

The Radio Sky

Lecture 3

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Size limits from variability

A source cannot vary coherently on a time-scale Δt shorter than its light-crossing time, so its radius $R \lesssim c \Delta t$.

$$\Omega_s \approx \frac{\pi R^2}{D^2} \quad \Rightarrow \quad T_b \gtrsim \frac{S_\nu D^2}{2\pi k_B \nu^2 (\Delta t)^2}.$$

- **Causality argument:** gives size without resolving the source.
- Assumes no relativistic bulk motion. Doppler factor δ relaxes the limit as $R \lesssim c \delta \Delta t$, and intrinsic T_b falls by a power of δ (the exact power depends on geometry).
- Assumes Δt is intrinsic, not caused by propagation (scintillation can mimic variability).

Worked example: a millisecond burst

An FRB: $S_\nu = 1 \text{ Jy}$ at $\nu = 1.4 \text{ GHz}$, duration $\Delta t = 1 \text{ ms}$, distance $D = 1 \text{ Gpc}$ (illustrative values).

$$S_\nu D^2 = 10^{-26} \times (3.09 \times 10^{25})^2 = 9.5 \times 10^{24} \text{ W Hz}^{-1}$$

$$2\pi k_B \nu^2 (\Delta t)^2 = 2\pi (1.38 \times 10^{-23}) (1.4 \times 10^9)^2 (10^{-3})^2 = 1.7 \times 10^{-10} \text{ J K}^{-1}$$

$$T_b \gtrsim 6 \times 10^{34} \text{ K}$$

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- 22 orders of magnitude above the incoherent limit.
- No plausible Doppler factor bridges that gap: **the emission must be coherent.**

Concept test

Question

An unresolved radio burst has an inferred brightness temperature far above 10^{12} K. Which broad emission mechanism is immediately favoured?

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Answer: coherent emission

1. Incoherent synchrotron sources are limited near 10^{12} K by inverse-Compton losses.
2. Coherent emission adds amplitudes, not intensities, so T_b can far exceed any particle energy.

Spectral index and optical depth

Spectral index, convention for this course:

$$S_\nu \propto \nu^\alpha, \quad \alpha = \frac{\ln(S_1/S_2)}{\ln(\nu_1/\nu_2)}$$

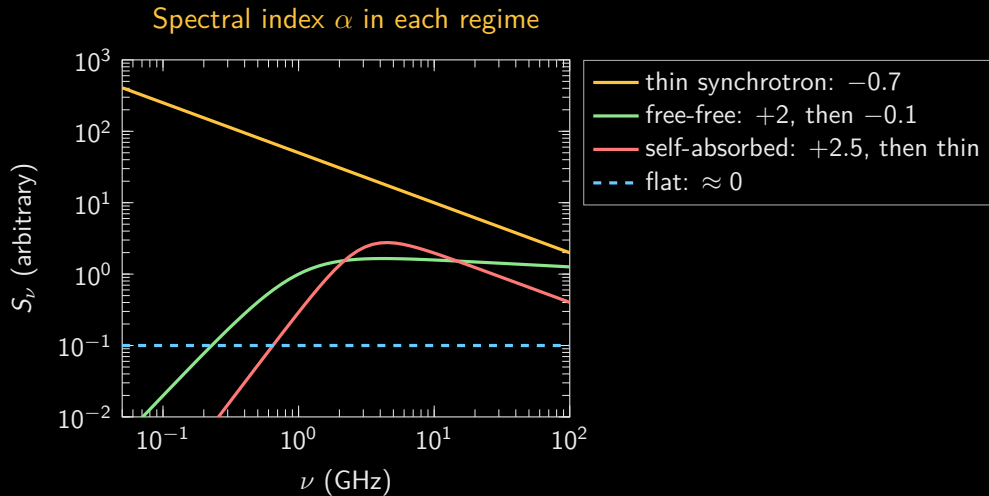
- Measured from two or more bands.
- Some literature uses $S_\nu \propto \nu^{-\alpha}$: **check the sign convention.**

Optical depth along a path:

$$\tau_\nu = \int \kappa_\nu ds$$

- κ_ν : absorption coefficient (m^{-1}), the fraction of intensity absorbed per unit path length ds .
- $\tau_\nu \ll 1$: every emitter is seen; emission adds.
- $\tau_\nu \gg 1$: only the surface layer is seen.
- The value of α often reveals which regime applies (Lectures 4–5).

