Galaxies: Structure, Formation and Evolution
Assignment 2

Due: 1700 hours, May 4, 2022 (*Total: 300 Points, Weightage : 30%*)

General Instructions

(1) You should try to obtain solutions to these questions by yourself. If you are really stuck, you may discuss with your colleagues but the final solution should be obtained by you. Any suspicion of plagiarism will warrant deduction of points (2) The deadline for submission of the answers is fixed. No credit for late submissions. (3) The number of points allotted for each question is given in square brackets at the end of the question. (4) Submit your solutions to Yash Bhusare before the deadline.

1. a) An astronomer performs optical spectroscopy on a faint galaxy. The spectrum shows a strong continuum with absorption lines and some emission lines superimposed. What is the morphological type of the galaxy? Justify your answer.

   b) The astronomer observes a second galaxy and finds very strong emission lines super-imposed on a continuum and some absorption lines. How does the star formation rate of this second galaxy compare with that in the first? What morphological type might the second galaxy be? Again, justify your answer. [10 points]

2. Observations of a part of the interstellar medium of the Galaxy show that a region of hot ionised gas (with a temperature 500,000 K, number density of ions 6000 m^{-3}), a region of cold neutral gas (temperature 50 K, number density of molecules $2 \times 10^7 m^{-3}$), and a region of warm neutral gas (temperature 10,000 K, number density of atoms $1 \times 10^5 m^{-3}$) are in contact with each other. Which, if any, of these are in pressure equilibrium with the others? [20 points]

3. A star lying in the Galactic plane is observed to have a visual magnitude of $V = 13.60$ mag and a colour index $(B-V) = 0.98$ mag. Its spectrum shows it to be a dwarf star of spectral type G6 with a solar composition. Stars of this type are known to have an intrinsic colour of $(B-V)_0 = 0.76$ mag and an absolute visual magnitude of $M_V = +5.20$. What is the extinction by the interstellar medium in the V band between us and the star? What is the distance of the star? What is the mean extinction per unit distance in the direction of the star expressed in mag kpc^{-1} for the V band? Will this extinction per unit distance be the same for other stars in the sky? What would you expect the extinction to be towards the star in the I and K photometric bands (which have central wavelengths of 790 nm and 2.2 μm respectively)? (You may refer to the book by Henden & Kaitchuck titled *Astronomical Photometry* for understanding color indices and the concept of extinction.) [35 points]

4. For gravitational lensing, for very distant sources (i.e., $D_S \gg D_L$), we can write the expression for the Einstein angular radius as

   $$\theta_E = k\sqrt{M/D_L}$$

   where k is a constant. Find the value of k in arcsec if M is measured in solar masses and D_L in parsecs. [25 points]

5. A weakly-interacting massive particle (WIMP) with a mass of 1000 m_p , where $m_p = 1.6726 \times 10^{-27} kg$ is the mass of the proton, lenses the light of a star in the Large Magellanic Cloud, which

is situated 50 kpc from the Earth. Calculate the Einstein angular radius of the WIMP if it lies at a distance 20 kpc from the Earth. How does this figure compare with the angular radius of the star if it has the same radius, 6.96×10^8 m, as the Sun? Will the microlensing effect of the WIMP be noticeable? Are dark matter microlensing surveys sensitive to the lensing of stars by WIMPs? What is the Einstein angular radius of a brown dwarf with a mass of $0.05 M_\odot$ at the same location as the WIMP? Will the lensing effect of the brown dwarf on the background star be noticeable if there is a suitable alignment? [25 points]

6. Ionized gas in a galaxy is observed to be moving at a speed of $10,000 kms^{-1}$ at a radius of 2 pc. Assuming that the gas is on a circular orbit, estimate the mass enclosed inside this radius. Why is this massive central object considered to be a black hole rather than a massive compact stellar cluster? [25 points]

7. Background: The spiral galaxy M31 is at a distance of 770 kpc and has an inclination of 77 degrees. Assume that it has a flat rotation curve of $v_{rot} = 250 kms^{-1}$ out to a radius of $r_{max} = 30\ kpc$. Assume also that the galaxy has a dark halo made up of MACHOs (MAssive Compact Halo Objects), and that the halo is spherically symmetric and obeys a $1/r^2$ density law out to r_{max}. Recall that the total mass of any spiral galaxy is related to its rotational velocity by

$$M = \alpha \frac{v_{rot}^2 r}{G}$$

where α is of the order of unity. (This is obvious from the virial theorem, as well.)

