Galaxies: Structure, formation and evolution
Lecture 4

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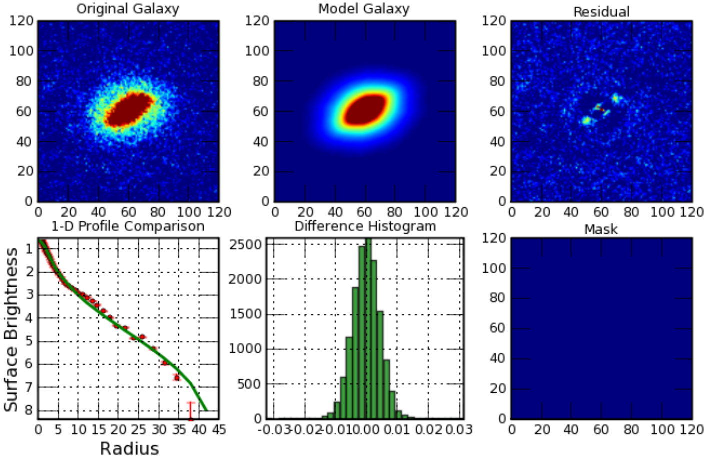
Mar-Apr 2022

Why do we need quantitative morphological classification?

Modern CCD imaging surveys generate vast numbers of galaxy images. Too many to classify by eye, even with citizen science projects like Galaxy Zoo (Lintott et al. 2008).
There is a need for fast, objective, robust classification.

Quantitative morphology: bulge-disk decomposition



Wadadekar et al. (1999)

χ^2 minimisation

$$\chi^2 = \sum \frac{(I_i - I_0)^2}{\sigma_i^2}$$

where I_i, I_0 and σ_i are the observed flux, model flux and the error in I_i respectively. The **error** depends on the signal to noise ratio of the data. Exactly how this is to be computed should be covered in your Astronomical Techniques I course.

Analytics model for the disk

Exponential Disk:

$$\begin{aligned} I_{disk}(x,y) &= I_s e^{-r_{disk}/r_s}, \\ r_{disk} &= \sqrt{x^2 + y^2/(1-e_d)^2}. \\ e_d &= 1 - \cos(i), \end{aligned}$$

r_{disk} is the galactocentric radius [putting it more accurately, the length of the semi-major axis of the (elliptical!) isophote], r_d the length scale of the disk and I_S refers to intensity at the centre and i is the inclination angle.

Inner and outer disk - double exponentials

It is possible to fit two different disks (inner and outer disks) that share the same position angle and ellipticity, but have different central brightness and length scales. This ability tends to become very useful since galaxies with double exponential disks are being now frequently found with ever deeper and finer images.

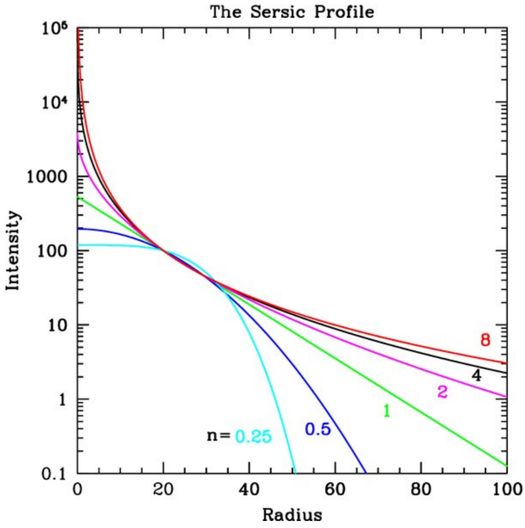
Analytic model for the bulge

Sérsic Bulge:

$$\begin{aligned} I_{bulge}(x,y) &= I_e e^{-b_n[(r_{bulge}/r_e)^{1/n}-1]}, \\ r_{bulge} &= \sqrt{x^2 + y^2/(1-e_b)^2}, \end{aligned}$$

Where e refers to effective values and n is the Sérsic index. For n=4 it becomes the **de Vaucouleurs function**; for n=1 an exponential, and when n=0.5, a Gaussian! For values in the range 1-4, approximately, it describes bulges in late-type spiral galaxies (or pseudo-bulges) to bulges in early-type spirals (or classical bulges) and elliptical galaxies. It is easy to realize that the larger the value of n the more concentrated is the light (and mass!) of the bulge (or elliptical) in the center.
The effective radius of a galaxy is the one that contains half of the total light emitted by the galaxy. The numerical constants b_n is chosen so that the brightness at the effective radius is the effective brightness, and depends only on n.

