

The Radio Sky

Lecture 5

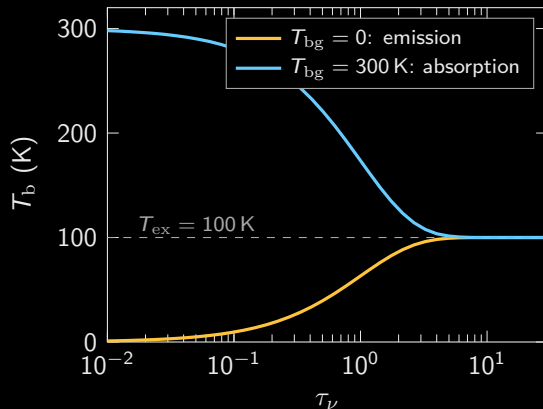
Yogesh Wadadekar

National Centre for Radio Astrophysics (NCRA-TIFR), Pune

PHY-236.3



Brightness temperature against optical depth



- Thin ($\tau_\nu \ll 1$):
 $\Delta T_b \approx (T_{\text{ex}} - T_{\text{bg}})\tau_\nu$. Signal $\propto$ column of emitters.
- Thick ($\tau_\nu \gg 1$): $T_b \rightarrow T_{\text{ex}}$ whatever the background. Signal saturates; column is lost.
- The same cloud emits or absorbs depending on what lies behind it.

Uniform slab, $T_{\text{ex}} = 100$ K (illustrative).

Concept test

Question

An optically thick cloud with $T_{\text{ex}} = 50 \text{ K}$ is observed (a) against the 2.7 K CMB and (b) against a background continuum source with $T_{\text{bg}} = 1000 \text{ K}$. Is it seen in emission or in absorption in each case?

Concept test

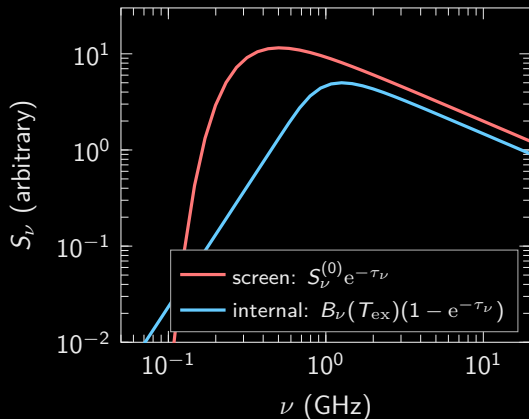
Question

An optically thick cloud with $T_{\text{ex}} = 50$ K is observed (a) against the 2.7 K CMB and (b) against a background continuum source with $T_{\text{bg}} = 1000$ K. Is it seen in emission or in absorption in each case?

Answer: (a) emission, (b) absorption

1. $\Delta T_{\text{b}} = (T_{\text{ex}} - T_{\text{bg}})(1 - e^{-\tau_{\nu}})$: the sign is set by $T_{\text{ex}} - T_{\text{bg}}$, not by the cloud alone.
2. (a) $50 > 2.7$ K: emission. (b) $50 < 1000$ K: absorption. In both cases $T_{\text{b}} \rightarrow 50$ K, since a thick cloud hides its background.

Foreground absorption versus internal absorption



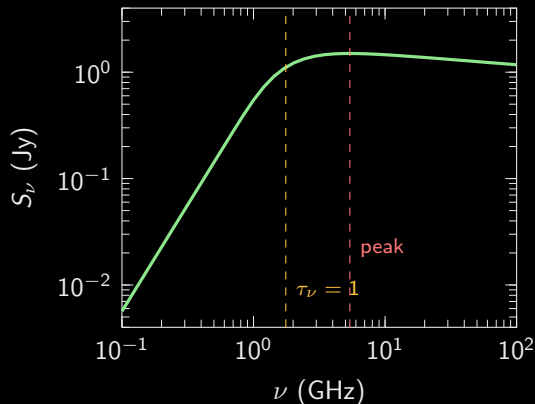
- **External screen:** a cold ionized foreground with no emission of its own. $S_\nu = S_\nu^{(0)} \exp[-\tau_\nu]$, with $\tau_\nu \propto \nu^{-2.1}$: an *exponential* cutoff, steeper than any power law.
- **Internal (self-) absorption:** emitter and absorber are the same population; the thick side is a power law set by $\nu^2 T_{\text{ex}}(\nu)$.
- Both curves: thin synchrotron $\alpha = -0.7$ at high frequency (schematic).