1. Show that any object projected within the Einstein ring of a Schwartzschild lens is amplified by at least 34%.

2. For microlensing to work (or, at least obey the amplification equations) a source must fit entirely within the Einstein ring of the lensing object. For MACHOs in M31's halo, what is the minimum mass that will microlens a bright main-sequence O-star, which has $\ln T_{eff} = 4.6$ and $\log L = 5.8$? What is the mass needed to microlens a red M0 supergiant ($\log T_{eff} = 3.5, logL = 4.7$)? Suppose MACHOs in the halo of our own Galaxy ($\sim$ 30 kpc distant) were to microlens stars in M31. What is the minimum mass MACHO that can be probed with this type of experiment?

3. If the MACHO's in M31's halo are moving at 250 km s^{-1} , how long will a microlensing event last on a red supergiant, and a main sequence O-star, for a $1 M_\odot$ lens? For a $10^{-5} M_\odot$ lens?

4. The "optical depth" to lensing is defined as the probability of an object falling within the Einstein ring of some lens and being magnified by at least 34%. Suppose M31's dark halo is composed of 1 $M_\odot$ neutron stars. What is the probability of an object in M31's plane being microlensed by some M31 MACHO positioned along the line of sight? (In this case, since the probabilities involved are small, the total probability is simply proportional to the total area covered by all MACHOs along the line-of-sight.) How does this probability change with position along the minor axis? How does the probability change if the lensing objects are 10^{-5} $M_\odot$ brown dwarfs? Note that since we are considering only lensing in M31's halo, the distances to the source and the lens are about equal. [50 points]

8. In this problem you will derive the Fundamental Plane of early-type galaxies and one of its projections, the Faber–Jackson relation. Assume an elliptical galaxy with a constant and isotropic velocity dispersion σ and a constant mass-to-light ratio M/L throughout the galaxy, with no dark matter.

(a) Show that the kinetic energy $KE = 3M\sigma^2/2$. (See the last paragraph on page 121 of Sparke & Gallagher (Second edition) for a hint.) Assume that the galaxy is a uniform sphere of radius R_e . Use the result of Problem 3.11 in Sparke & Gallagher to write $PE = -3GM^2/(5R_e)$ and then use the Virial Theorem to show that $M = 5\sigma^2 R_e/G$.

(b) If all elliptical galaxies followed the Sérsic law with the same value of n, then show that their luminosities must scale as $L \propto I_e R_e^2$ and their mass-to-light ratios as $M/L \propto \sigma^2/(I_e R_e)$.

(c) Show that the Faber–Jackson relation, $L \propto \sigma^4$, only holds if M/L and I_e are constant for *all* elliptical galaxies.

(d) Show that the observed Fundamental Plane, $R_e \propto \sigma^{1.2} I_e^{-0.8}$ implies $I_e \propto R_e^{-1.25}\sigma^{1.5}$ and hence that $M/L \propto \sigma^{0.5} R_e^{0.25} \propto M^{0.25}$, so that the mass-to-light ratio increases with increasing galaxy mass.

[50 points]

9. **Chemical Evolution**:

You need to modify the "closed box" model for chemical evolution we discussed in class, by allowing a source/sink of gas; continue to assume a time-independent, metal yield y, complete and instantaneous mixing, and $Z << 1$. Let the box contain a gas mass $M_g(t)$, a stellar mass $M_s(t)$, and let gas with metallicity Z_f be be added to or ejected from the box at rate $\dot{M}_f$. Let $M_s(t=0)=0$ and $Z(t=0)=0$ (no stars or heavy elements in the primordial gas).

a) Show that the equations for the evolution of Z are

$$\dot{M}_g + \dot{M}_s = \dot{M}_f$$

$$\dot{Z}M_g - y\dot{M}_s = (Z_f - Z)\dot{M}_f$$

b) Outflow model: gas is blown out of the box at a rate which is a constant fraction η of the star formation rate $\dot{M}_s$, $\dot{M}_f = -\eta\dot{M}_s$. Show that if the gas fraction is $f_g = M_g/(M_g + M_s)$,

$$Z = \frac{y}{1+\eta}\log\left[1 + (1+\eta)\left(\frac{1}{f_g} - 1\right)\right]$$

c) Inflow model: pristine ($Z = 0$) gas flows into the box at a rate which is a constant fraction η of the star formation rate: $\dot{M}_f = +\eta\dot{M}_s$. Derive an expression analogous to the one in part (b) for Z as a function of the gas fraction f_g.

d) In the inflow model of part (c), derive an expression for the fraction of stars with metallicity less than Z at a time when the metallicity is Z_1 (the expression should involve only the variables f_g, Z, Z_1 and η). What value of η is required to solve the "G-dwarf problem" in the local solar neighborhood, i.e. give 2% of F,G dwarf stars with $Z < (1/6)Z_1$, ($Z1 = 0.03 = 1.5Z_\odot$ is the present metallicity of gas in the solar neighborhood)? Is this a good solution? Would there be a better solution if the fraction of stars with $Z < (1/6)Z_1$ were much less than 2%?

[60 points]