

The Radio Sky

Lecture 4

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PHY-236.3



Radio continuum comes from accelerated electrons

Larmor formula (non-relativistic charge q , acceleration a):

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}$$

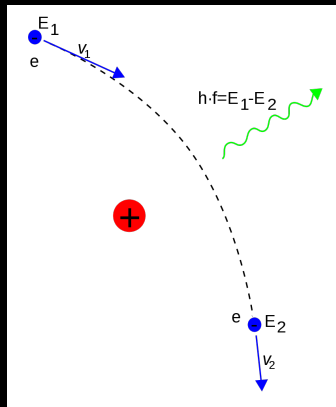
(SI); $P = 2q^2 a^2 / 3c^3$ in cgs.

- Dipole pattern $\propto \sin^2 \Theta$ about $\mathbf{a}$.
- Spectrum: the Fourier transform of $a(t)$. An acceleration lasting Δt radiates up to $\nu \sim 1/\Delta t$.
- $\gamma \gg 1$: emission is beamed into a cone of half-angle $\sim 1/\gamma$ along $\mathbf{v}$.

	Free-free	Synchrotron
Force	Coulomb field of an ion	Lorentz force of $\mathbf{B}$
Electrons	Maxwellian at $T_e \sim 10^4$ K; non-relativistic	power law in energy; $\gamma \sim 10^3$ – 10^4
Emission	brief, random encounters	continuous spiralling
Sources	H II regions, ionized winds, diffuse ionized gas	SNRs, radio galaxies, galaxy discs

Their observable signatures: the diagnostics table of Lecture 3. Lines: Lectures 7–9. Gyrosynchrotron and coherent emission: Lecture 6.

Free-free radiation: the physical picture



- An electron is accelerated in the Coulomb field of an ion and radiates. Its initial and final states are both unbound: **free-free**.
- **Thermal**: electrons have a Maxwellian distribution at T_e . Collisions keep it so, so LTE holds and $S_\nu = B_\nu(T_e)$.
- The ion barely moves (mass ratio $\gtrsim 1836$); electron–electron encounters have no dipole radiation.
- Radio photons ($h\nu \sim 10^{-5}$ eV) come from distant, weak encounters: $h\nu \ll k_B T_e \approx 0.9$ eV.

Free-free emissivity (cgs)

Thermal free-free emissivity of a hydrogen plasma, integrated over solid angle [Rybicki & Lightman eq. 5.14b]:

$$4\pi j_\nu = 6.8 \times 10^{-38} Z^2 n_e n_i T_e^{-1/2} e^{-h\nu/k_B T_e} \bar{g}_{\text{ff}}(\nu, T_e) \text{ erg s}^{-1} \text{ cm}^{-3} \text{ Hz}^{-1}$$

- $n_e n_i$: two-body process. For pure H, $n_i = n_e$, so $j_\nu \propto n_e^2$.
- $T_e^{-1/2}$: faster electrons spend less time near each ion.
- $e^{-h\nu/k_B T_e} \approx 1$ at radio frequencies: the spectrum would be exactly flat except for...
- ...the velocity-averaged **Gaunt factor**, a quantum-mechanical correction of order unity in the X-ray band but ≈ 5 in the radio ($h\nu \ll k_B T_e$):

$$\bar{g}_{\text{ff}} \approx \frac{\sqrt{3}}{\pi} \left[\ln \frac{(2k_B T_e)^{3/2}}{\pi e^2 \nu m_e^{1/2}} - \frac{5\gamma_E}{2} \right] \quad (\gamma_E = 0.577)$$

Z : ion charge; n_i : ion density.

Free-free absorption and optical depth

Kirchhoff's law (LTE, Lecture 3): $\kappa_\nu = j_\nu / B_\nu(T_e)$. In the RJ limit [R&L §5.3]:

$$\kappa_\nu \approx 0.018 Z^2 n_e n_i T_e^{-3/2} \nu^{-2} \bar{g}_{\text{ff}} \quad \text{cm}^{-1} \quad (\text{cgs})$$