Turnover signatures: a reference table

Process	Low-frequency side	Other discriminants
Free-free self-absorption (uniform)	$\alpha = +2$; $T_b \rightarrow T_e \sim 10^4 \text{ K}$	thin $\alpha \approx -0.1$; unpolarized; RRLs
Foreground free-free screen	exponential cutoff	thin side is that of the background source
Synchrotron self-absorption (later in this lecture)	$\alpha = +2.5$ (uniform)	compact; $T_b \sim 10^{10}\text{--}10^{12} \text{ K}$; variable
Low-energy cutoff in electrons	$\alpha \rightarrow +\frac{1}{3}$	no T_b constraint; gentle curvature
Inhomogeneous source	$0 < \alpha < 2$	frequency-dependent size

- Slopes are asymptotic. With only two or three points near the peak, most of these cannot be told apart.
- Angular size gives T_b , and T_b is often the decisive discriminant.

Worked example 1: an H II-region turnover



Illustrative: $T_e = 10^4$ K,
 $EM = 10^7 \text{ pc cm}^{-6}$, uniform disc of
 diameter $10''$.

ν/GHz	τ_ν	T_b/K	S_ν/Jy
0.3	41	10^4	0.05
1	3.3	9600	0.55
3	0.33	2800	1.4
10	0.026	260	1.5
30	0.0026	26	1.3

- $\nu_t \approx 1.8$ GHz; flux peak ≈ 5 GHz.
- Thick side measures $T_e \Omega_s$; thin side measures $EM T_e^{-0.35} \Omega_s$.

τ_ν from the free-free optical-depth approximation (Lecture 4) [ERA Ch. 4].

What the turnover buys you: three parts of one spectrum

Uniform, isothermal source; three unknowns T_e , EM, Ω_s :

$$T_b(\nu) = T_e (1 - e^{-\tau_\nu}), \quad \tau_\nu = 3.28 \times 10^{-7} T_4^{-1.35} \nu_{\text{GHz}}^{-2.1} \text{EM}, \quad S_\nu = \frac{2k_B \nu^2}{c^2} T_b \Omega_s$$

Part	T_b	What it fixes	Worked example 1
Thick, $\nu \ll \nu_t$	$\rightarrow T_e$	$T_e \Omega_s$; T_e if resolved	0.3 GHz: $T_b = 10^4$ K
$\tau_\nu = 1$ at ν_t	$0.63 T_e$	EM $T_e^{-1.35}$; no size needed	$\nu_t = 1.76$ GHz
Thin, $\nu \gg \nu_t$	$\approx T_e \tau_\nu$	EM $T_e^{-0.35} \Omega_s$; EM $T_e^{-0.35}$ if resolved	10 GHz: $T_b = 260$ K

- T_e is the same at all ν , while τ_ν changes by $\sim 10^3$ over a factor 30 in ν . Thick side: the **temperature**; thin side: the **amount of gas**.
- One frequency gives one combination; both sides together separate them, because the ν dependence of τ_ν is *known*.

What the turnover buys you: the inference, step by step

Resolved source (Worked example 1, Ω_s from the image):

1. Thick side: $T_b = 9600$ K at 1 GHz, 10^4 K at 0.3 GHz; it has stopped rising $\Rightarrow T_e = 10^4$ K.
2. Thin side: $\tau_\nu = T_b/T_e = 260/10^4 = 0.026$ at 10 GHz. Invert the τ_ν formula:
 $EM = \tau_\nu \nu_{\text{GHz}}^{2.1} T_4^{1.35} / 3.28 \times 10^{-7} \Rightarrow EM = 1.0 \times 10^7 \text{ pc cm}^{-6}$.
3. Check: these predict $\nu_t = 1.76$ GHz. Agreement with the measured turnover tests the uniform, isothermal model.

