Cosmology
Lecture 6
The standard model of cosmology

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Constituents of our universe: radiation

- Let us now discuss the various constituents of our universe.
- We observe a blackbody radiation (CMB) at a temperature of 2.73 K, hence the corresponding energy density is

$$\rho_{r,0} = a_B T^4 = 7.56 \times 10^{-15} \text{erg cm}^{-3} \text{K}^{-4} (2.73)^4 \text{K}^4 \approx 4.2 \times 10^{-13} \text{erg cm}^{-3}.$$

- Converting this to equivalent mass density, we get

$$\rho_{r,0} \approx \frac{4.2 \times 10^{-13}}{(3 \times 10^8)^2} \text{gm cm}^{-3} = 4.6 \times 10^{-34} \text{gm cm}^{-3}.$$

- Thus

$$\Omega_{r,0} \approx \frac{4.6 \times 10^{-34}}{1.88 \times 10^{-29} h^2} = 2.45 \times 10^{-5} h^{-2}.$$

- There are also neutrinos which contribute to the relativistic matter density. We will show later that their contribution can be computed theoretically. However, the cosmological neutrino is yet to be observed directly.
- If we include relativistic neutrinos too, then the value goes up to

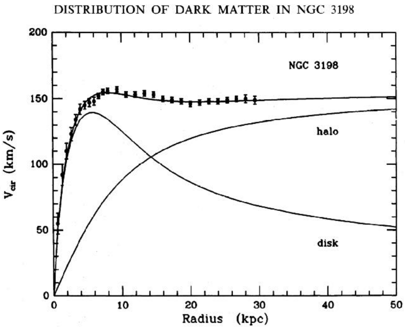
$$\Omega_{r,0} \approx 4.3 \times 10^{-5} h^{-2}.$$

- Clearly $\Omega_{r,0} \ll 1$. However, since $\rho_r \propto a^{-4}$, their contribution will be significant at early times.

Non-relativistic matter: baryons

- Galaxy survey and CMB fluctuation data indicate that for non-relativistic matter $\Omega_{m,0} \approx 0.15\ h^{-2}$.
- However, the baryon density can be estimated from abundances of light elements produced in Big Bang nucleosynthesis (BBN). These imply $\Omega_{b,0} \approx 0.02\ h^{-2}$ (only $\sim 15\%$ of total non-relativistic matter).
- The matter in stars can be computed from the luminosity function of galaxies (i.e., the number of galaxies per unit volume in a given luminosity range). These observations (mostly B-band) imply $L_{B,*} \approx 2 \times 10^8 h L_\odot \mathrm{Mpc}^{-3}$.
- If all stars had mass-to-light ratio like the Sun, this would imply a mass density of stars $\rho_{*,0} \approx 2 \times 10^8 h L_\odot \mathrm{Mpc}^{-3} \implies \Omega_{*,0} \approx 7 \times 10^{-4} h^{-1}$.
- In general, observations of galaxies imply a mass-to-light ratio of $2 - 10$, so even if we take a typical vale of 5, the stellar density will be $\Omega_{*,0} \approx 0.003 h^{-1}$.
- A substantial fraction of baryons are found as photoionized gas in the intergalactic space. These contribute $\Omega_{\mathrm{IGM},0} \approx 0.01\ h^{-3/2}$.
- There is (cold) gas locked up in atoms, molecules etc, detected via absorption/emission lines from galaxies. These make up to $\Omega_{\mathrm{cold},0} \approx 5 \times 10^{-4} h^{-1}$.
- There is also matter in hot gas in clusters, detected in X-ray, which give $\Omega_{\mathrm{cl},0} \approx 0.0016 h^{-3/2}$.
- There are also contributions from the circumgalactic medium and other sources. However, these do not add up to the value implied by the BBN. This is sometimes called the **missing baryon problem**.
- It is believed that the rest is in the **warm-hot intergalactic medium (WHIM)** which is difficult to detect directly.
- At high redshifts $z \sim 2 - 3$, the relative contributions of the components are different.