Spectral shapes: first-look reference



Schematic curves with arbitrary normalization; derived in Lectures 4–6.

Four axes for classifying radio emission

Axis	One side	Other side
Spectral form	Continuum : broadband	Line : discrete transition
Particle energy	Thermal : Maxwellian, $T_b \leq T_k$	Non-thermal : power-law, relativistic
Phase relation	Incoherent : intensities add	Coherent : amplitudes add, $T_b \gg$ particle energy
Time behaviour	Steady	Variable / transient

- The axes are independent: a maser is a *coherent line*; a pulsar is *coherent continuum*.
- Galactic versus extragalactic is a further, distance-based split.

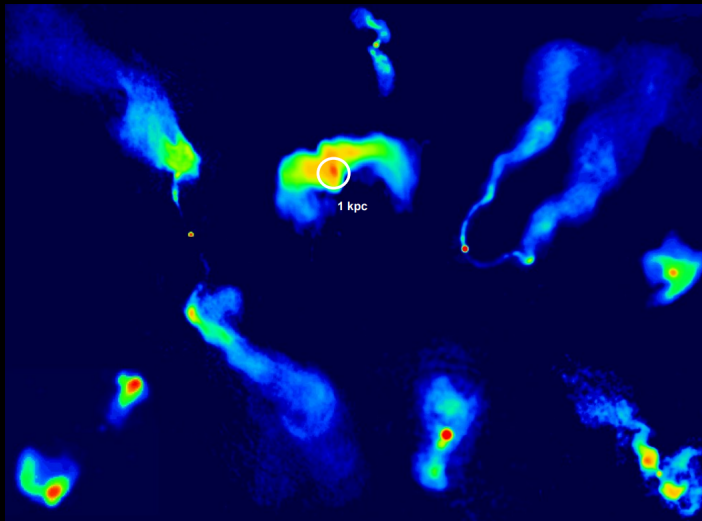
Diagnostics that separate the classes

	Thermal free-free	Synchrotron	Coherent
Thin α	≈ -0.1	≈ -0.5 to -1	varied; often narrow-band
Max T_b	$T_k \sim 10^4$ K	$\sim 10^{12}$ K	$\gg 10^{12}$ K
Linear pol.	≈ 0	up to $\sim 70\%$	varied
Circular pol.	≈ 0	small	often high
Variability	slow	months–years (compact)	ms–s

No single column is decisive

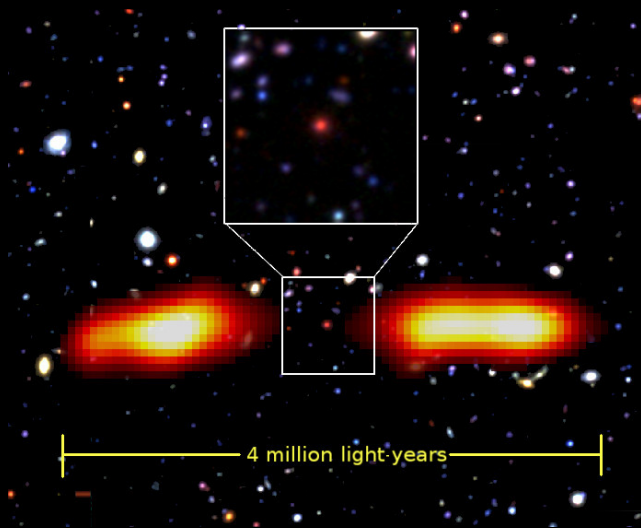
Depolarized synchrotron looks unpolarized; self-absorbed synchrotron is not steep.
Combine observables before naming a mechanism.

