# Galaxies: Structure, Formation and Evolution
# Assignment 1

Due: 1700 hours, April 18th, 2022 (*Total: 300 Points, Weightage : 30%*)

**General Instructions**

*(1) You should try to obtain solutions to these questions by yourself. If you are really stuck, you may discuss with your colleagues but the final solution should be obtained by you. Any suspicion of plagiarism will warrant deduction of points (2) The deadline for submission of the answers is fixed. No credit for late submissions. (3) The number of points allotted for each question are given in square brackets at the end of the question. (4) Some of your questions involve making plots. Please attach a print-out of the most relevant plot(s) and the code you used to make the plot(s). Make sure all plots are properly labeled and use sensible units. (5) Submit your solutions to your tutor Yash Bhusare (at NCRA, Phone: 25719278) before the deadline.*

1. This problem will help you understand the difficulties involved in measuring distances. The various distance indicators we use tend to be quite noisy, and this noise can lead to biased measurements of the distances. These distance measurements often involve an estimate of the absolute magnitude of the galaxy (e.g., using the Tully-Fisher relation for spirals, for example), and the errors of that absolute magnitude estimate are Gaussian-distributed, meaning that the error in the logarithm of the inferred distance $d$ is also Gaussian-distributed. Consider, therefore, a measurement of $\ln d$ , that has an intrinsic scatter $\sigma$, ; i.e., the probability distribution of measuring a distance $d$ when the true distance is $d_0$ is given by:

$$p(\ln d \mid \ln d_0) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\,\frac{(\ln \mathrm{d} - \ln \mathrm{d_0})^2}{\sigma^2}\right)$$

   What we want, of course, is to infer the true distance, given the measured distance. While the scatter appears symmetric in the above expression, there are more galaxies at larger $d_0$ than smaller (simply because space is three-dimensional). Thus the probability distribution of $p(\ln d_0 \mid \ln d)$ (i.e., the true distance, given a measurement) is skewed. Calculate this distribution, and use this to calculate the expectation value of distance; it differs from the true distance by the Malmquist bias. The Tully-Fisher relation predicts absolute magnitudes with a 1 $\sigma$ precision of 0.3 mag. If the Malmquist bias in distances this leads to is not corrected, by what factor will the Hubble Constant based on these distance measurements be biased? Will $H_0$ be biased high or low?

   (*Hint: If you get stuck, see the paper by Lynden-Bell et al. 1988, ApJ, 326, 19*)

   [25 points]

2. The present-day luminosity function of galaxies in the universe is often fit to the *Schechter* form: the number of galaxies per unit volume with absolute magnitude M in an interval dM is:

$$\hat{\Phi}(M)dM = 0.4\ln(10)\ \phi_* 10^{-0.4(M-M_*)(\alpha+1)}\ \exp(-10^{-0.4(\mathrm{M}-\mathrm{M}_*)})\mathrm{dM}$$

   (It has this complicated form because of the use of magnitudes rather than luminosities.) At an effective wavelength of 5600 Å, the parameters of this relation at the present are found to be:

$$\phi_* = 1.49 \times 10^{-2} MPc^{-3}, \quad M_* = -20.44, \quad \alpha = -1.05$$

These quantities assume a Hubble Constant of 100 km/s/Mpc; you'll have to convert to a more correct value of $H_0$. Note that the absolute magnitude of the Sun at 5600 Å is 4.83. Integrate this up to obtain the total luminosity density of galaxies, in units of solar luminosities per cubic Megaparsec. Compare with the critical density of the Universe to ask: what would the mass to light ratio of galaxies (expressed in solar units) have to be in order to reach the critical density? To reach $\Omega_m = 0.27$ (the current concordance value?) What about $\Omega_b = .044$? Compare to what you know about the mass-to-light ratios of stars; are all the baryons in stars?

Over the years, astronomers have suggested that there is an appreciable population of galaxies at very low luminosities ($M > -15$) that have been missed by redshift surveys (perhaps they are also of very low surface brightness), and therefore are not counted in the luminosity function; could there be a lot of mass there? By what factor would you have to multiply the numbers of such low-luminosity galaxies (above that expected given the above luminosity function) in order to make an appreciable contribution to the luminosity density of the Universe? Discuss.

Galaxy rotation curves are flat, as you know, which implies an increasing mass-to-light ratio as a function of scale. To what radius would the Milky Way rotation curve have to remain flat in order to give a mass-to-light ratio high enough to reach the critical density?

Finally, compare with the mass-to-light ratios in clusters of galaxies. Read and summarize the paper on the Coma Cluster by White et al (1993, Nature, 366, 42 9), and discuss whether these results are consistent with the current standard model.

[50 points]

3. Evalute the difference between mean luminosities of (i) a magnitude limited sample and (ii) a volume-limited sample of objects with luminosity given by the following equation. Assume that the population is homogeneously distributed in space.

$$\Phi(M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(M - M_0)^2}{2\sigma^2}\right)$$

In what situations would you encounter a luminosity function (or an absolute magnitude distribution function) of the above form?