Non-relativistic matter: (cold) dark matter



- Bulk of the non-relativistic matter ($\Omega_{m,0} - \Omega_{b,0} \approx 0.13h^{-2}$) is in the form of **dark matter**.
- The indication for dark matter was already known in various astrophysical observations. For example, virial theorem applied to (Coma) cluster gives $\langle v^2 \rangle = GM/R$. We can measure $\langle v^2 \rangle$ from redshifts and also measure the size $\Longrightarrow$ calculate M. Observations imply $M \sim 10 M_{\rm gas}$.
- For the rotation curve of spiral galaxies, we expect $v \propto R^{-1/2}$ beyond the galaxy (visible) mass. However, one observes "flat" rotation curves $\Longrightarrow$ require $\rho \propto R^{-2}$.
- All these observations indicate that there is a matter which does not emit or absorb light, but is more abundant than the normal matter. There is no viable candidate in the so-called *standard model of particle physics.*
- The matter should be non-relativistic (hence "cold"), and can have weak interactions (in addition to gravity). Yet to be detected directly.

Curvature of the Universe

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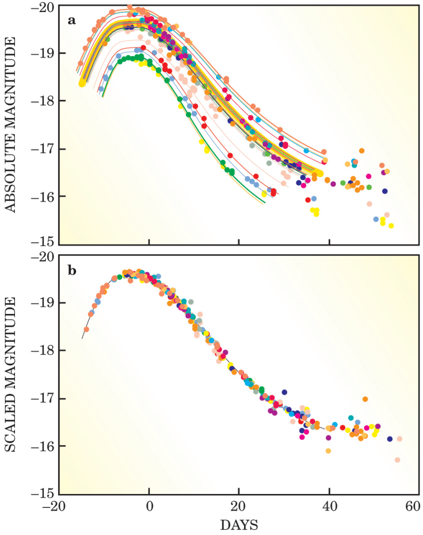
- It is possible to measure the curvature $\Omega_{k,0}$ using CMB observations. They indicate $\Omega_{k,0} \approx 0 \implies \Omega_{\text{tot},0} \approx 1$, a flat universe.
- Since $\Omega_{m,0} \approx 0.3$, it means that there must be a component which contributes the rest.
- Since this component does not manifest itself in galaxy surveys (i.e., observations of large-scale structure), it must not cluster at small scales. This implies that the component of matter behaves very differently under gravity compared to normal matter.
- A more direct evidence of such a component was found from observations of Supernova Type Ia (SN Ia).

Supernova lightcurves

- It is possible to measure distances to nearby SN Ia (e.g., using Cepheids) and hence obtain their luminosity.
- Their luminosities are highly (anti-)correlated with the time taken for the lightcurve to decline, i.e., brighter the supernova, slower the lightcurve declines.
- When corrected for this effect, the luminosities of all type Ia supernovae are the same. They are thus “standard candles”.
- Hence, just the observations of flux can allow measurement of luminosity distances. In addition, one can measure the redshifts from the spectra (e.g., Silicon absorption line). This allows us to obtain $d_L(z)$.



Perlmutter (2003)

The accelerating universe

- We have already seen that, for a flat model,

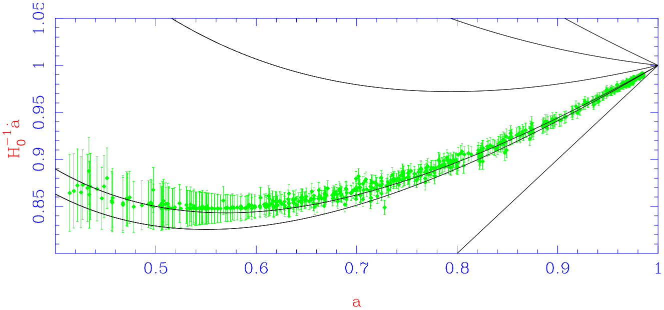
$$d_L(z) = R_0 S_k(\chi)(1+z) = R_0\ (1+z)\ \chi = R_0\ (1+z) \times \frac{1}{R_0}\int_0^z \frac{dz'}{H(z')} = (1+z)\int_0^z \frac{dz'}{H(z')}.$$

- This can be inverted to give

$$\frac{1}{H(z)} = \frac{d}{dz}\left[\frac{d_L(z)}{1+z}\right].$$

Thus, the observations can be used to compute $H(z)$, or $H(a)$, or $\dot{a}(a)$.

- The data imply acceleration at $a \gtrsim 0.6$.