Source-recognition gallery: radio galaxies

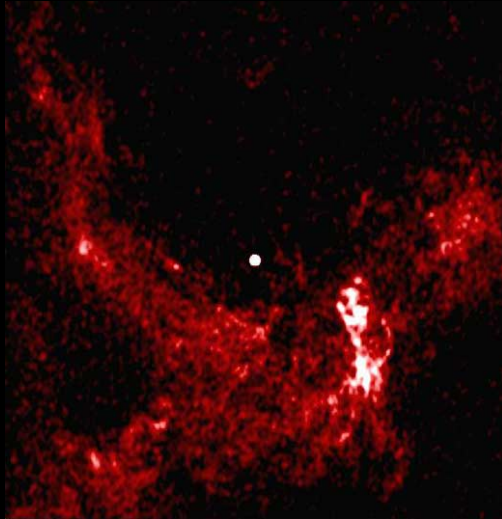


Credit: Radio images of radio galaxies, 1 kpc scale marked.

Source-recognition gallery: a giant radio galaxy with the GMRT



Source-recognition gallery: the Galactic Centre



Credit: Radio image of Sgr A* and surroundings.

Source-recognition gallery: what to ask of each image

1. What is plotted: intensity, polarization, velocity, or time?
2. Is the source resolved?
3. What frequency, and what spectral index?
4. Is there a counterpart at other wavelengths?

Classes in the course

H II region · shell SNR · H I velocity field ·
pulsar dynamic spectrum · radio galaxy ·
molecular cube · FRB waterfall

Worked example: three brightness temperatures

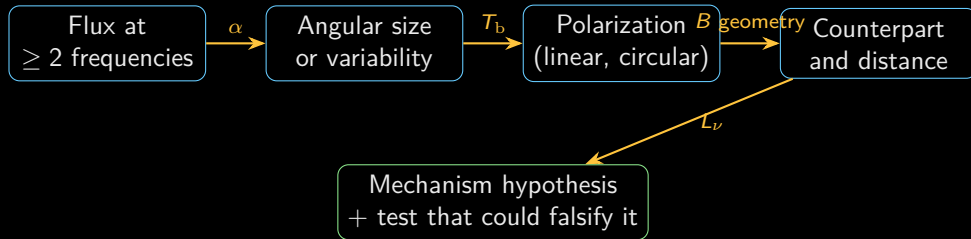
Source (illustrative)	S_ν	ν	Size	T_b
H II region	1 Jy	5 GHz	$30'' \times 30''$	≈ 54 K
Radio-galaxy lobe	100 mJy	1.4 GHz	$20'' \times 10''$	≈ 310 K
VLBI core	100 mJy	5 GHz	$1 \text{ mas} \times 1 \text{ mas}$	$\approx 5 \times 10^9$ K

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- H II region: $T_b \ll T_e \sim 10^4$ K $\Rightarrow$ optically thin at 5 GHz, not cold.
- Lobe: $T_b \sim 300$ K, far below the energies of radiating electrons: an optically thin synchrotron source, not a 300 K gas.
- Core: $T_b \sim 10^9$ – 10^{10} K excludes thermal emission; consistent with self-absorbed synchrotron.

Classifying an unknown source: the first observations



- Each step turns an **observable** into an **inference**.
- State explicitly which step carries the largest uncertainty.

End of topic question and reading

End of topic question

Name one observable that distinguishes coherent from incoherent emission, and state one way it can mislead.

- **Review:** ERA Ch. 1, *Introduction* (“The Discovery of Cosmic Radio Noise”, “A Tour of the Radio Universe”); ERA Ch. 2, “Brightness and Flux Density”.
- **Deepen:** TRA Ch. 1, *Radio Astronomical Fundamentals*.
- **Optional:** IRA introductory overview; Kraus on units and source taxonomy.
- **Online:** Condon & Ransom, *Essential Radio Astronomy*,
<https://www.cv.nrao.edu/~sransom/web/xxx.html>

After class

Compute T_b for a compact pulsar, an HII region and a radio-galaxy lobe; state what can and cannot be concluded in each case.