Unresolved source: only $T_e \Omega_s$ and $EM T_e^{-1.35}$. Two numbers, three unknowns: T_e must come from elsewhere (later in this topic).

Choose the frequencies with care

- Use the thin approximation only well above ν_t : at 3 GHz ($\tau_\nu = 0.33$), $T_e \tau_\nu = 3300$ K but $T_b = 2800$ K.
- “Flat” near the peak is not “thick”: $T_b < T_e$ there.

$T_4 \equiv T_e/10^4$ K. EM gives a density only with a path length (Emission measure section).

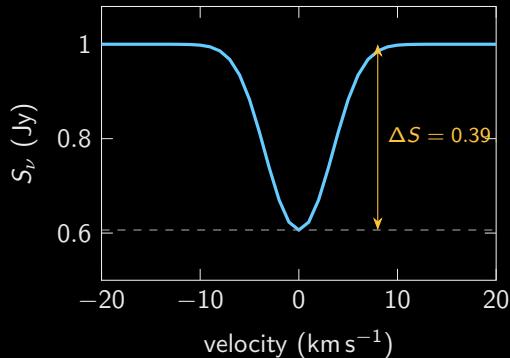
Worked example 2: H I absorption against a continuum source

Background $T_c \gg T_s$; cloud covers it:

$$S_\nu(\nu) = S_c e^{-\tau(\nu)}$$

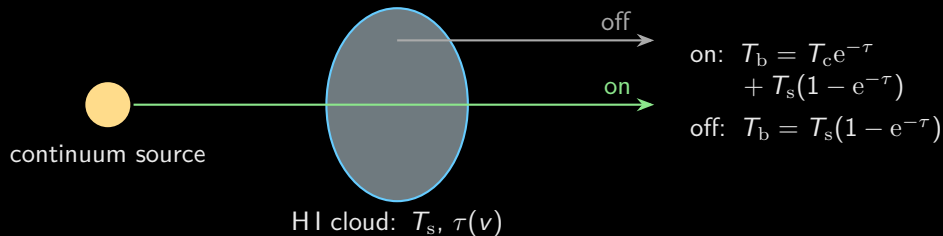
$$\tau(\nu) = -\ln \left[1 - \frac{|\Delta S(\nu)|}{S_c} \right]$$

Illustrative: $S_c = 1 \text{ Jy}$, line depth 0.39 Jy
 $\Rightarrow \tau_0 = 0.50$.



- τ is a ratio: needs neither the source size nor a distance.
- Assumes full covering; cloud emission negligible.
- T_s needs a second measurement: emission just off the source (Lecture 8).

Absorption geometry: why a compact background helps



- On minus off: $T_c e^{-\tau}$, which yields τ . Then the off spectrum yields T_s .
- Assumes the cloud has the same τ and T_s on and off the sightline. **Small-scale structure violates this.**
- The fractional depth $1 - e^{-\tau}$ saturates near 1: for $\tau \gtrsim 3$ the depth no longer measures τ well.

Schematic, not to scale. T_c : brightness temperature of the background continuum within the beam.

Concept test

Question

A cold H I cloud lies in front of a bright continuum source. If its optical depth increases, what happens to the observed intensity at line centre?

Concept test

Question

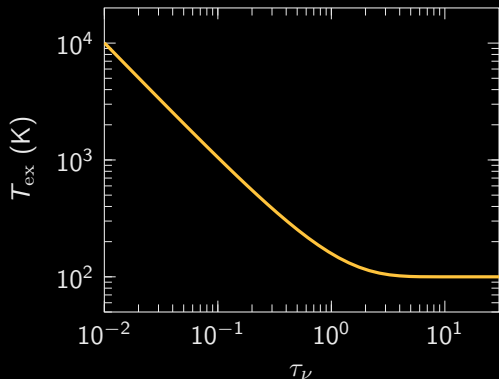
A cold H I cloud lies in front of a bright continuum source. If its optical depth increases, what happens to the observed intensity at line centre?