Padmanabhan & **TRC** (2003); updated 2013

Dark energy

- The acceleration of the Universe implies

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) > 0 \Longrightarrow \rho + 3P < 0.$$

- Assuming $P = w\rho$ and ρ positive, we get

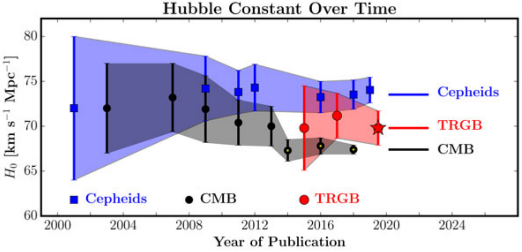
$$w < -\frac{1}{3}.$$

  Such a component is called **dark energy**.
- The cosmological constant with $w = -1$ is a candidate for this. Indeed observations favour $w \approx -1$.
- In fact, acceleration for $a > 0.6$ for a flat model implies

$$\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}} \approx 0.4 \Longrightarrow \Omega_{m,0} \approx 0.3,\ \Omega_{\Lambda,0} \approx 0.7.$$

The Hubble constant

- The next parameter of interest in the standard cosmological model is the Hubble constant H_0. Hubble measured its value to be ≈ 500 km s^{-1} Mpc^{-1}.
- Better observational data in recent times has given a much smaller value $H_0 \approx 70$ km s^{-1} Mpc^{-1}.
- The value can be measured from distance measurement to nearby galaxies using Cepheids.
- It an also be measured using cosmological observations, e.g., CMB.
- There seems to be a mismatch between the two measurements: **Hubble tension** or H_0 **tension**.



Freedman et al (2019)

- Independent measurement from the brightest red giant stars ("Tip of the Red Giant Branch").

The standard model: flat ΛCDM

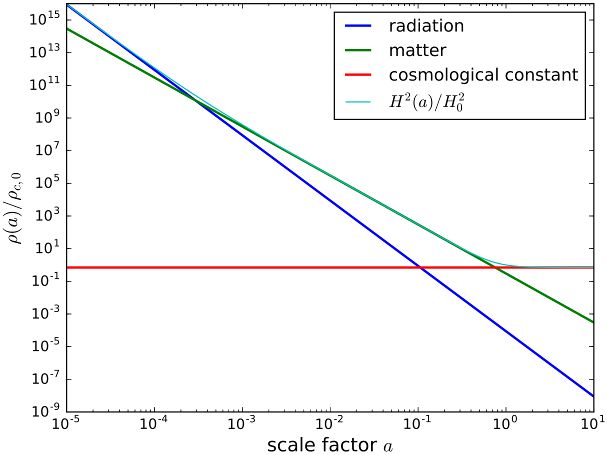
- Several observations, led by the CMB fluctuations and complemented by Supernova Ia, Big Bang nulceosynthesis, galaxy surveys, and so on have led to a cosmological model that agrees with "almost all" observations.
- The model can be described by *six* parameters.
- Three of them are: H_0 (Hubble parameter today), $\Omega_{m,0}$ (matter density), $\Omega_{b,0}$ (baryon density). Flatness implies that $\Omega_{\Lambda,0} = 1 - \Omega_{m,0}$.
- Two are related to the primordial fluctuations in the density field: n_s (the power-law slope) and A_s (the normalization).
- The sixth is related to the formation of the first stars: τ (the optical depth to reionization).
- The present constraints are (from *Planck-2018*)

$\Omega_{m,0} \approx 0.3111 \pm 0.0056$	$n_s \approx 0.9665 \pm 0.0038$
$\Omega_{b,0} \approx 0.0490 \pm 0.0009$	$A_s \approx (2.105 \pm 0.030) \times 10^{-9}$
$H_0 \approx (67.66 \pm 0.42)\text{km s}^{-1}\text{ Mpc}^{-1}$	$\tau \approx 0.0561 \pm 0.0071$

- Note: $\Omega_{\rm DM,0} \approx 0.2621$, $\Omega_{\Lambda,0} \approx 0.6889$, $t_0 \approx 13.787$ Gyr.

Evolution of different components

$$H^2(a) = H_0^2 \left[\frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} \right].$$



Three phases: radiation-dominated, matter-dominated and Λ-dominated.

