

The Radio Sky

Lecture 6

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PHY-236.3



Central question

What physical information is encoded in a synchrotron spectrum and a polarization map?

From Lecture 4

- Larmor formula, relativistic beaming
- Single-electron synchrotron power and spectrum $F(x)$
- Power-law-ensemble results: $\alpha(p)$, polarization

SI units unless stated; $S_\nu \propto \nu^\alpha$ throughout.

New here

- Reading p and B geometry from data, with assumptions
- Faraday rotation and depolarization (preview)

Recall: the synchrotron critical frequency (Lecture 4)

An electron of Lorentz factor γ gyrating in a field B at pitch angle ϑ :

- Non-relativistic gyrofrequency: $\nu_G = \frac{eB}{2\pi m_e} = 2.80 \text{ MHz } (B/\text{G})$.
- Beaming compresses each pulse by $\sim \gamma^{-3}$, so the spectrum extends up to

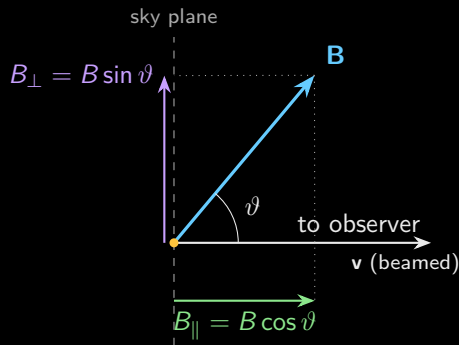
$$\nu_c = \frac{3}{2} \gamma^2 \nu_G \sin \vartheta = 4.2 \text{ MHz } \gamma^2 \left(\frac{B}{\text{G}} \right) \sin \vartheta$$

- $\nu_c \propto \gamma^2 B$: a γ ten times larger radiates at a frequency a hundred times higher.
- Radiation from one electron peaks near ν_c , so frequency labels electron energy (next slide).

Why $B_{\perp}$ and not B ?

- Only the field component *perpendicular to the electron's velocity* bends its path, $\propto B \sin \vartheta$. The component along $\mathbf{v}$ does nothing: the electron just spirals along it.
- Radiation is beamed within $\sim 1/\gamma$ of $\mathbf{v}$, so the electrons we see are those moving *towards us*.
- For them, “perpendicular to $\mathbf{v}$ ” means perpendicular to the line of sight: the sky-plane component $B_{\perp} = B \sin \vartheta$.

Hence ν_c , the emitted power ($\propto B_{\perp}^2$) and the polarization direction all depend on $B_{\perp}$ only. The component $B_{\parallel}$ is invisible in synchrotron emission; Faraday rotation measures it.



Side view; ϑ is both the pitch angle and the angle between $\mathbf{B}$ and the line of sight.

Which electrons do we see?

Invert $\nu \approx \nu_c$ (order of magnitude; using the peak $0.29\nu_c$ raises γ by ≈ 1.9):

$$\gamma \approx \left(\frac{\nu}{4.2 \text{ MHz } (B/\text{G})} \right)^{1/2}, \quad E = \gamma m_e c^2 = 0.511 \text{ MeV } \gamma$$

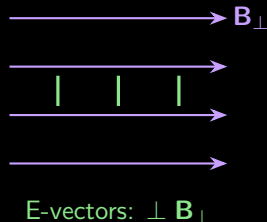
Setting (illustrative)	B	ν	$\gamma ; E$
Galactic disc, GMRT band	$10 \mu\text{G}$	150 MHz	$1.9 \times 10^3 ; 1 \text{ GeV}$
Galactic disc, L band	$10 \mu\text{G}$	1.4 GHz	$5.8 \times 10^3 ; 3 \text{ GeV}$
SNR shell	$100 \mu\text{G}$	1.4 GHz	$1.8 \times 10^3 ; 0.9 \text{ GeV}$

- Radio synchrotron samples GeV electrons: the same population as the cosmic-ray electrons measured near Earth.
- $\gamma \propto (\nu/B)^{1/2}$: each frequency probes a different electron energy, *for an assumed B* .

Synchrotron radiation is linearly polarized

Electrons gyrating in an ordered field radiate linearly polarized light, up to $\sim 70\%$ for typical power-law electrons.

- The electric vector is **perpendicular** to the sky-projected field $\mathbf{B}_\perp$.
- So a polarization map shows the orientation of the magnetic field.
- Observed fractions are much lower (depolarization, below).



