

The Radio Sky

Lecture 7

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PHY-236.3



Question: why only electrons?

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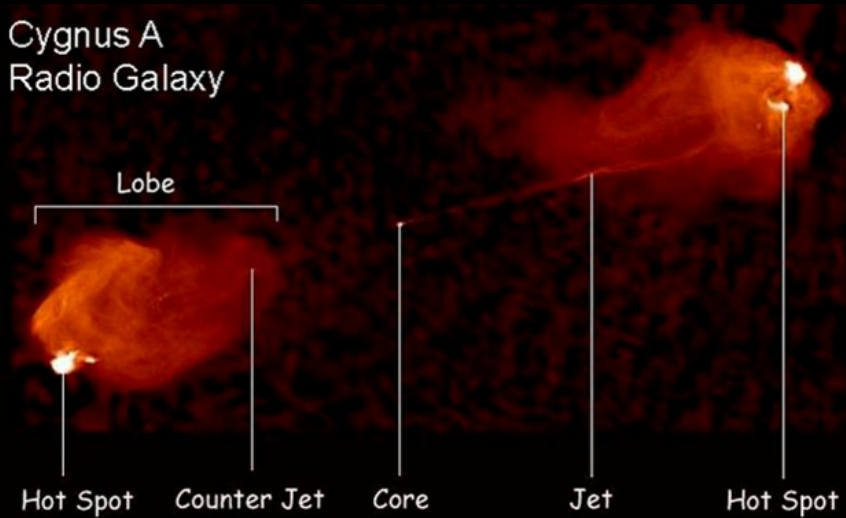
Answer: electrons are shaken far more than ions

The wave pushes every charge with the same force qE , so the motion is $a = qE/m$. A proton is 1836 times heavier than an electron: a bowling ball next to a ping-pong ball.

- **DM**: the plasma's effect on the wave scales as q^2/m . Ions contribute $\sim 1/1836$ of the electrons.
- **RM**: Faraday rotation scales as $q^3 B/m^2$. Ions contribute $\sim 10^{-7}$.
- **EM**: radiated power $\propto a^2 \propto 1/m^2$. Ions barely radiate. They matter only as targets: the emission is really $\int n_e n_i dl$, and $n_i \approx n_e$.

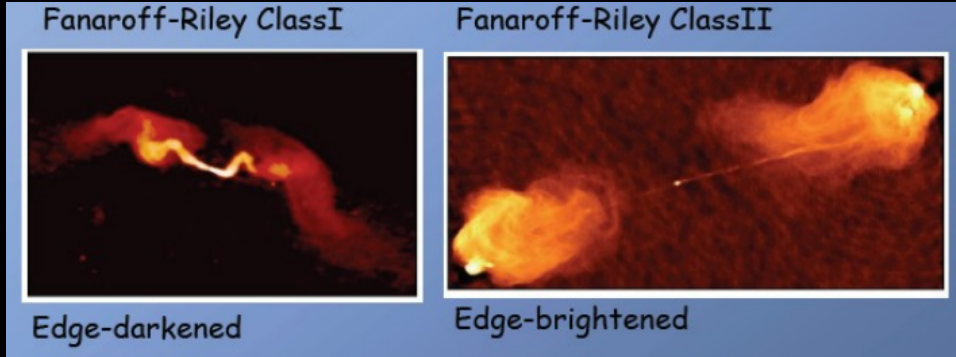
Neutral atoms have no free charges and contribute nothing: these measures trace ionized gas only.

Total intensity: a radio-galaxy lobe, Cygnus A



Credit: VLA radio image of Cygnus A with labelled components. NRAO/AUI/NSF.

Two kinds of radio-galaxy morphology: FR I and FR II

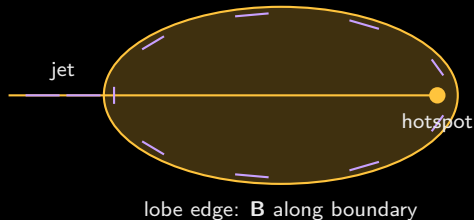


FR I: brightest near the nucleus, fading outward (edge-darkened).

FR II: brightest at the outer ends, in compact hotspots (edge-brightened).