[50 points]

4. Show that, if the distribution of stars in space is uniform, the distribution of stars over absolute magnitude in a magnitude limited survey is independent of the survey's limiting magnitude.

[25 points]

5. One way of fitting an ellipse to an isophote is to perform a least square fit of the polar equation for an ellipse, $R^{-2} = C - A\cos 2\phi - B\sin 2\phi$ to the points $(R_i, \phi_i))$ that lie on the isophote. Express the semi-major axis length, $a$, the ellipticity $\epsilon$, and the position angle of the major axis $\phi_0$, in terms of A, B and C.

[25 points]

6. The surface brightness profile of an elliptical galaxy can be described by giving its effective radius ($r_e$ ) and surface brightness at $r_e$ via log or it can be given simply as

$$\log\left(\frac{I}{I_e}\right) = -3.33071\left(\left(\frac{r}{r_e}\right)^{1/4} - 1\right)$$

or it can be simply given as $m = a + br^{1/4}$, where $m$ is the surface brightness in magnitudes. How are $r_e$ and $I_e$ related to a and b? (Remember that $I_e$ is related to $m_e$ through an arbitrary constant, C.)

Using the above equation, answer the following questions.

(a) The effective radius of the bulge of M31 is $r_e = 18$'; the surface brightness at $r_e$ is $B_e = 22.9$ mag per square arcsec. The disk of M31 has a scale length of $r_d = 30$' , and its estimated central surface brightness is B(0) = 21.7 mag per square arcsec. At what radius does the disk begin to dominate? (In other words, at what point does the disk become brighter than the bulge?)

(b) What is the total B magnitude of M31's disk?

(c) What is the total B magnitude of M31's $r^{1/4}$ -law component?

(d) According to these laws, what is the total magnitude of M31?

[50 points]

7. Vist the following URL:

   https://skyserver.sdss.org/dr14/en/proj/advanced/galaxies/clusters.aspx

   Do exercises 3, 4 & 5 and answer the questions below the exercises on this page. While the instructions ask that you use Microsoft Excel for plotting, you are strongly encouraged to use a plotting package within the programming language of your choice. While making the plots, you may encounter a few outlier points which will cause the plots to become senseless. You should manually remove these outliers from your data file to achieve reasonable results.

   [25 points]

8. This exercises requires you to use the NASA Extragalactic Database (NED) to find properties of the following 5 galaxies: NGC 720, NGC 4772, NGC 5248, SMC, and M32. You can access the NED service from the following URL:

   http://ned.ipac.caltech.edu/

   First view images of each galaxy as follows: Use the "By Name" link on the NED home page; enter the object name and submit the query. On the resulting page, scroll down to the section "INDEX for ...." and on the right hand side, click on the link "Images". This link will show you images at different wavelengths. Notice how the morphology of the galaxy changes from optical (e.g., B and R band images from 4360 to 6400 Å to infrared (e.g., J, H, K images at 1.1, 1.65, A and 2.2 $\mu m$) to radio (e.g., 20 cm or 21 cm maps). For the 5 given galaxies, look up the properties listed below from the Third Reference Catalogue of Bright Galaxies (RC3; de Vaucouleurs et al. 1991) as follows: return to the section "INDEX for ...." and click on the link 'RC3 data' and you will see a set of RC3 data entries. The 'Help' button at the top of this page explains the data entries. Present your result as a table with the following columns (1) to (7). Note that your answer must be in the units requested to get credit.

   (1) The galaxy name.

   (2) The coded revised Hubble type from RC3. [5x1=5 pts] (Example: For NGC 1300, this is ".SBT4")

   (3) The revised Hubble type in standard format. [5x3=15 pts] You can get this by decoding your answer in (2) using the conversion table in section 3.2 of the Second Reference Catalogue of Bright Galaxies (RC2, de Vaucouleurs G. et al. 1976). On the course website, under section "Extra Class Resources" I have posted a copy of this table, entitled "Orignal table from RC2", along with a simplified guideline on how to use it for converting the coded revised Hubble type in RC3 into standard format. (Example: For NGC 1300, ".SBT4" translates into SB(rs)bc)

   (4) The major axis D25 in arcminutes from RC3. [5x2=10 pts] D25 represents the diameter of the isophote where the galaxy surface brightness is 25 mag square arcsec .

   (5) (B-V )T , the total B-V color index from RC3. [5x1=5 pts] (Example: For NGC 1300, (B-V )T is 0.68.)

   (6) VGSR in km/s from RC3. [5x1=5 pts] This is the weighted mean of neutral hydrogen and optical velocities, reduced to the "Galactic standard of rest." (Example: For NGC 1300, VGSR is 1496 km/s .)

   (7) Distance D in Mpc, derived as below. [5x2=10 pts] Derive D by applying Hubble's law and assuming that the recession speed is equal to the value in (6). For M32, why is this approach not valid? (Example: For NGC 1300, D is 21.4 Mpc.)

   [Total 50 points]