Cosmology
Lecture 10
Nucleosynthesis and recombination

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Primordial nucleosynthesis

- The formation of nuclei, i.e., bound states of protons and neutrons, can occur when the temperature is lower than the nuclear binding energy $\sim$ MeV. Most of the heavy nuclei were thus synthesized during $1 - 10^2$ s.
- The most basic nucleus is hydrogen which just a proton. The next one is helium which consists of 2 protons and 2 neutrons.
- However, one cannot produce helium directly through many-body reactions like $2n + 2p \longrightarrow {}^4\text{He} + \gamma$ because the number densities are too low at these temperatures. Hence one has to rely on a chain of reactions, e.g.,

$$p + n \longrightarrow \text{D} + \gamma, \quad \text{D} + n \longrightarrow {}^3\text{H} + \gamma, \quad {}^3\text{H} + p \longrightarrow {}^4\text{He} + \gamma,$$

$$p + n \longrightarrow \text{D} + \gamma, \quad \text{D} + p \longrightarrow {}^3\text{He} + \gamma, \quad {}^3\text{He} + n \longrightarrow {}^4\text{He} + \gamma,$$

  and so on.
- It is possible to produce heavier nuclei ^{7}Be and ^{7}Li through such two-body interactions.
- However, nuclei heavier than this cannot be produced with only two-body interactions. The main reason is that there are no stable nuclei of atomic weight 5 or 8.
- In the interiors of heavy stars, this mass gap is superseded by the triple-alpha reaction ${}^4\text{He} + {}^4\text{He} + {}^4\text{He} \longrightarrow {}^{12}\text{C} + \gamma$. This reaction, however, involves a three-body process and cannot take place under the conditions of the early Universe. All heavy nuclei starting from ^{12}C are produced only inside stars.

Neutron-to-proton ratio

- Since the first non-trivial nucleus to form was deuterium, we need to compute its abundance before others. This abundance will depend on n/p ratio.
- In equilibrium, the protons and neutrons take part in reactions like $n + \nu_e \longleftrightarrow p + e$ and $n + e^+ \longleftrightarrow p + \bar{\nu}_e$, mediated by weak interaction.
- To find the decoupling of neutrons, we need to find out the reaction rate Γ_ν. It can be shown to be $\Gamma_\nu / H \approx (k_B T / 0.8 \text{ MeV})^3$.
- Thus neutrinos decouple at $t_D \sim 4$ s. Both p and n are non-relativistic at this epoch ($m_{n,p} \sim 940$ MeV).
- For $k_B T \gg 0.8$ MeV, we can apply the equilibrium formula for number density of non-relativistic species

$$n_A^{(0)} = g_A \left(\frac{m_A k_B T}{2\pi\hbar^2} \right)^{3/2} \mathrm{e}^{(\mu_A - m_A)/k_B T} \approx g_A \left(\frac{m_A k_B T}{2\pi\hbar^2} \right)^{3/2} \mathrm{e}^{-m_A/k_B T}.$$

- It follows that the ratio of these number densities in equilibrium will be

$$\frac{n_n^{0)}}{n_p^{(0)}} = \left(\frac{m_n}{m_p} \right)^{3/2} \mathrm{e}^{-(m_n - m_p)/k_B T} \approx \mathrm{e}^{-(m_n - m_p)/k_B T}.$$

  Note that we set $m_p = m_n$ in the prefactor, but their difference is crucial in the exponential.
- There is a very simple way of understanding the above. We can regard the proton as the ground state for a baryon and the neutron as the first excited state. Then the relation can be regarded merely as the Boltzmann distribution law applied to this situation.