Credit: Radio images of an FR I and an FR II source. [source and credit to be confirmed] Classes: Fanaroff & Riley 1974; details in Lecture 18.

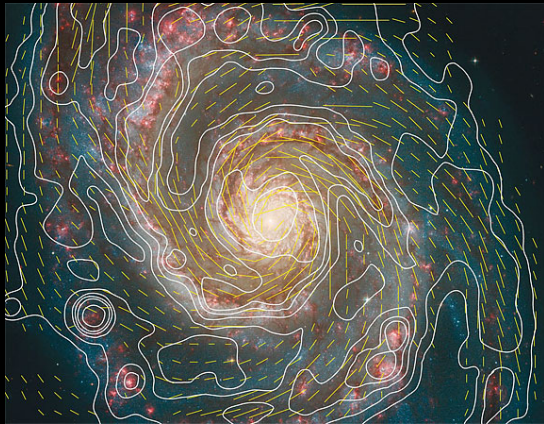
Lobes and jets: field geometry from polarization



Schematic B-vectors; not a specific source.

- Lobe edges: the field follows the boundary, as expected from compression and shear.
- Jets: in powerful (FR II) jets the field is mostly along the axis; in many FR I jets it turns perpendicular downstream (Bridle & Perley 1984, [doi:10.1146/annurev.aa.22.090184.001535](https://doi.org/10.1146/annurev.aa.22.090184.001535)).

Reading a polarization map: field vectors over a galaxy (M51)



Credit: M51 radio contours and B-vectors on an optical image. MPIfR Bonn and Hubble Heritage Team (data: Fletcher et al. 2011, [doi:10.1111/j.1365-2966.2010.18065.x](https://doi.org/10.1111/j.1365-2966.2010.18065.x)).

A uniformly moving charge does not radiate

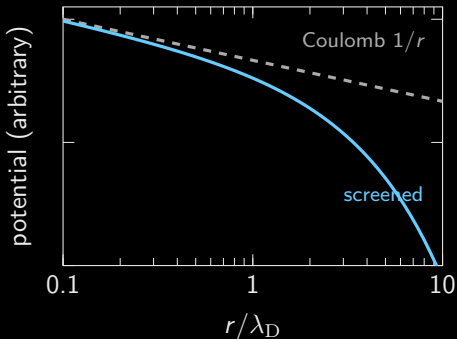
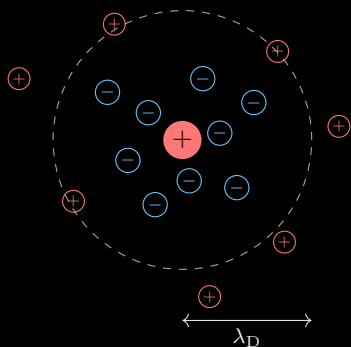
A fast charge in empty space has an electric field, but radiates nothing.

- Boost to the charge's rest frame: only a static Coulomb field, which carries no energy away. Radiation is frame-independent, so the lab frame sees none either.
- The lab-frame field is squeezed towards the plane perpendicular to $\mathbf{v}$ (enhanced by γ), but it travels with the charge and falls as $1/r^2$: its energy flux ($\propto 1/r^4$) never reaches infinity.
- Radiation fields fall as $1/r$ and are proportional to the **acceleration** (Larmor, Lecture 4).

Which accelerations matter in astrophysical plasmas?

- An electric field can accelerate a charge too: an electron deflected by an ion's Coulomb field radiates (free-free, Lecture 3).
- But free charges screen static electric fields over a Debye length, so large-scale electric fields do not survive in a plasma (next two slides).
- Magnetic fields are frozen into the plasma. A magnetic force changes only the *direction* of $\mathbf{v}$: a relativistic electron keeps accelerating, with no energy gain, for as long as the field lasts.