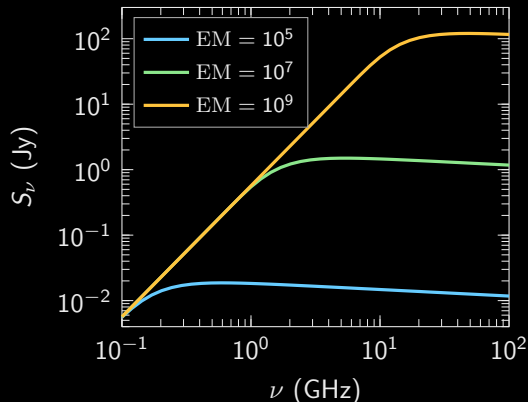
Absorbing the weak ν and T_e dependence of $\bar{g}_{\text{ff}}$ into power laws gives the widely used approximation [ERA Ch. 4; Mezger & Henderson 1967, [doi:10.1086/149030](https://doi.org/10.1086/149030)]:

$$\tau_\nu \approx 3.28 \times 10^{-7} \left(\frac{T_e}{10^4 \text{ K}} \right)^{-1.35} \left(\frac{\nu}{\text{GHz}} \right)^{-2.1} \left(\frac{\text{EM}}{\text{pc cm}^{-6}} \right)$$

Emission measure $\text{EM} \equiv \int n_e^2 dl$: the column of n_e^2 along the line of sight.

- An *approximation* to the Gaunt-factor result, adequate for estimates at $T_e \sim 10^4 \text{ K}$ and GHz frequencies; not exact.
- $T_e^{-1.35}$: cool plasma is more opaque. $\nu^{-2.1}$: opacity rises steeply to low frequency.

The free-free spectrum



$10''$ disc, $T_e = 10^4$ K; EM in pc cm^{-6} (illustrative).

Uniform source, solid angle Ω_s , no background (RJ; slab solution of Lecture 3):

$$S_\nu = \frac{2k_B T_e \nu^2}{c^2} \Omega_s (1 - e^{-\tau_\nu})$$

- Thick: $T_b \rightarrow T_e$, $\alpha = +2$, common to all three curves.
- Thin: $T_b \approx T_e \tau_\nu \propto T_e^{-0.35} \nu^{-2.1} \text{EM}$; flux level $\propto \text{EM}$.
- Turnover ($\tau_\nu = 1$): $\nu_t \approx 0.82 \text{ MHz } (T_e/10^4 \text{ K})^{-0.64} \text{EM}^{0.48}$: 0.2, 1.8, 16 GHz for $\text{EM} = 10^5, 10^7, 10^9$.

Thermal continuum: slopes and the turnover

For the uniform source of the previous slide, with $\tau_\nu \propto \nu^{-2.1}$:

$$\tau_\nu \gg 1 : S_\nu \propto \nu^2 \quad (\alpha = +2); \quad \tau_\nu \ll 1 : S_\nu \propto \nu^2 \tau_\nu \propto \nu^{-0.1} \quad (\alpha \approx -0.1)$$

- $\alpha = +2$ is the steepest rise any *homogeneous thermal* source can show: it is the RJ blackbody slope.
- **Pitfall:** because the thin slope is so shallow, the flux maximum lies at $\tau_\nu \approx 0.1$, a factor ~ 3 above the turnover ν_t ($\tau_\nu = 1$). Do not read ν_t off the peak.

Synchrotron radiation: the physical picture

- An electron in a magnetic field spirals about the field line. The Lorentz force is perpendicular to $\mathbf{v}$, so the electron is accelerated continuously and radiates.
- The regime depends on the electron's Lorentz factor γ :

Electrons	Name	Spectrum
$\gamma \approx 1$	cyclotron	a line at the gyrofrequency
$\gamma \sim \text{a few}$	gyrosynchrotron (Lecture 6)	low harmonics of the gyrofrequency
$\gamma \gg 1$	synchrotron	broad continuum, far above the gyrofrequency

- The radiating electrons are **non-thermal**: a power law in energy, typically from shock acceleration (Lecture 13). No temperature describes them, so Kirchhoff's law with a single T does not apply.
- Radiation from an ordered field is **linearly polarized**.

Gyration, beaming and the critical frequency

An electron of Lorentz factor γ in a uniform field B , pitch angle ϑ between velocity and field:

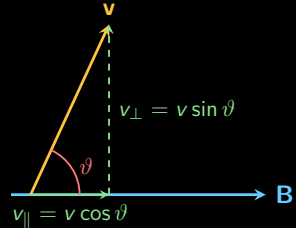
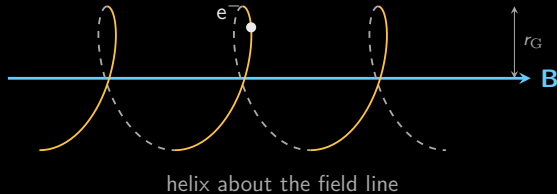
- Non-relativistic gyrofrequency: $\nu_G = \frac{eB}{2\pi m_e} = 2.80 \text{ MHz} \left(\frac{B}{G} \right)$. Relativistic gyration: ν_G/γ .
- Because of beaming, the observer sees one short pulse per orbit, compressed in time by $\sim \gamma^{-3}$.
- The spectrum extends to the **critical frequency** [Rybicki & Lightman §6.2]:

$$\nu_c = \frac{3}{2} \gamma^2 \nu_G \sin \vartheta = 4.2 \text{ MHz} \gamma^2 \left(\frac{B}{G} \right) \sin \vartheta$$

- Note the inversion: a *low* orbital frequency produces radiation at $\gamma^2 \nu_G$, far above it. This is purely a beaming and time-compression effect.