Neutron abundance

- As the temperature keeps decreasing, we expect the Boltzmann relation to be valid till $k_B T \approx 0.8$ MeV, after which the reactions will no longer be able to maintain thermodynamic equilibrium and the ratio n_n/n_p will be approximately frozen (except for the fact that free neutrons decay).
- Since $m_n - m_p = 1.29$ MeV, this frozen ratio is given by

$$\frac{n_n}{n_p} \approx \mathrm{e}^{-1.29/0.8} \approx 0.2.$$

- Let us define the **neutron fraction** or **neutron abundance** as

$$X_n = \frac{n_n}{n_n + n_p}.$$

- The equilibrium value is

$$X_n^{(0)} = \frac{\mathrm{e}^{-(m_n - m_p)/k_B T}}{1 + \mathrm{e}^{-(m_n - m_p)/k_B T}},$$

  which at neutron decoupling becomes

$$X_n(t_D) = X_n^{(0)}(t_D) \approx 0.17.$$

- The free neutrons decay through the reaction $n \longrightarrow p + e^- + \bar{\nu}_e$ with a time-scale $\tau_n \approx 887$ s. Hence we can write

$$n_n(t) = n_n(t_D)\, \mathrm{e}^{-(t-t_D)/\tau_n} \Longrightarrow X_n(t) = X_n(t_D)\, \mathrm{e}^{-(t-t_D)/\tau_n} \approx 0.17\, \mathrm{e}^{-(t-t_D)/\tau_n}.$$

Deuterium abundance in equilibrium

- The neutron decay will be halted only when it gets bound to deuterium.
- At early times, D is in equilibrium via $n + p \longleftrightarrow \mathrm{D} + \gamma$. Note that $\mu_n + \mu_p = \mu_D$.
- In equilibrium, we can calculate the number densities of each of the species. We can then show that the ratio

$$\frac{n_D^{(0)}}{n_p^{(0)} n_n^{(0)}} = \frac{g_D}{g_p g_n} \left(\frac{2\pi\hbar^2}{k_B T}\right)^{3/2} \left(\frac{m_D}{m_n m_p}\right)^{3/2} \mathrm{e}^{-(m_D - m_n - m_p)/k_B T}.$$

- It is known that

$$m_D = m_p + m_n - B_D, \quad B_D = 2.22 \text{ MeV},$$

  and $g_D = 3,\ g_p = g_n = 2$, hence

$$\frac{n_D^{(0)}}{n_p^{(0)} n_n^{(0)}} \approx \frac{3}{4} \left(\frac{4\pi\hbar^2}{m_p\, k_B T}\right)^{3/2} \mathrm{e}^{B_D/k_B T}.$$

- We can then write

$$\frac{n_D^{(0)}}{n_p^{(0)}} \approx n_n^{(0)} \times \frac{3}{4} \left(\frac{4\pi\hbar^2}{m_p\, k_B T}\right)^{3/2} \mathrm{e}^{B_D/k_B T}.$$

Formation of deuterium

- For simplicity, let us do a quick order of magnitude estimate of n_D. Let us take

$$n_n^{(0)} \sim 0.1 n_b = 0.1\, \eta n_\gamma, \quad \eta \equiv \frac{n_b}{n_\gamma} \sim 10^{-9}$$

  is the baryon to photon ratio.
- Also, $n_\gamma \sim (k_B T)^3/(\hbar c)^3$, so

$$\frac{n_D^{(0)}}{n_p^{(0)}} \sim \eta \left(\frac{k_B T}{m_p c^2}\right)^{3/2} \mathrm{e}^{B_D/k_B T}.$$

- Now, we are working at a regime where $k_B T \lesssim 1$ MeV, so $k_B T/m_p c^2 \sim 10^{-3}$, and

$$\frac{n_D^{(0)}}{n_p^{(0)}} \sim 10^{-13}\, \mathrm{e}^{B_D/k_B T}.$$

- Clearly, to produce appreciable amount of deuterium, T needs to become small enough so that the exponential factor can compete with the small pre-factor. The value of $k_B T$ where $n_D \sim n_p$ turns out to be ~ 0.07 MeV, which corresponds to $t \sim 300$ s.
- Once deuterium is formed, the helium formation proceeds rapidly. This is because the binding energy of helium nuclei is 28.3 MeV which is considerably larger than that of deuterium. Hence the exponential factor assists in efficient formation of helium.
- Note that although neutrons form around 4 s after the Big Bang, however, one has to wait till ~ 300 s for helium to form. This is known as the **deuterium bottleneck**.