Debye screening: a plasma cancels electric fields



- Put a positive charge in a plasma: mobile electrons crowd round it, ions are pushed away. Beyond a distance λ_D the cloud cancels the charge's field:

$$\phi = \frac{q}{4\pi\epsilon_0 r} e^{-r/\lambda_D}.$$

- $\lambda_D = \sqrt{\epsilon_0 k_B T_e / (n_e e^2)} \approx 6.9 \text{ cm } (T_e/\text{K})^{1/2} (n_e/\text{cm}^{-3})^{-1/2}.$

Why large-scale electric fields do not survive

Plasma (illustrative)	T_e (K)	n_e (cm $^{-3}$)	λ_D
Warm interstellar gas	8×10^3	0.3	11 m
Solar corona	10^6	10^9	2 mm
Cluster gas	10^8	10^{-3}	22 km

- λ_D is tiny compared with any radio source (parsecs to Mpc). A net electric field over a size $L \gg \lambda_D$ would need a large charge separation, which the plasma cancels.
- It cancels fast: charges rearrange on the plasma timescale $\sim 1/\nu_p$, about 0.2 ms in the warm interstellar gas.
- In the plasma's own frame $\mathbf{E} \approx 0$. Bulk motion through a field gives $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$, which is just the magnetic field seen from another frame.

Exceptions are local and brief: shocks, current sheets, pulsar magnetospheres. These are where particles are accelerated.

Where the scaling comes from, and what K is

K sets how many electrons there are: $N(E) dE = K E^{-p} dE$ (number per unit volume per unit energy), so $n_e \approx K E_{\min}^{1-p} / (p - 1)$.

1. One electron: power $P \propto E^2 B^2$, radiated mostly at $\nu \approx \gamma^2 \nu_G \propto E^2 B$. So $E \propto (\nu/B)^{1/2}$ and $dE/d\nu \propto \nu^{-1/2} B^{-1/2}$.
2. The emissivity collects the electrons that radiate at ν : $j_\nu d\nu \propto P N(E) dE$, so

$$j_\nu \propto E^2 B^2 \cdot K E^{-p} \cdot \nu^{-1/2} B^{-1/2} \Rightarrow j_\nu \propto K B^{(p+1)/2} \nu^{-(p-1)/2}$$

3. Optically thin source of volume V at distance D : $S_\nu = j_\nu V / D^2$.

Delta-function approximation; the exact convolution with $F(x)$ gives the same exponents (R&L Ch. 6). Hence $\alpha = -(p - 1)/2$.

The particle–field degeneracy

For a given source (V , D fixed), the steps on the previous slide give:

$$S_\nu \propto \frac{V}{D^2} K B^{(p+1)/2} \nu^{-(p-1)/2} \quad (p = 2.4 : S_\nu \propto K B^{1.7})$$

- One observable, two unknowns: doubling B and dividing K by $2^{1.7} = 3.2$ leaves the spectrum unchanged.
- Polarization gives the direction of $\mathbf{B}_\perp$ and how ordered or tangled it is, never its strength.

Adding a second equation

Each route adds an assumption:

- Inverse-Compton X-rays from the same electrons (Lecture 8)
- Self-absorption turnover plus angular size, for compact sources (Lecture 8)
- Minimum energy / equipartition: an assumption, not a measurement (Lecture 8)
- Faraday rotation with DM: $\langle B_{\parallel} \rangle$ in the *thermal* gas, not in the emitting region (Lecture 15)

Retrieval: electrons that cool first

From Lecture 4

Power-law electrons, $N(\gamma) \propto \gamma^{-p}$. Each electron radiates mainly near $\nu \approx \gamma^2 \nu_G$.

1. The synchrotron power of one electron grows as γ^2 . Which electrons lose their energy first?
2. No fresh electrons are injected. How does the spectrum change with time, and at which frequencies?

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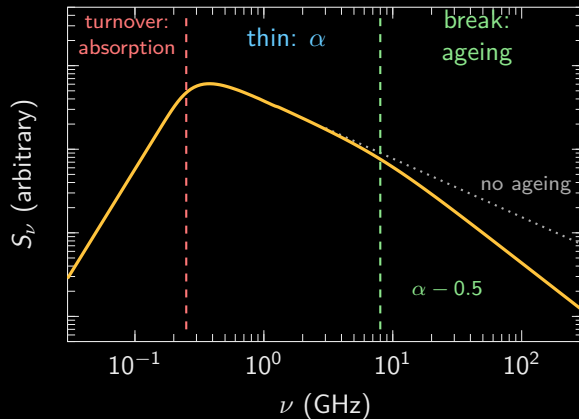
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Answer: the highest- γ electrons cool first. They emit at the highest frequencies, so the spectrum **steepens above a break frequency** ν_b , which moves lower as the source ages.