The Sérsic profile



Other equivalent forms of the Sérsic function

$$
\begin{aligned}
I_{bulge}(x,y) &= I_0 10^{-c_n[(r_{bulge}/r_e)^{1/n}]} \\
I_{bulge}(x,y) &= I_e 10^{-d_n[(r_{bulge}/r_e)^{1/n}-1]}
\end{aligned}
$$

Derived quantitities - the B/T flux/luminosity ratio

$$B/T = \frac{f_b}{f_b + f_d} \tag{1}$$

where f_b and f_d are the total flux enclosed by the bulge and disk components. Using the structural parameters involved in the Sérsic profile the total flux of the bulge can be found analytically using the following formula

$$\begin{aligned} f_b &= \int_0^\infty 2\pi r I(r) dr \\ &= \frac{2\pi \exp(b_n)}{b_n^{2n}} I_e n r_e^2 \Gamma(2n) \end{aligned} \tag{2}$$

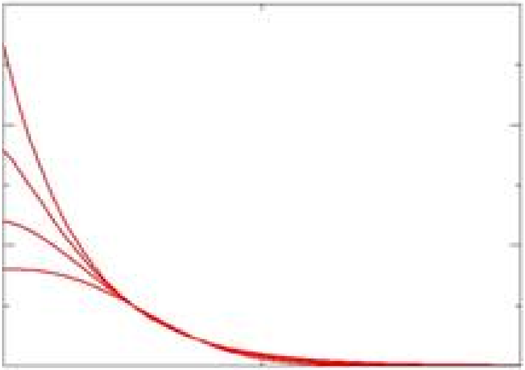
Similarly the total light enclosed by disk part is

$$f_d = 11.948 I_d r_d^2 \tag{3}$$

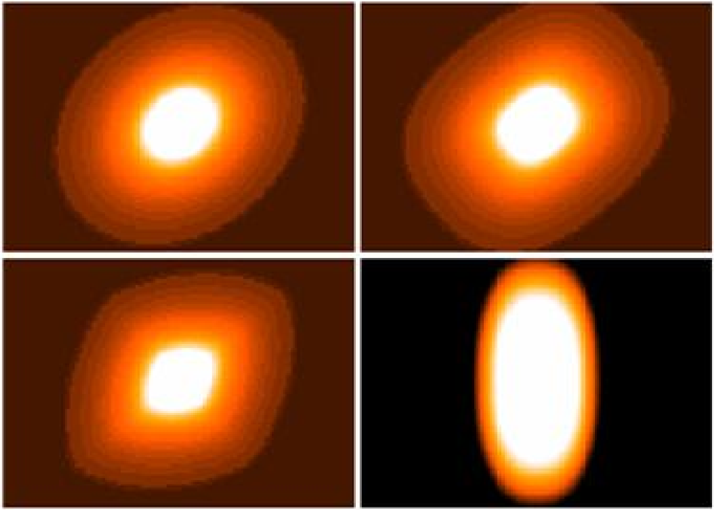
Fitting the bar

The Sérsic function can also be used to describe bars,with $0.4 < n < 1$. A bar in a late-type galaxy can be well fitted by an exponential ($n = 1$), whereas bars in early-type galaxies have a flatter luminosity profile ($n = 0.6$, say). The plot on the next page shows the Sérsic function for n=0.4, 0.6, 0.8 and 1 (upwards).
Bars are hard to fit; you need to use **boxy isophotes**, generally with a somewhat high ellipticity. The effective radius of the Sérsic function describing the bar must also be carefully fitted. In many cases too, you will need an outer cutoff radius.

Low Sérsic index profiles



Boxy and disky isophotes



Mathematical expression for boxiness and diskiness

$$r = \left(|x|^{c+2} + \left| \frac{y}{q} \right|^{c+2} \right)^{1/(c+2)}$$

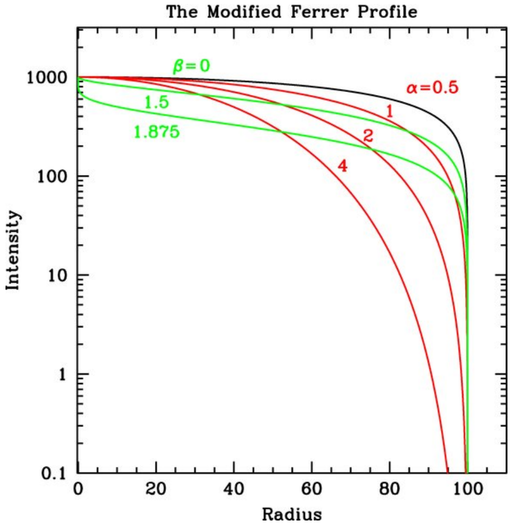
where $q = b/a = 1 - e_{bar}$ and a pure ellipse has $c = 0$. For $c > 0$ one has a boxy ellipse, or isophote. In this case there is a deficit of light in the directions of the major and minor axes. For $c < 0$ one gets a disky isophote, where there is an excess of light in the directions of the major and minor axes.