Helium fraction

- We will take $t \sim 300$ s as the helium formation time.
- By this time, the neutron fraction decreases to

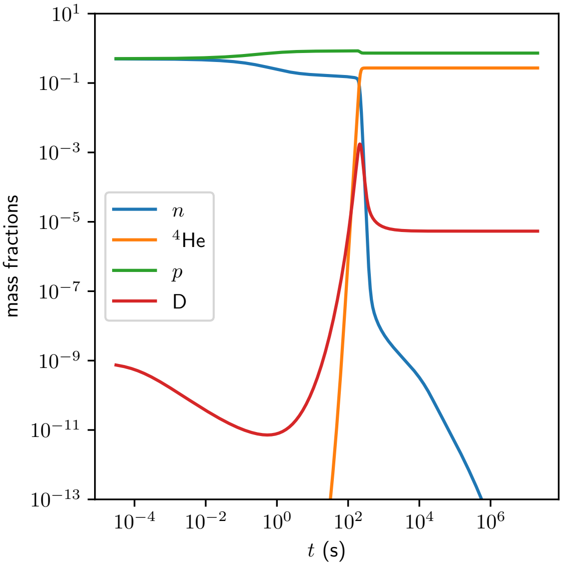
$$X_n(t) \approx 0.17\, \mathrm{e}^{-(t-t_D)/\tau_n} \approx 0.17\, \mathrm{e}^{-300/886} \approx 0.12.$$

- All the neutrons surviving till this time can be assumed to be used up to synthesize helium (a very small fraction remains in D and/or is used for heavier elements). Let us assume that n_n neutrons in the unit volume combine with n_n protons to synthesize $n_{\mathrm{He}} = n_n/2$ helium nuclei and the other $n_p - n_n$ protons remain as protons (hydrogen).
- Then the helium mass fraction should be given by

$$Y_{\mathrm{He}} = \frac{4n_{\mathrm{He}}}{n_n + n_p} = \frac{2n_n}{n_p + n_n} = 2X_n \approx 0.24.$$

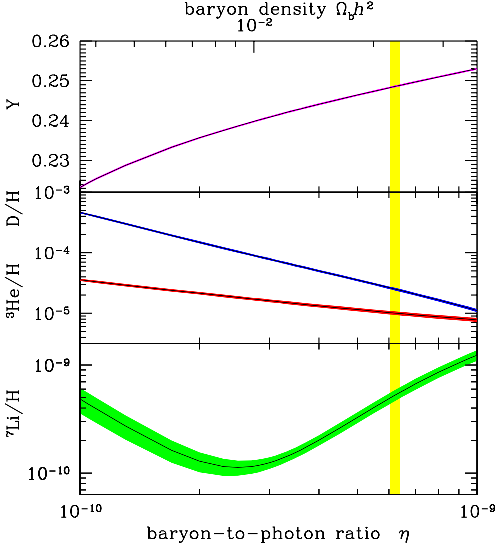
- It turns out that the helium mass fraction is not very sensitive to $\Omega_{b,0}h^2$, though it becomes slightly smaller for low $\Omega_{b,0}h^2$. The reason behind this is that a low $\Omega_{b,0}h^2$ implies a low baryon number density, which means that the nuclear reactions which build up helium nuclei would proceed at a slower rate and more neutrons would decay before being bound up inside helium nuclei, thereby leading to a lower helium mass fraction.
- If $\Omega_{b,0}h^2$ is high and the nuclear reaction rate is fast, then most of the deuterium would get converted into helium during the primordial nucleosynthesis era, and vice-versa.
- It therefore follows that the deuterium fraction falls sharply (exponentially) with the increase in $\Omega_{b,0}h^2$. This allows us to determine $\Omega_{b,0}h^2$ to a high accuracy from observations of D-abundance.