Central question: what does a curved spectrum tell us?



- **High-frequency break:** radiative age of the electrons.
- **Low-frequency turnover:** compactness (self-absorption) or an ionized foreground screen.
- **Normalization:** energy content, but only with a magnetic-field assumption.

Schematic spectrum. Dotted: the unbroken power law.

Energy losses of a relativistic electron

For $\gamma \gg 1$, isotropic pitch angles, Thomson-regime scattering:

$$P_{\text{syn}} = \frac{4}{3}\sigma_{\text{T}}c\gamma^2 U_B, \quad U_B = \frac{B^2}{2\mu_0}; \quad P_{\text{IC}} = \frac{4}{3}\sigma_{\text{T}}c\gamma^2 U_{\text{rad}}$$

$$\frac{d\gamma}{dt} = -b\gamma^2, \quad b = \frac{4\sigma_{\text{T}}}{3m_e c} (U_B + U_{\text{rad}})$$

- σ_{T} : Thomson cross-section; U_{rad} : radiation energy density. Both losses scale as γ^2 .
- Neglected: adiabatic expansion, Coulomb and bremsstrahlung losses (important at low γ or high density).

SI units; B quoted in G or μG ($1\text{ G} = 10^{-4}\text{ T}$). Synchrotron power: Lecture 4; inverse-Compton power quoted without derivation [R&L Ch. 7].

The CMB as the radiation field

For extended sources U_{rad} is usually the CMB:

$$U_{\text{CMB}} = aT_0^4(1+z)^4 = 4.17 \times 10^{-14}(1+z)^4 \text{ J m}^{-3}$$

where $a = 4\sigma_{\text{SB}}/c = 7.566 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$ is the radiation constant (σ_{SB} : Stefan–Boltzmann constant) and $T_0 = 2.725 \text{ K}$.

- The field with the same energy density: $B_{\text{IC}} = 3.24(1+z)^2 \mu\text{G}$.
- For $B \gg B_{\text{IC}}$ synchrotron losses dominate; for $B \ll B_{\text{IC}}$ inverse-Compton losses do.
- B_{IC} grows as $(1+z)^2$: at high redshift the CMB dominates losses in most radio sources.

Radiative lifetime at an observed frequency

Cooling time: $t_{\text{cool}} = \gamma/|d\gamma/dt| = 1/(b\gamma)$. With $\gamma \approx (\nu/\nu_G)^{1/2}$ for the electrons seen at rest-frame frequency ν (order-of-magnitude coefficient):

$$t_{\text{cool}} \approx 1.3 \times 10^3 \text{ Myr} \frac{(B/\mu\text{G})^{1/2}}{(B/\mu\text{G})^2 + (B_{\text{IC}}/\mu\text{G})^2} \left(\frac{\nu}{\text{GHz}} \right)^{-1/2}$$

- Synchrotron-only limit ($B \gg B_{\text{IC}}$): $t_{\text{cool}} \propto B^{-3/2} \nu^{-1/2}$.
- IC-only limit ($B \ll B_{\text{IC}}$): $t_{\text{cool}} \propto B^{1/2}$, because weaker B requires higher γ at fixed ν .

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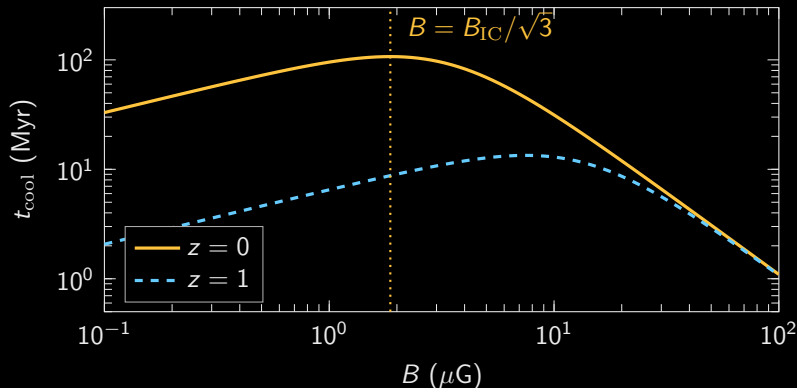
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Numbers at 1.4 GHz, $z = 0$ (computed from the formula)

$B = 10 \mu\text{G}$: $\approx 31 \text{ Myr}$. $B = 1 \mu\text{G}$: $\approx 95 \text{ Myr}$. $B = 0.16 \text{ G}$ (compact source, 5 GHz): $\approx 9 \text{ yr}$.