The pitch angle is written ϑ here to keep α for the spectral index.

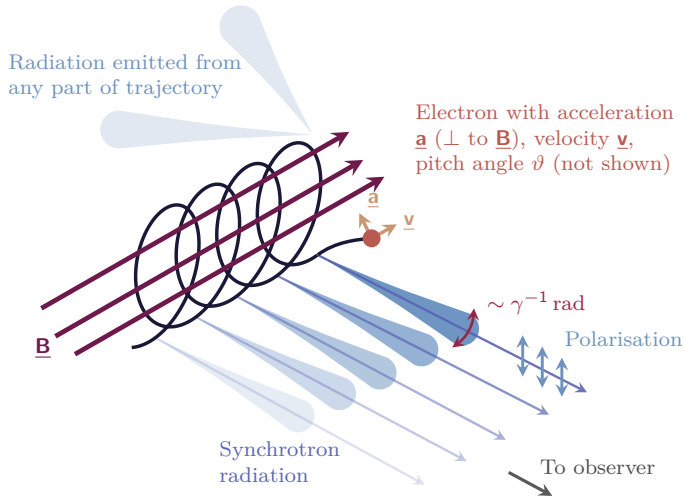
The pitch angle



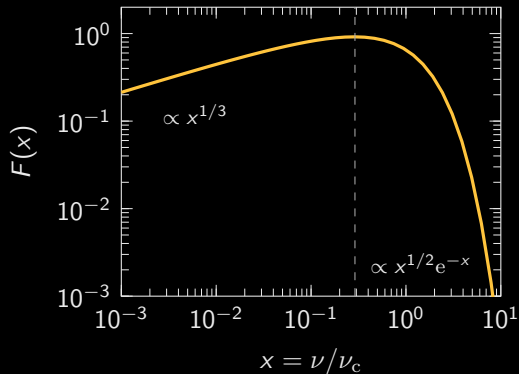
- **Pitch angle** ϑ : the angle between the electron's velocity **v** and the magnetic field **B**.
- $v_{||}$ carries the electron along the field unchanged; $v_{\perp}$ makes it circle the field. Together: a helix.
- $\vartheta = 90^\circ$: a circle; $\vartheta \rightarrow 0$: motion along the field, no gyration and no radiation.

Schematic, not to scale.

Synchrotron geometry



Single-electron spectrum and power

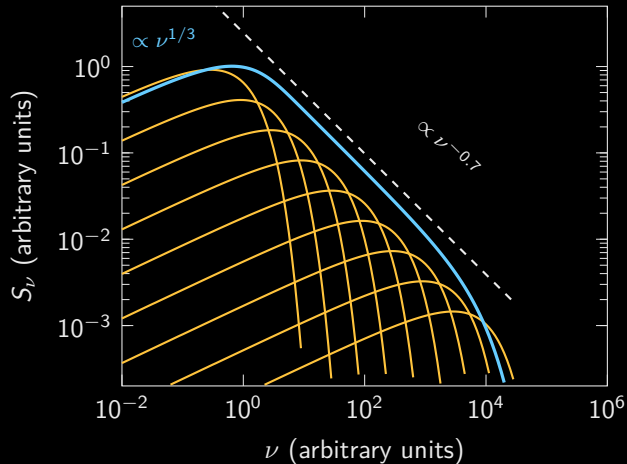


$$F(x) = x \int_x^\infty K_{5/3}(\xi) d\xi$$

- Peak at $x = 0.29$; broad: $\Delta \ln \nu \sim$ a few.
- Power (exact, SI):
 $P = 2\sigma_T c \gamma^2 \beta^2 U_B \sin^2 \vartheta$,
 $U_B = B^2/2\mu_0$.
- Isotropic pitch angles:
 $\langle P \rangle = \frac{4}{3}\sigma_T c \gamma^2 \beta^2 U_B$.