Simple numerical model



- Simple model containing elements only upto Helium.
- Rates from Wagoner, Fowler & Hoyle (1967).
- More accurate calculations include heavier elements.

Dependence on baryon density



compare with CMB (WMAP) results Cyburt et al (2008)

Observational results

- **Helium-4:**
  - Observationally, helium is detected in the intergalactic medium.
  - However, it is possible that not all the helium observed is primordial, some can be created within stars as well. Usually, we expect that regions which have low abundances of metals (i.e., higher order elements) like C, N and O are less enriched by stars. Hence, the helium abundances within these regions are a good indication of primordial helium.
  - The actual helium abundance is best determined from the observations of HeII $\to$ HeI recombination lines in extragalactic HII regions.
  - Observationally, it has been found that $Y_{\rm He} \approx 0.23 - 0.25$, which is consistent with the theoretical calculations.
- **Deuterium:**
  - Note that D can be destroyed in stellar evolution but cannot be created, hence any observation of D-abundance is a lower bound to the primordial abundance. This allows us to put an upper bound on $\Omega_{b,0} h^2$.
  - The D-abundance is measured from observations of high redshift Lyman-α systems and also from local interstellar medium (ISM).
- In addition, more constraints are obtained from ^{3}He and Lithium abundance. Interestingly, the Lithium abundance, measured from stellar atmospheres, shows inconsistency with the implied value of $\Omega_{b,0} h^2$ from other observations.

Formation of atoms

- After nucleosynthesis, the universe mainly consists of protons (hydrogen), electrons, photons and (decoupled) neutrinos. There is also some helium and negligible heavier elements which we will ignore for the moment.
- When $k_B T \lesssim 0.5$ MeV, all the baryonic particles are non-relativistic. The electrons, protons and photons can remain in equilibrium through electromagnetic interactions, e.g., Compton scattering, recombinations and free-free interactions (bremsstrahlung).
- Statistical equilibrium says that neutral hydrogen will form sometime after the temperature drops below the binding energy of hydrogen, which is $B_H = 13.6$ eV.
- In equilibrium, we have $p + e \longleftrightarrow \mathrm{H} + \gamma$, hence $\mu_p + \mu_e = \mu_H$. Also, $m_p + m_e - m_H = B_H$. Using the equilibrium number densities for non-relativistic species, we can write

$$\frac{n_H^{(0)}}{n_p^{(0)} n_e^{(0)}} = \frac{g_H}{g_p g_e} \left(\frac{m_H k_B T}{2\pi\hbar^2}\right)^{3/2} \left(\frac{m_p k_B T}{2\pi\hbar^2}\right)^{-3/2} \left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-3/2} \mathrm{e}^{(\mu_H - m_H - \mu_p + m_p - \mu_e + m_e)/k_B T}$$

$$\approx \frac{g_H}{g_p g_e} \left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-3/2} \mathrm{e}^{B_H/k_B T},$$

  where we have used $m_p \approx m_H$. This is nothing but the **Saha equation**.
- Define $x_e \equiv n_e/n_b = n_e/(n_H + n_p)$, then $n_p = n_e = x_e n_b$ (charge neutrality) and $n_H = n_b - n_p = (1 - x_e) n_b$. Also, $g_p = g_e = 2, g_H = 4$. Then

$$\frac{1 - x_e^{(0)}}{x_e^{(0)2}} = n_b \left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-3/2} \mathrm{e}^{B_H/k_B T}.$$

Recombination epoch

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- $$\frac{1 - x_e^{(0)}}{x_e^{(0)2}} = n_b \left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{-3/2} e^{B_H/k_B T}.$$
- Now, $n_b = n_{b,0} a^{-3} = \Omega_{b,0} \rho_c a^{-3}/m_p = 1.12 \times 10^{-5}\ \mathrm{cm}^{-3} \Omega_{b,0} h^2 a^{-3}$.
- We can eliminate a in terms of T using $a = 2.73\ \mathrm{K}/T$, then

$$n_b = 2.09 \times 10^{41} (\mathrm{erg\ cm})^{-3} (k_B T)^3 \Omega_{b,0} h^2.$$