Answer: it falls: the continuum is more strongly absorbed

1. The transmitted background is $T_c e^{-\tau}$.
2. Larger τ gives smaller transmission; the cloud's own emission, $T_s(1 - e^{-\tau})$, cannot compensate while $T_s < T_c$.

Why an unresolved flux density cannot give a temperature

All give $T_b = 100$ K



1. Unresolved: Ω_s is only bounded above, so T_b is a **lower limit** (Lecture 2).
2. Even resolved: $T_b = T_{\text{ex}}(1 - e^{-\tau_\nu})$. One number, two unknowns: the curve at left.
3. Only $T_{\text{ex}} \geq T_b$ is guaranteed (no background, thermal emitter).

Consequence

A flux density gives a physical temperature only if the source is resolved *and* demonstrably thick.

Breaking the temperature–optical-depth degeneracy

- **Multi-frequency continuum**: the turnover method of Worked example 1 ($\tau_\nu \propto \nu^{-2.1}$ for free-free).
- **On–off absorption**: a background source supplies a known T_{bg} (Worked example 2; H I, Lecture 8).
- **Higher resolution**: converts a T_{b} lower limit to a value; tests whether the thick side is genuinely $T_{\text{b}} \approx T_{\text{ex}}$.
- **Independent thermometers**: recombination-line widths and line-to-continuum ratios give T_{e} (this lecture and Lecture 12).

State the assumption you are adding

Each method trades the degeneracy for a model assumption: isothermality, uniform covering, or a known frequency scaling. Say which.

End of topic question and reading

End of topic question

A resolved cloud has $T_b = 40$ K at one frequency. Name one observation that would separate its excitation temperature from its optical depth, and the assumption that observation adds.

- **Review:** ERA Ch. 2, “Radiative Transfer”, “Blackbody Radiation”, “Brightness and Flux Density”; Radiative Processes notes on transfer, LTE and non-LTE.
- **Deepen:** TRA Ch. 10, *Emission Mechanisms of Continuous Radiation*; TRA Ch. 12, *Spectral Line Fundamentals* (line transfer, excitation temperature).
- **Optional:** IRA sections on radiation temperature and radio transfer.
- **Online:** ERA, <https://www.cv.nrao.edu/~sransom/web/xxx.html>

After class

Model a homogeneous thermal source with a given angular diameter and emission measure. Identify the band in which its optically thin flux can be measured reliably.

Retrieval (5 minutes)

From earlier in this lecture

A dense, compact H II region (illustrative EM $\sim 10^8\text{--}10^9 \text{ pc cm}^{-6}$, $T_e = 10^4 \text{ K}$) is observed at 1 GHz and at 30 GHz. At which frequency is it brighter? Which assumption controls your answer?

Retrieval (5 minutes)

From earlier in this lecture

A dense, compact H II region (illustrative EM $\sim 10^8$ – 10^9 pc cm $^{-6}$, $T_e = 10^4$ K) is observed at 1 GHz and at 30 GHz. At which frequency is it brighter? Which assumption controls your answer?

1. At 1 GHz, $\tau_\nu \approx 33$ – 330 : thick, $S_\nu \propto \nu^2$. At 30 GHz, $\tau_\nu \approx 0.03$ – 0.3 : nearly thin.
2. Hence $S_\nu(30 \text{ GHz})/S_\nu(1 \text{ GHz}) \approx 20$ – 200 : **brighter at 30 GHz**.

Controlling assumption: a homogeneous, isothermal source of the same solid angle at both frequencies. A density gradient makes the source larger at low frequency and the ratio smaller.

Central question

How can radio continuum diagnose ionized gas and massive-star formation?

Why radio for ionized gas?

- Free-free emission is unaffected by dust.
- Its emissivity depends on n_e^2 and weakly on T_e .
- Thin flux counts recombinations, hence ionizing photons.