$F(x)$ computed numerically. $K_{5/3}$: modified Bessel function; σ_T : Thomson cross-section. [R&L Ch. 6]

A power law built from single-electron spectra



- **Thin curves:** single-electron spectra $F(\nu/\nu_c)$ for electron energies a factor $10^{1/4}$ apart, each weighted by $N(E) \propto E^{-2.4}$.
- **Thick curve:** their sum over a continuous distribution from $E_{\min}$ to $E_{\max} = 100 E_{\min}$.
- **Dashed:** $\nu^{-0.7}$, i.e. $\alpha = -(p-1)/2$ for $p = 2.4$.

Computed from $F(x)$; $\nu_c(E_{\min}) = 1$.

Synchrotron from a power-law population: the results

Electrons with $N(E) dE = KE^{-p} dE$ (number density per unit energy), uniform B , isotropic pitch angles [R&L Ch. 6]. K : normalization; electron density $n_e \approx KE_{\min}^{1-p}/(p-1)$ for $p > 1$ (not the Bessel function $K_{5/3}$).

Quantity	Result
Thin emissivity	$j_\nu \propto K B^{(p+1)/2} \nu^{-(p-1)/2}$
Thin spectral index	$\alpha = -(p-1)/2$; typically $p \approx 2-3$ ($\alpha \approx -0.5$ to -1)
Self-absorbed (thick)	$S_\nu \propto \nu^{5/2}$, independent of p (Lecture 6)
Polarization (uniform $\mathbf{B}$)	$\Pi_{\max} = (p+1)/(p+7/3)$
Outside the power law	$S_\nu \propto \nu^{1/3}$ below $\nu_c(E_{\min})$; exponential cutoff above $\nu_c(E_{\max})$

- Each electron radiates near its own $\nu_c \propto E^2 B$, so the electron power law maps onto a frequency power law.
- j_ν depends on the product $K B^{(p+1)/2}$, not on K and B separately.

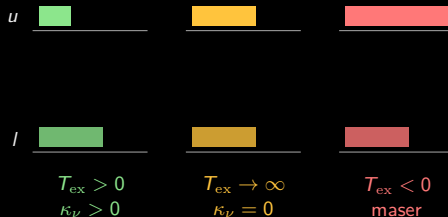
Excitation temperature: what it means

Definition. For a transition between levels u (upper) and l (lower), T_{ex} is the temperature that fits the level populations to the Boltzmann law:

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-h\nu/k_B T_{\text{ex}}}$$

g_u, g_l : **statistical weights** (number of sub-states per level), $g = 2J + 1$; H I hyperfine: $g = 2F + 1$, $g_u = 3$, $g_l = 1$. Then $j_\nu/\kappa_\nu = B_\nu(T_{\text{ex}})$.

- One transition, not the gas.
- Radiation pulls T_{ex} towards the radiation temperature, collisions towards T_k (LTE: $T_{\text{ex}} = T_k$ for $n \gtrsim n_{\text{crit}}$).
- Not a physical temperature: can exceed T_k , be infinite, or negative.



Bar length: population per state, n/g (illustrative). A maser needs $n_u/g_u > n_l/g_l$.

Three temperatures that are easily confused

Temperature	Defined by	Equals the others when
Kinetic T_k (or T_e)	Maxwellian particle velocities	— (a property of the gas)
Excitation T_{ex}	Level populations (Boltzmann law, previous slide)	collisions dominate (LTE): $T_{\text{ex}} = T_k$
Brightness T_b	Measured intensity: $T_b = c^2 I_\nu / 2k_B \nu^2$	$\tau_\nu \gg 1$, isothermal, no background, RJ: $T_b = T_{\text{ex}}$

- Free-free: electrons are Maxwellian, so $T_{\text{ex}} = T_e$ (LTE; Kirchhoff's law, Lecture 3).
- Lines: $T_{\text{ex}} < T_k$ below the critical density; masers in Lecture 9.
- Synchrotron: electrons are non-thermal; no T_k exists. An *effective* temperature still limits T_b (Lecture 6).

Reading

- **Review:** ERA Ch. 4, *Free-Free Radiation*, “Thermal and Nonthermal Emission”; ERA Ch. 5, *Synchrotron Radiation*, “Magnetobremssstrahlung”, “Synchrotron Power” and “Synchrotron Spectra”.
- **Deepen:** TRA Ch. 10, *Emission Mechanisms of Continuous Radiation* (free-free and synchrotron sections); Rybicki & Lightman, synchrotron chapter: single-particle emission, power-law spectra and polarization.