Lifetime versus field: there is a maximum age



- Electrons radiating at rest-frame 1.4 GHz. Maximum at $B = B_{\text{IC}}/\sqrt{3}$: $\approx 1.9 \mu\text{G}$ at $z = 0$.
- At high z the CMB shortens all lifetimes: steep-spectrum sources fade faster (relevant to Lecture 19).

From cooling to a spectral break

Solve $d\gamma/dt = -b\gamma^2$ for an electron injected with γ_0 :

$$\gamma(t) = \frac{\gamma_0}{1 + b\gamma_0 t} \Rightarrow \gamma(t) < \gamma_{\max}(t) = \frac{1}{bt} \text{ for every } \gamma_0.$$

- After time t no electron is left above $\gamma_{\max}$, whatever its initial energy (exact for constant b).
- Electrons near $\gamma_{\max}$ radiate at the **break frequency**

$$\nu_b \approx \gamma_{\max}^2 \nu_G = \frac{\nu_G}{(bt)^2} \propto \frac{B}{(B^2 + B_{IC}^2)^2 t^2}$$

- Inverting gives the **spectral age**: the lifetime formula with $\nu \rightarrow \nu_b$.
- Example ($B = 5 \mu\text{G}$, $z = 0$): $t = 10 \text{ Myr}$ gives $\nu_b \approx 67 \text{ GHz}$; $t = 30 \text{ Myr}$ gives $\approx 7 \text{ GHz}$. The break sweeps down through the radio band in tens of Myr.

Why the spectrum is unchanged below the break

Energy loss is $\propto \gamma^2$: low-energy electrons cool very slowly.

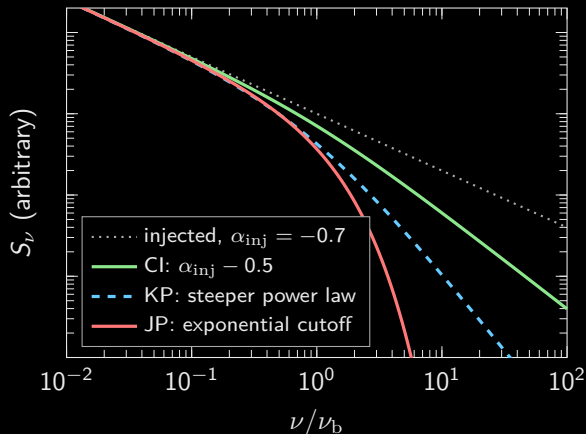
- For $\gamma \ll \gamma_{\max} = 1/(bt)$, $b\gamma t \ll 1$ and $\gamma(t) \approx \gamma_0$: the electron has barely changed energy.
- Electrons now at γ started at $\gamma_0 = \gamma/(1 - b\gamma t)$. Conserving number, $N d\gamma = N_0 d\gamma_0$:

$$N(\gamma, t) = K\gamma^{-p} \left(1 - \frac{\gamma}{\gamma_{\max}}\right)^{p-2}$$

- For $\gamma \ll \gamma_{\max}$ the bracket is ≈ 1 : the original power law. Electrons that cool out of a given γ are replaced by electrons cooling in from above.
- As $\gamma \rightarrow \gamma_{\max}$, $N \rightarrow 0$: the break.

No injection after $t = 0$; K is the initial normalization.

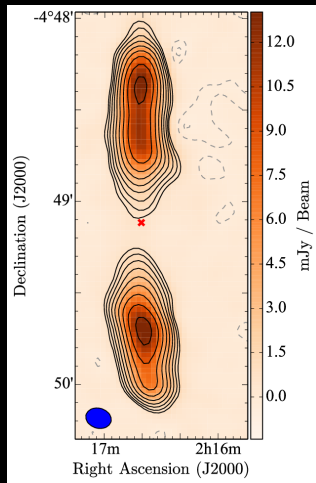
Injection history shapes the spectrum above the break



Schematic shapes, not exact model spectra.