Conventions

- Emissivity, opacity and spectrum: Lecture 4 (cgs)
- Luminosities converted to W Hz^{-1}
- EM in pc cm^{-6} ; $S_\nu \propto \nu^\alpha$

Why the thin spectrum has $\alpha \approx -0.1$

In the thin limit $S_\nu \propto j_\nu \propto \bar{g}_{\text{ff}}$, and the radio Gaunt factor (Lecture 4) falls logarithmically with ν . The local thin spectral index is the logarithmic slope of $\bar{g}_{\text{ff}}$:

$$\alpha = \frac{d \ln S_\nu}{d \ln \nu} = \frac{d \ln \bar{g}_{\text{ff}}}{d \ln \nu} = -\frac{\sqrt{3}/\pi}{\bar{g}_{\text{ff}}}$$

ν (GHz), $T_e = 10^4$ K	0.1	1	5	10	100
$\bar{g}_{\text{ff}}$	7.2	6.0	5.1	4.7	3.4
α	-0.08	-0.09	-0.11	-0.12	-0.16

- $\alpha \approx -0.1$ is not a power law: it steepens slowly with ν .

Emission measure, density and filling factor

- $\text{EM} = \int n_e^2 \, dl$ is distance-independent: it follows from T_b (a surface brightness) and T_e .
- Uniform path length L : $n_e = \sqrt{\text{EM}/L}$. L needs an angular size *and a distance*.

Emission measure, density and filling factor

- $\text{EM} = \int n_e^2 dl$ is distance-independent: it follows from T_b (a surface brightness) and T_e .
- Uniform path length L : $n_e = \sqrt{\text{EM}/L}$. L needs an angular size *and a distance*.

Clumping

If gas of density n_e fills a fraction f of the path: $\text{EM} = f n_e^2 L$, so

$$n_e = \frac{n_{\text{rms}}}{\sqrt{f}}, \quad n_{\text{rms}} \equiv \sqrt{\text{EM}/L}.$$

- Radio gives n_{rms} , a **lower limit** on local n_e .
- Ionized mass $\propto f n_e V \propto \sqrt{f}$: overestimated by a smooth model.
- Recombination rate $\propto f n_e^2 V$: independent of f .
- f needs another density probe (e.g. line ratios).

Concept test

Question

For the same path length, if the electron density doubles, by what factor does the free-free emission measure change?

Concept test

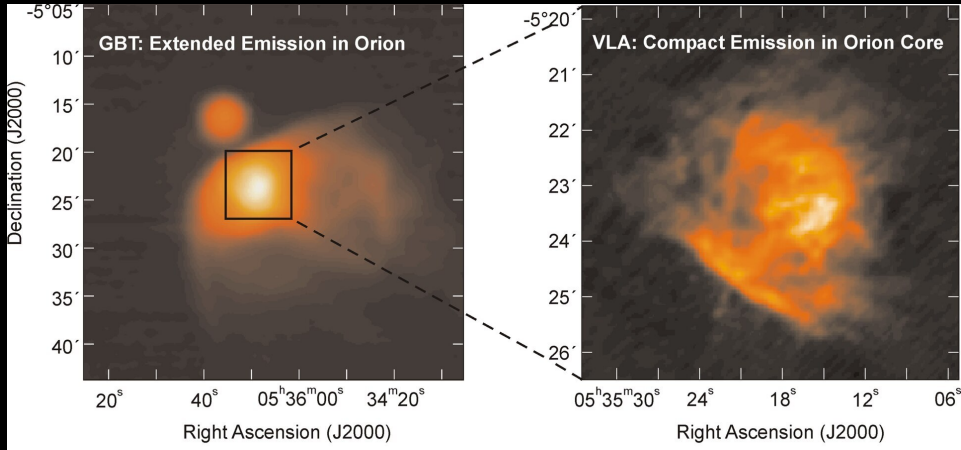
Question

For the same path length, if the electron density doubles, by what factor does the free-free emission measure change?

Answer: four

1. $EM = \int n_e^2 dl$.
2. $(2n_e)^2 = 4n_e^2$; the total flux density also rises by four.

A resolved H II region at 3.6 cm: Orion Nebula



Credit: GBT (left) and VLA (right) 3.6 cm continuum of the Orion Nebula; D. Shepherd, R. Maddalena, J. McMullin. Image courtesy of NRAO/AUI, CC BY 3.0, via [Wikimedia Commons](#).