Central question

How do optical depth and source function shape a radio spectrum?

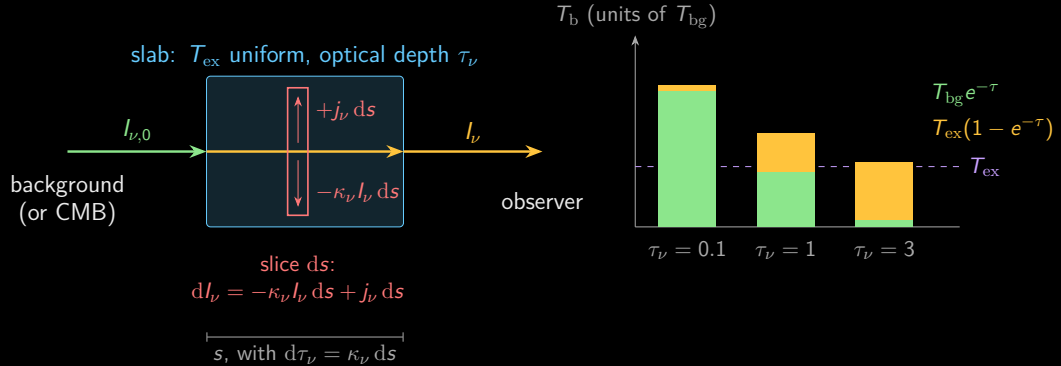
This topic (Lectures 3–5)

- Transfer equation in temperature units
- Kinetic, excitation and brightness temperatures
- Spectral signatures of thin, thick and absorbed sources
- Why a flux density is not a temperature

Units and conventions

- SI throughout; janskys for flux density
- $S_\nu \propto \nu^\alpha$ (course convention)
- Single-frequency results are *reviews* of Radiative Processes; the new material is their use as radio diagnostics

Picture of the transfer equation



Each layer removes a fraction of what passes through and adds its own emission: thin $\rightarrow$ background survives; thick $\rightarrow$ only T_{ex} is seen. *Illustrative values:* $T_{\text{ex}} = 0.4 T_{\text{bg}}$.

The transfer equation: review

Along a ray with path length s :

$$\frac{dl_\nu}{ds} = -\kappa_\nu l_\nu + j_\nu, \quad d\tau_\nu \equiv \kappa_\nu ds, \quad \frac{j_\nu}{\kappa_\nu} \equiv B_\nu(T_{\text{ex}}).$$

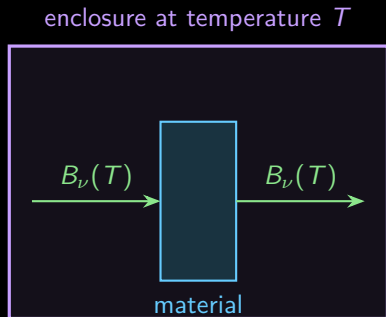
- j_ν : emission coefficient ($\text{W m}^{-3} \text{Hz}^{-1} \text{sr}^{-1}$); κ_ν : absorption coefficient (m^{-1}), *net* of stimulated emission; j_ν/κ_ν : source function, written as a Planck function $B_\nu(T_{\text{ex}})$ (this *defines* T_{ex}).

For a uniform slab (T_{ex} constant) with background intensity $l_{\nu,0}$:

$$l_\nu = l_{\nu,0} e^{-\tau_\nu} + B_\nu(T_{\text{ex}}) (1 - e^{-\tau_\nu})$$

- Exact for constant T_{ex} . Two terms: attenuated background plus self-absorbed foreground emission.
- In LTE, $T_{\text{ex}} = T_{\text{k}}$ (Kirchhoff's law, next slide); out of LTE they differ.