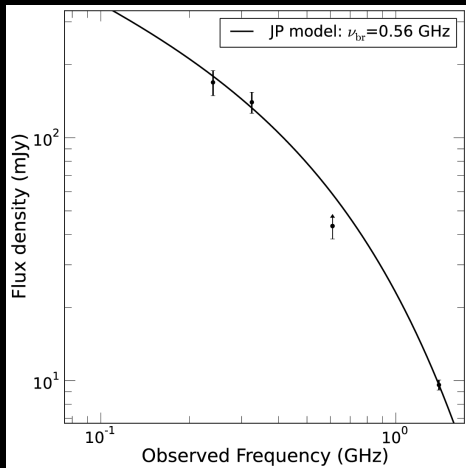
- **CI** (continuous injection, Kardashev 1962): old and new electrons mix; above ν_b , $\alpha \rightarrow \alpha_{inj} - 0.5$.
- **KP** (single injection, fixed pitch angles): small-pitch-angle electrons survive; tail tends to $\alpha \rightarrow \frac{4}{3}\alpha_{inj} - 1$.
- **JP** (single injection, pitch angles isotropized; Jaffe & Perola 1973): all electrons cool at the mean rate, so an exponential cutoff.

A real example: a dying radio galaxy at $z = 1.33$



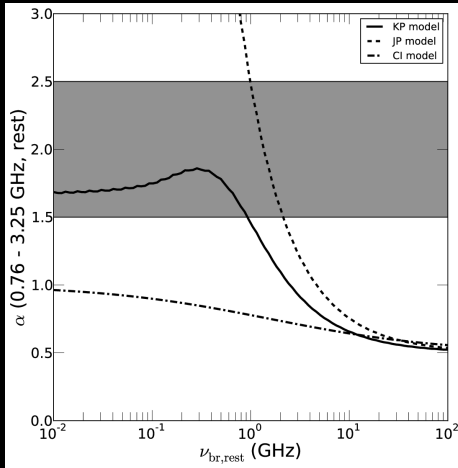
- J021659–044920: GMRT image at 325 MHz. Two lobes, $2.4'$ end to end: ≈ 1.2 Mpc projected, a giant radio galaxy.
- No core, no jets, no hot spots (core $\lesssim 5\%$ of the lobes at 325 MHz). The host (cross) lies midway.
- The jet has switched off: no fresh electrons are injected. This is a relic, the case the single-injection models describe.
- At $z = 1.33$ the CMB is strong: $B_{\text{IC}} = 3.24(1+z)^2 \approx 17.5 \mu\text{G}$, far above the lobe field ($\approx 3.3 \mu\text{G}$, measured in Lecture 8). Inverse-Compton losses dominate by $(B_{\text{IC}}/B)^2 \approx 28$.

Its spectrum: a sharp high-frequency cutoff



- Integrated flux densities at 0.24–1.4 GHz (0.61 GHz: a lower limit). Indices in the course convention (the paper uses $S_\nu \propto \nu^{-\alpha}$).
- $\alpha_{0.24}^{0.325} = -0.6 \pm 0.5$ but $\alpha_{0.325}^{1.4} = -1.83 \pm 0.08$: the spectrum is **convex**. Curvature > 1.19 ; relics are usually selected by a curvature ≥ 1 .
- Fit: JP approximated as $S_\nu = A\nu^{\alpha_{inj}}e^{-\nu/\nu_b}$ with $\alpha_{inj} = -0.5$ fixed.
- $\nu_b = 0.57 \pm 0.02$ GHz observed, i.e. $\nu_b(1+z) = 1.32$ GHz in the rest frame.

Which injection model fits?



- Curves: index between rest-frame 0.76 and 3.25 GHz (the observed 0.325 and 1.4 GHz) against the rest-frame break, for $\alpha_{\text{inj}} = -0.5$. Plotted values are $-\alpha$.
- Grey band: the range observed across the lobes, -1.5 to -2.5 .
- **CI** never gets steeper than $\alpha_{\text{inj}} - 0.5 = -1$: **excluded**, as expected once injection has stopped.
- **JP** covers the whole band for $\nu_{\text{b,rest}} \approx 1\text{--}2.2$ GHz, consistent with the fitted 1.32 GHz. **KP** levels off near -1.9 .