Reading the Orion images

- **Observed:** smooth extended emission (GBT) and a bright, clumpy, sharp-edged core (VLA) at 3.6 cm, at the position of the optical nebula M42; a separate compact region to the north (M43).
- **Inferred:** at 3.6 cm most of the region is optically thin, so the brightness maps EM projected on the sky.
- The sharp south-eastern edge: an ionization front against the molecular cloud behind (a “blister” geometry).
- Brightness fluctuations on all scales imply $f < 1$.

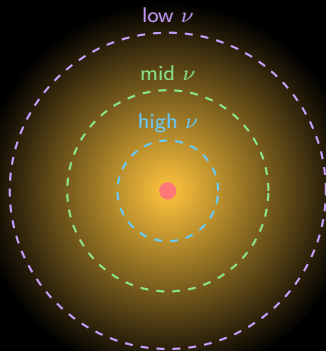
Reading the Orion images

- **Observed:** smooth extended emission (GBT) and a bright, clumpy, sharp-edged core (VLA) at 3.6 cm, at the position of the optical nebula M42; a separate compact region to the north (M43).
- **Inferred:** at 3.6 cm most of the region is optically thin, so the brightness maps EM projected on the sky.
- The sharp south-eastern edge: an ionization front against the molecular cloud behind (a “blister” geometry).
- Brightness fluctuations on all scales imply $f < 1$.

Instrumental caveat

An interferometer alone misses smooth large-scale emission; a single dish alone misses fine structure. The two panels are not two different nebulae. (Details: PHY-336.3.)

Frequency-dependent photospheres



$\tau_\nu = 1$ surfaces in
 $n_e \propto r^{-2}$ (schematic)

- With a density gradient, the radius at which the line of sight reaches $\tau_\nu \approx 1$ depends on frequency.
- The observed size grows toward low frequency, so the thick side rises *more slowly* than ν^2 .
- Spherical wind at constant speed ($n_e \propto r^{-2}$):

$$S_\nu \propto \nu^{0.6}, \quad \theta \propto \nu^{-0.7}$$

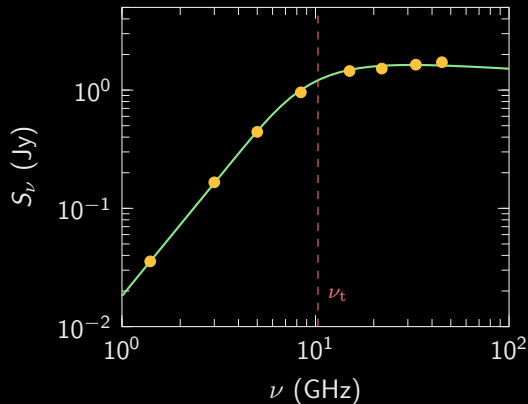
[Wright & Barlow 1975,
[doi:10.1093/mnras/170.1.41](https://doi.org/10.1093/mnras/170.1.41)]

Four thermal ionized environments

Environment	Typical EM (pc cm^{-6})	α at GHz	Remarks
Diffuse warm ionized gas	$\sim 1\text{--}10^3$ (plane)	≈ -0.1	thick only below $\sim 3\text{--}25$ MHz; absorbs low- ν Galactic synchrotron resolved; $\nu_t \lesssim 1$ GHz
Classical H II region	$10^3\text{--}10^6$	-0.1 (thin)	
Ultracompact H II	$\gtrsim 10^7$	$+2 \rightarrow -0.1$	$\nu_t \gtrsim 2$ GHz; size $\lesssim 0.1$ pc
Ionized wind / jet	gradient	$+0.6$ (wind); -0.1 to $+2$ (jet)	unresolved or elongated; jets: Reynolds 1986, doi:10.1086/164209

EM ranges are indicative orders of magnitude, not survey statistics. Diffuse-gas turnovers computed with the formula above for $\text{EM} = 10\text{--}10^3$ and $T_e = 8000$ K.