Maps often plot E-vectors rotated by 90° (“B-vectors”). **Check which.**

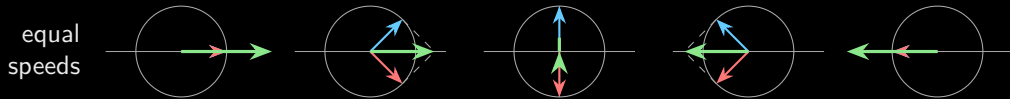
Faraday rotation: the picture

Radio waves from the source cross magnetized plasma (e.g. the Galaxy's interstellar medium) on the way to us.

1. A linearly polarized wave is two circularly polarized waves, one turning each way.
2. In a plasma with a field along the line of sight, the two travel at slightly different speeds.
3. When they recombine, the plane of polarization has been **twisted** by an angle that grows with distance travelled.
 - More electrons, a stronger field along the line of sight, or a longer path: a bigger twist.
 - Long wavelengths are twisted much more than short ones.

A linear wave is two circular waves

Looking at the wave head-on at five successive instants (time runs left to right):



Right-handed and left-handed arrows turn in opposite directions; their sum only grows and shrinks along one line. Top: that line stays fixed. Bottom: delaying one wave tilts the line, which is Faraday rotation.

Faraday rotation: what we measure

The twist grows as the wavelength squared:

$$\chi = \chi_0 + \text{RM} \lambda^2, \quad \text{RM} = 0.812 \int \frac{n_e}{\text{cm}^{-3}} \frac{B_{\parallel}}{\mu\text{G}} \frac{dl}{\text{pc}} \text{ rad m}^{-2}$$

RM (rad m ⁻²)	6 cm	21 cm	2 m
10	2°	25°	2300°
100	21°	250°	—

- The angle is only known modulo π , so measure χ at several wavelengths and fit a line against λ^2 .
- The slope is RM; the intercept χ_0 is the original direction, before rotation.
- Derivation: Rybicki & Lightman, Ch. 8.

Gaussian units (R&L): $\text{RM} = \frac{e^3}{2\pi m_e^2 c^4} \int n_e B_{\parallel} dl$. $B_{\parallel}$ is positive toward the observer; RM values illustrative.

Dispersion measure

The same plasma also delays radio waves, by an amount that depends on frequency. Its column of electrons is the **dispersion measure**:

$$DM \equiv \int_0^D n_e dl \quad (\text{pc cm}^{-3})$$

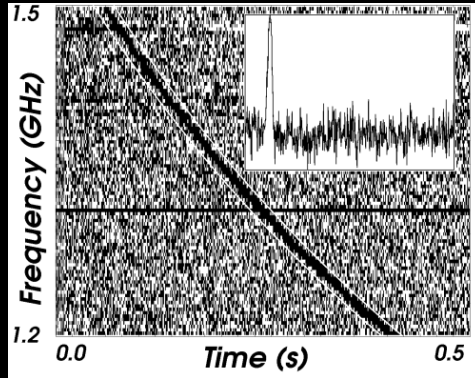
- In a plasma, low-frequency waves travel more slowly (group velocity $v_g = c\sqrt{1 - \nu_p^2/\nu^2}$, with plasma frequency $\nu_p \approx 8.98 \text{ kHz } (n_e/\text{cm}^{-3})^{1/2}$).
- The extra arrival delay, relative to the vacuum travel time, is

$$t_{DM} = 4.149 \text{ ms} \left(\frac{DM}{\text{pc cm}^{-3}} \right) \left(\frac{\nu}{\text{GHz}} \right)^{-2} \approx 46 \text{ ms} \left(\frac{DM}{\text{pc cm}^{-3}} \right) \left(\frac{\lambda}{\text{m}} \right)^2$$

- Like Faraday rotation, the effect grows as λ^2 : it is largest at long wavelengths.
- A pulse seen at two frequencies arrives at different times; $DM = 50$ gives a 1.7 s lag between 600 and 300 MHz.

Derivation and pulsar applications: Lecture 15; Rybicki & Lightman, Ch. 8.

Dispersion in action: the Lorimer burst



A single radio pulse arrives first at high frequency and later at low frequency, sweeping across the band by ≈ 0.35 s ($DM \approx 375$). Inset: the dedispersed pulse.

