA/Kraus on synchrotron self-absorption, spectral breaks and minimum-energy fields; Beck & Krause 2005 on revised equipartition.

After class

Estimate radiative lifetimes at 150 MHz and 1.4 GHz for $B = 1 \mu\text{G}$ and $B = 10 \mu\text{G}$ at $z = 0$. Explain why each result is model dependent.