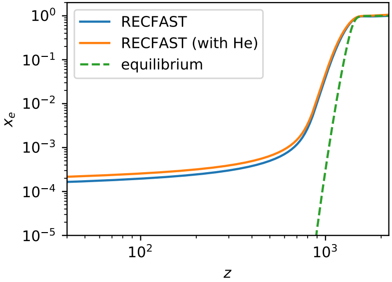
- Also, $m_e = 3.76 \times 10^4 B_H/c^2$, then

$$\frac{1 - x_e^{(0)}}{x_e^{(0)2}} = 1.43 \times 10^{-14} \Omega_{b,0} h^2 \left(\frac{k_B T}{B_H}\right)^{3/2} e^{B_H/k_B T}.$$

  This can be solved for $k_B T/B_H$ given x_e.
- If $x_e^{(0)} = 0.9$, we get $k_B T \approx 0.025\ B_H \approx 0.34$ eV. If $x_e^{(0)} = 0.1$, then $k_B T \approx 0.022\ B_H \approx 0.3$ eV.
- Hence, the transition from ionized to neutral occurs quite rapidly. The corresponding values of redshift are $z_{x_e=0.9} \approx 1445$ and $z_{x_e=0.1} \approx 1275$. This is known as **cosmological recombination** (although this is the first time 'combination' is occurring in the universe).
- Note that $k_B T \ll B_H$, i.e., the temperature must be substantially smaller than the hydrogen binding energy for the ionizations to stop. This is because there are many more photons than baryons and there can be sufficient photons with energies > 13.6 eV in the blackbody exponential tail so as to keep the hydrogen ionized.

Non-equilibrium evolution

- With expansion, the densities decrease, hence recombination and ionization rates become smaller and the species go out of equilibrium $\implies$ need to solve the differential equations.
- There are various possibilities regarding recombination:
  1. **Direct recombination to the ground state:** In this case, the electron recombines directly to $n = 1$ state. This is accompanied by a photon with energy $> B_H$ (called Lyman-continuum photon). Such a photon immediately ionizes another neutral hydrogen and hence there is no net decrease in the ionization fraction.
  2. **Cascade from excited states:** The electron can be captured in an excited state and then it cascades to the ground state. In this case, one obtains Lyman-series photons, in particular Lyman-α. These photons can re-excite hydrogen atoms from their ground states, which can subsequently lead to reionization. Some of these Lyman-series photons may redshift to lower frequencies because of expansion and hence may not excite other atoms. This process is thus partially effective.
  3. **Two-photon process:** The other effective process in cosmological recombination is the two-photon process where the electrons can decay from $2s$ to $1s$ state (otherwise forbidden by selection rules) by emission of two photons. These photons are below the ionization threshold and hence cannot be absorbed.
- Detailed calculations show that the electron fraction deviates from the equilibrium solutions and approaches a 'freeze-out' value.



Decoupling

- The formation of neutral atoms is followed by decoupling of photons. Photons are coupled to the matter mainly through the Thomson scattering off free electrons $e + \gamma \longleftrightarrow e + \gamma$.
- The interaction rate is given by $\Gamma_\gamma = c n_e \, \sigma_T = c x_e n_b \, \sigma_T$, where $\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 \approx 6.65 \times 10^{-25} \text{cm}^2$ is the Thomson cross section. Clearly the interaction rate decreases as x_e decreases during recombination.
- Let us assume that decoupling occurs when $x_e \ll 1$. Using the equilibrium approximation, we find

$$\Gamma_\gamma(T) \approx 361.23 \text{ s}^{-1} \left(\Omega_{b,0} h^2\right)^{1/2} \left(\frac{B_H}{k_B T} \right)^{-9/4} \mathrm{e}^{-B_H/2k_B T}.$$