Credit: The Planetary Society. Lorimer burst (FRB 010724), Parkes 1.4 GHz; Lorimer et al. 2007, [doi:10.1126/science.1147532](https://doi.org/10.1126/science.1147532)

EM, DM and RM: three line-of-sight integrals

	Definition	Units	Observable effect
EM	$\int n_e^2 dl$	pc cm^{-6}	$\tau_\nu \approx 3.28 \times 10^{-7} \text{ EM } \nu_{\text{GHz}}^{-2.1}$
DM	$\int n_e dl$	pc cm^{-3}	$t = 4.149 \text{ ms DM } \nu_{\text{GHz}}^{-2}$
RM	$0.812 \int n_e B_{\parallel} dl$	rad m^{-2}	angle $\chi = \chi_0 + \text{RM } \lambda^2$

- Effects: free-free optical depth ($T_e \approx 10^4 \text{ K}$), pulse delay, polarization-angle rotation. RM uses n_e in cm^{-3} , $B_{\parallel}$ in μG , dl in pc.
- All three probe the thermal electrons along the path; only RM involves the magnetic field.
- For one source, RM/DM gives the density-weighted mean field:
 $\langle B_{\parallel} \rangle = 1.232 \mu\text{G RM/DM}$ (Lecture 15).

Depolarization: why $\Pi_{\text{obs}} \ll 70\%$

- **Tangled field:** an isotropic random component b added to a uniform field B_0 (sky-plane components):

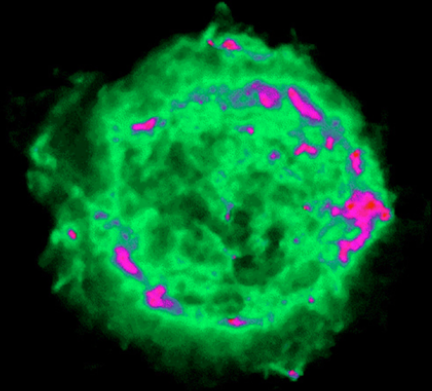
$$\Pi \approx \Pi_{\text{max}} \frac{B_{0\perp}^2}{B_{0\perp}^2 + \langle b_{\perp}^2 \rangle}$$

(Burn 1966, [doi:10.1093/mnras/133.1.67](https://doi.org/10.1093/mnras/133.1.67); approximate, exact for $p = 3$).

- **Beam depolarization:** different χ within one resolution element average out. Higher resolution recovers Π .
- **Faraday depolarization:** emission and rotation mixed in the same volume, or a turbulent foreground screen. For a uniform slab: $\Pi = \Pi_0 |\sin(\text{RM}\lambda^2)/(\text{RM}\lambda^2)|$, where RM is the total through the slab. Worse at long λ .
- **Bandwidth depolarization:** χ rotates across the channel.

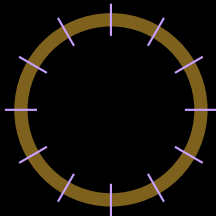
Review: Sokoloff et al. 1998, [doi:10.1046/j.1365-8711.1998.01782.x](https://doi.org/10.1046/j.1365-8711.1998.01782.x); RM synthesis: Brentjens & de Bruyn 2005, [doi:10.1051/0004-6361:20052990](https://doi.org/10.1051/0004-6361:20052990).

Total intensity: a young shell remnant, Cas A

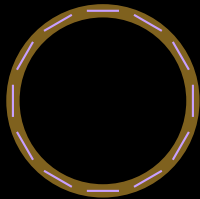


Credit: VLA radio image of Cassiopeia A. NRAO/AUI/NSF; public domain, via [Wikimedia Commons](#).

Shell remnants: what polarization maps add



young: radial $\mathbf{B}$



older: tangential $\mathbf{B}$

Schematic $\mathbf{B}$ -vectors ($\mathbf{E}$ -vectors rotated by 90°).

- **Observed** tendency: young remnants show predominantly radial fields and low Π (few %); older remnants show tangential fields and higher Π (Dickel & Milne 1976, [doi:10.1071/ph760435](https://doi.org/10.1071/ph760435)).
- **Inferred**: tangential field from shock compression of the ambient field; radial field from stretching by Rayleigh–Taylor instabilities at the contact discontinuity.
- Interpretation is model dependent; Lecture 13.