Kirchhoff's law: emission and absorption are linked



$$dI_\nu/ds = 0 \Rightarrow j_\nu = \kappa_\nu B_\nu(T)$$

- Thermodynamic equilibrium: $I_\nu = B_\nu(T)$ everywhere, inside the material too, so $dI_\nu/ds = -\kappa_\nu B_\nu + j_\nu = 0$:

$$j_\nu = \kappa_\nu B_\nu(T)$$

- j_ν , κ_ν depend only on the local state of the matter. Particles in equilibrium at T (**LTE**): the law holds even if the radiation is *not* blackbody.
- **A good absorber is a good emitter**: a medium cannot emit at ν without absorbing at ν .

Kirchhoff 1860; R&L §1.5. Microscopically, the same result follows from the Einstein relations (Lecture 7).

What Kirchhoff's law gives a radio astronomer

Consequences (LTE)

- **Ceiling:** $I_\nu \leq B_\nu(T)$, so $T_b \leq T$ for any thermal emitter without a brighter background.
- **Thick limit:** $\tau_\nu \gg 1$ gives $I_\nu \rightarrow B_\nu(T)$ whatever j_ν and κ_ν are individually.
- **Opacity from emissivity:** compute j_ν , then $\kappa_\nu = j_\nu / B_\nu(T)$. Used for free-free absorption in Lecture 4.

What LTE requires, and when it fails

- Requires: Maxwellian particles and level populations set by collisions. It does *not* require blackbody radiation.
- Free-free: electrons Maxwellian, so it holds with $T = T_e$.
- Lines below n_{crit} : $j_\nu / \kappa_\nu = B_\nu(T_{\text{ex}})$ with $T_{\text{ex}} \neq T_k$.
- Synchrotron: no single T ; j_ν / κ_ν rises as $\nu^{5/2}$ (Lecture 6).
- Masers: inverted populations, $\kappa_\nu < 0$ (Lecture 9).

Rewrite in brightness-temperature form

In the Rayleigh–Jeans limit every intensity is linear in a temperature: $I_\nu = 2k_B\nu^2 T_b/c^2$, $B_\nu(T_{\text{ex}}) = 2k_B\nu^2 T_{\text{ex}}/c^2$, $I_{\nu,0} = 2k_B\nu^2 T_{\text{bg}}/c^2$. Divide through by $2k_B\nu^2/c^2$:

$$T_b = T_{\text{bg}} e^{-\tau_\nu} + T_{\text{ex}} (1 - e^{-\tau_\nu})$$

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$$T_b = T_{\text{bg}} e^{-\tau_\nu} + T_{\text{ex}} (1 - e^{-\tau_\nu})$$

Subtract the background (what an on–off or continuum-subtracted measurement gives):

$$\Delta T_b \equiv T_b - T_{\text{bg}} = (T_{\text{ex}} - T_{\text{bg}}) (1 - e^{-\tau_\nu})$$

- **Emission** if $T_{\text{ex}} > T_{\text{bg}}$; **absorption** if $T_{\text{ex}} < T_{\text{bg}}$. The sign is set by the temperature contrast, not by the gas alone.
- Outside Rayleigh–Jeans, replace each T by $J_\nu(T) = (h\nu/k_B)/(e^{h\nu/k_B T} - 1)$.

When is Rayleigh–Jeans safe?

Ratio of the true Planck intensity to its RJ form:

$$\frac{B_\nu(T)}{2k_B T \nu^2 / c^2} = \frac{x}{e^x - 1},$$
$$x = \frac{h\nu}{k_B T} = \frac{0.048 (\nu/\text{GHz})}{T/\text{K}}.$$

ν	T	$x/(e^x - 1)$
1.4 GHz	10 K	0.997
30 GHz	10 K	0.93
230 GHz	10 K	0.55
100 GHz	2.725 K	0.37

- cm-wave continuum of 10^4 K gas: RJ is excellent.
- Cold molecular gas at mm wavelengths (CO 2–1 at 230 GHz): RJ underestimates the needed temperature by nearly a factor of 2.
- The CMB at 100 GHz is far from RJ.

Convention

T_b is always *defined* with the RJ factor (Lecture 2), even when RJ fails. Then $T_b \neq T$ for a blackbody.