- The decoupling epoch is given by $\Gamma_\gamma(T_D) = H(T_D)$. Since the decoupling occurs in the matter-dominated era, we can write

$$H^2 = H_0^2 \, \Omega_{m,0} a^{-3} = H_0^2 \, \Omega_{m,0} \left(\frac{T}{2.73 \text{ K}} \right)^3 \Longrightarrow H(T) \approx 4.5 \times 10^{-11} \text{ s}^{-1} \left(\Omega_{m,0} h^2\right)^{1/2} \left(\frac{k_B T}{B_H} \right)^{3/2}.$$

- These equations would give $k_B T_D \approx 0.019 \, B_H \approx 0.26$ eV. This leads to a decoupling redshift $z_D \approx 1097$.
- Hence the decoupling occurs shortly after recombination. The decoupled radiation free streams to us and produces the CMB.

Photon visibility function

- Consider a CMB photon originating at z and reaching us at $z = 0$. The optical depth of Thomson scattering will be

$$\tau(z) = \int_0^z \mathrm{d}z' \, \frac{c \, \mathrm{d}t}{\mathrm{d}z'} \, n_e \, \sigma_T.$$

  The optical depth increases with z and is very high at $z \gtrsim z_D$.

- The probability that a photon gets scattered in a infinitesimal length $\mathrm{d}l$ is $n_e \, \sigma_T \, \mathrm{d}l \equiv \mathrm{d}\tau$.
- For a distance equivalent to $\tau = \lim_{N \to \infty} \sum_{i=1}^{N} \mathrm{d}\tau_i = \lim_{N \to \infty} N \, \mathrm{d}\tau$, the probability that a photon does *not* encounter any scattering is

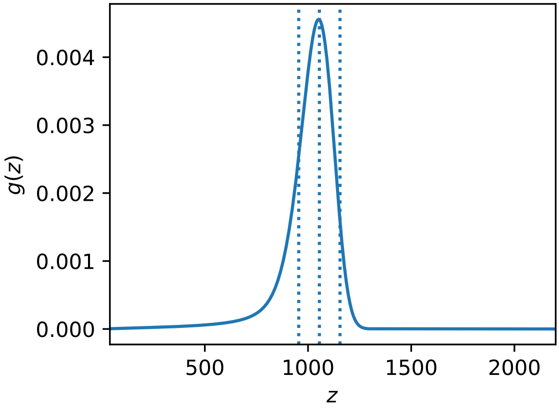
$$\lim_{N \to \infty} (1 - \mathrm{d}\tau_1)\,(1 - \mathrm{d}\tau_2) \ldots (1 - \mathrm{d}\tau_N) = \lim_{N \to \infty} \left(1 - \frac{\tau}{N}\right)\left(1 - \frac{\tau}{N}\right) \ldots \left(1 - \frac{\tau}{N}\right) = \lim_{N \to \infty} \left(1 - \frac{\tau}{N}\right)^N = \mathrm{e}^{-\tau}.$$

- So, the probability that a photon is scattered between $[z, z + \mathrm{d}z]$ and is not scattered at redshifts $< z$ is

$$g(z) \, \mathrm{d}z = \mathrm{d}z \, \frac{\mathrm{d}\tau}{\mathrm{d}z} \, \mathrm{e}^{-\tau(z)}.$$

  This is the the probability that a photon was *last scattered* in $[z, z + \mathrm{d}z]$ and is known as the **visibility function**.

Last scattering surface



- The visibility function, for RECFAST $x_e(z)$, peaks around $z \approx 1055$ and has a width $\Delta z \approx 100$.
- This corresponds to the surface of last scattering of photons, same as the surface probed by CMB.
- One cannot see beyond the last scattering surface $z \gtrsim 1100$ as photons will be scattered multiple times.