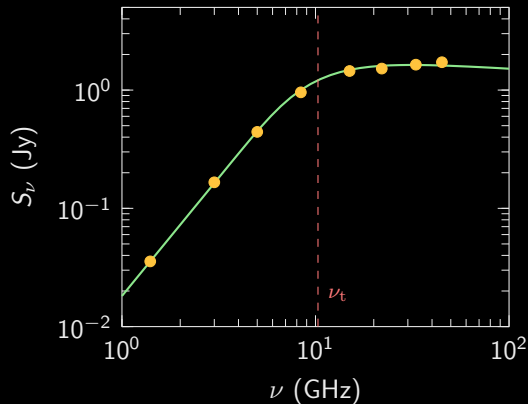
Worked example: a compact thermal source



Simulated data (5% scatter) and model.
Unresolved; $D = 5$ kpc (illustrative).

1. Low- ν slope: $\alpha = +2.0$ between 1.4 and 3 GHz. Thick, homogeneous, thermal.
2. High- ν slope: ≈ 0 , flattening toward -0.1 .
3. Assume $T_e = 8000$ K (e.g. from a recombination line).

Worked example: a compact thermal source



Simulated data (5% scatter) and model.
Unresolved; $D = 5$ kpc (illustrative).

1. Low- ν slope: $\alpha = +2.0$ between 1.4 and 3 GHz. Thick, homogeneous, thermal.
2. High- ν slope: ≈ 0 , flattening toward -0.1 .
3. Assume $T_e = 8000$ K (e.g. from a recombination line).

What do the two sides tell us?

Worked example: the inference chain

1. **Size from the thick side.** $S_\nu = 2k_B T_e \nu^2 \Omega_s / c^2$ with $S_{1.4} = 0.036$ Jy, $T_e = 8000$ K gives $\Omega_s \Rightarrow$ disc diameter $\theta \approx 2''$. Size without resolving the source, given T_e .
2. **EM from the turnover.** Fit gives $\nu_t \approx 10$ GHz ($\tau_\nu = 1$):
 $EM = \nu_t^{2.1} (T_e / 10^4)^{1.35} / 3.28 \times 10^{-7} \approx 3 \times 10^8$ pc cm $^{-6}$.
3. **Density needs distance.** $\theta = 2''$ at 5 kpc is 0.048 pc;
 $n_{\text{rms}} = \sqrt{EM/L} \approx 8 \times 10^4$ cm $^{-3}$.
4. **Classification.** Size < 0.1 pc, $n_e \gtrsim 10^4$ cm $^{-3}$: an ultracompact H II region, not a classical one.

Worked example: the inference chain

1. **Size from the thick side.** $S_\nu = 2k_B T_e \nu^2 \Omega_s / c^2$ with $S_{1.4} = 0.036$ Jy, $T_e = 8000$ K gives $\Omega_s \Rightarrow$ disc diameter $\theta \approx 2''$. Size without resolving the source, given T_e .
2. **EM from the turnover.** Fit gives $\nu_t \approx 10$ GHz ($\tau_\nu = 1$):
 $EM = \nu_t^{2.1} (T_e/10^4)^{1.35} / 3.28 \times 10^{-7} \approx 3 \times 10^8$ pc cm $^{-6}$.
3. **Density needs distance.** $\theta = 2''$ at 5 kpc is 0.048 pc;
 $n_{\text{rms}} = \sqrt{EM/L} \approx 8 \times 10^4$ cm $^{-3}$.
4. **Classification.** Size < 0.1 pc, $n_e \gtrsim 10^4$ cm $^{-3}$: an ultracompact H II region, not a classical one.

Assumptions carried

Homogeneous, isothermal sphere-like source; T_e from elsewhere; no dust emission yet at 45 GHz (dust rises as $\sim \nu^3$ – ν^4 and dominates above ~ 100 GHz for embedded sources); no synchrotron component.

Reading

- **Review:** ERA Ch. 4, *Free-Free Radiation*: “Thermal and Nonthermal Emission”, “H II Regions”, “Free-Free Radio Emission from H II Regions”.
- **Deepen:** TRA Ch. 10 (continuous emission); TRA Ch. 11 (thermal sources, H II regions).