Ferrer function for bars

$$\Sigma(r) = \Sigma_0(1 - (r/r_{out})^{2-\beta})^{\alpha}$$

which is only defined for $r \leq r_{out}$.The sharpness of the truncation is governed by the parameter α, whereas the central slope is controlled by the parameter β. It is well approximated by a Sérsic function with $n \leq 0.5$

Ferrer profile

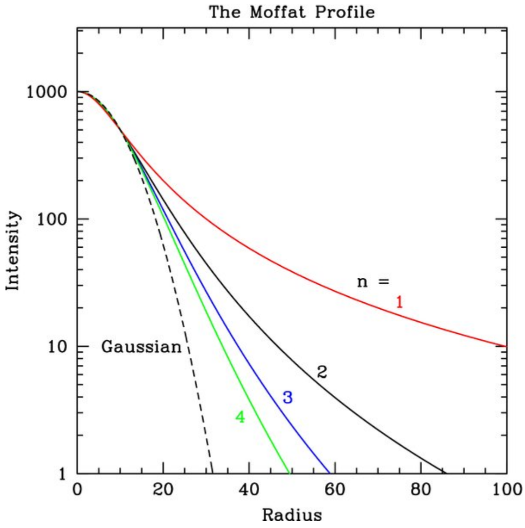


The Moffat function for nuclear source and PSF

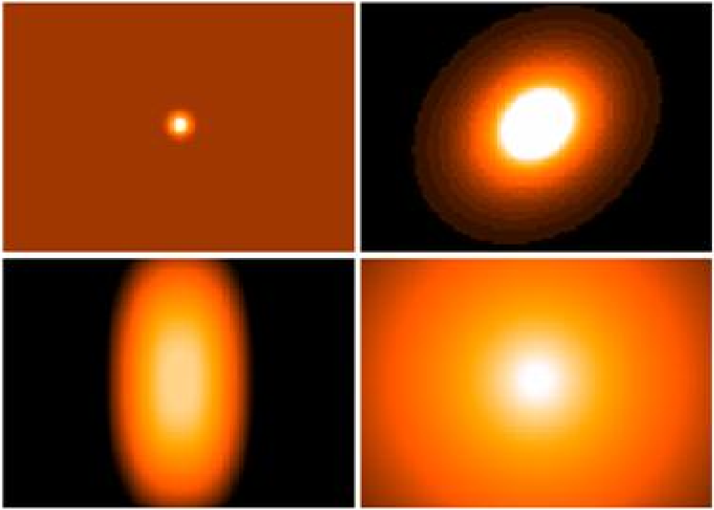
$$\Sigma(r) = \frac{\Sigma_0}{[1 + (r/r_d)^2]^n}$$

r_d is related to FWHM and n=4.765 (for ground based observations)Why?. For $n \to \infty$ it becomes a Gaussian.
If your galaxy has a central source and you don't include it in the fit you can get a wrong (too large!) value for the Sérsic index of the bulge.
Various other degeneracies exist and quantitative morphology remains a black art.

Fitting a point source and convolution



The complete fit - point source, bulge, disk

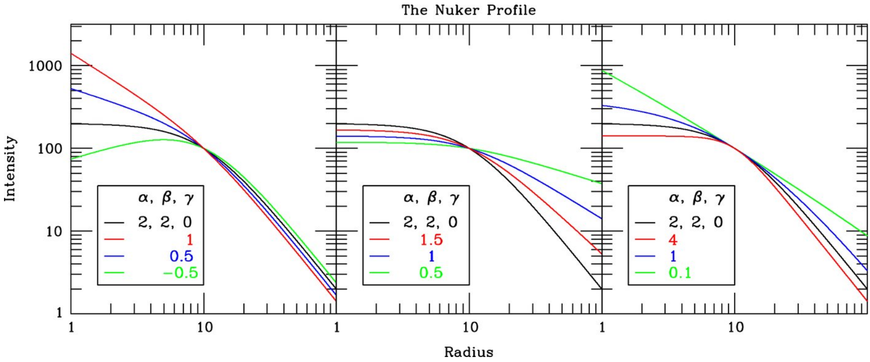


The Nuker profile

$$I(r) = I_b 2^{\frac{\beta-\gamma}{\alpha}} \left(\frac{r}{r_b}\right)^{-\gamma} \left[1 + \left(\frac{r}{r_b}\right)^{\alpha}\right]^{\frac{\gamma-\beta}{\alpha}}$$

Here β is the outer power-law slope, γ is the inner slope, and α controls the sharpness of the transition. The motivation for using this profile is that the nuclei of many galaxies appear to be fit well in 1D (see Lauer et al. 1995) by a double power law.

Nuker profile



Fitting spiral arms - why it is usually not a good idea

Why it is mostly not necessary.

Spiral arm profiles