Epochs of equality

- At very small a, the universe is radiation-dominated.
- The value of a when $\Omega_m(a) = \Omega_r(a)$ is given by

$$\Omega_{m,0} a_{\rm eq}^{-3} = \Omega_{r,0} a_{\rm eq}^{-4} \Longrightarrow a_{\rm eq}^{-1} = 1 + z_{\rm eq} = \frac{\Omega_{m,0}}{\Omega_{r,0}} \approx 7000 h^2 \approx 3500.$$

  This is known as the epoch of **matter-radiation equality**.
- At $a \gg a_{\rm eq}$, the universe becomes matter-dominated, until the cosmological constant takes over.
- One can obtain the epoch when matter density became equal to the cosmological constant as

$$\Omega_{m,0} a_{\rm eq}^{-3} = \Omega_{\Lambda,0} \Longrightarrow a_{\rm eq} = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3} \approx 0.75 \Longrightarrow a_{\rm eq}^{-1} = 1 + z_{\rm eq} \approx 1.33.$$

- In terms of z, the Hubble parameter is given by

$$H(z) = H_0 \sqrt{\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}}$$

- Interestingly, if we consider the epoch, say $2 < z < 1000$, we can see that the radiation term is sub-dominant, while the matter term dominates over the Λ-term. Hence, one can approximate

$$H(z) \approx H_0 \sqrt{\Omega_{m,0}}\, (1+z)^{3/2}.$$

  This is almost identical to an **Einstein-deSitter Universe**.

Difficulties/problems with the standard model

- We have already discussed a serious tension in the value of H_0, the so-called "Hubble tension".
- Both "Λ" and "CDM" are not understood in the ΛCDM model.
- There exist some difficulties related to the initial conditions. These are the horizon problem and the flatness problem.
- In addition, there are several problems with the value of Λ.

Horizon size at recombination epoch

- Recall that we have already calculated the size of the horizon at cosmic time t (assuming $a(t) = (t/t_0)^\alpha$)

$$d_{\rm hor}(t) = a(t) \int_0^t \frac{dt'}{a(t')} = \frac{t}{1-\alpha}, \quad \text{when } \alpha < 1,$$

  and $d_{\rm hor}(t) \to \infty$ when $\alpha \geq 1$.
- Let us now compute the horizon size at the epoch of recombination $t_{\rm rec}$, i.e., the epoch at which the universe became neutral and CMB originated. In terms of redshift, this corresponds to $z_{\rm rec} \approx 1100$.
- For simplicity, let us assume that the universe was radiation dominated till $t = t_{\rm rec}$ (in reality, the radiation domination ended at $z \approx 3500$).
- During radiation-dominated era, $a(t) \propto t^{2/[3(1+w)]} \propto t^{1/2}$, hence

$$d_{\rm hor}(t_{\rm rec}) = 2t_{\rm rec} \longrightarrow 2ct_{\rm rec}.$$

- Now, to estimate the horizon size, we need to know $t_{\rm rec}$. For that we assume that the universe is matter-dominated from $t_{\rm rec}$ to today t_0 (i.e., we ignore Λ for the time being). In that case, we know $a(t) \propto t^{2/3}$, i.e.,

$$a_{\rm rec} = \left(\frac{t_{\rm rec}}{t_0}\right)^{2/3} \Longrightarrow t_{\rm rec} = t_0\, a_{\rm rec}^{3/2} \approx 4 \times 10^5 \text{ years},$$

  assuming $t_0 = 13.8$ Gyr.
- So, $d_{\rm hor}(t_{\rm rec}) \approx 0.25$ Mpc.

Angular size of causally connected regions

- Hence, points that are separated by distances larger than $d_{\rm hor}(t_{\rm rec}) \sim 0.25$ Mpc could not have any causal connection when the CMB originated.
- Since we observe the CMB in terms of angles in the sky, let compute the angular size of $d_{\rm hor}(t_{\rm rec})$. This is

$$\theta_{\rm hor}(t_{\rm rec}) \equiv \frac{d_{\rm hor}(t_{\rm rec})}{d_A(t_{\rm rec})}.$$

- The angular diameter distance is

$$d_A(z_{\rm rec}) = \frac{1}{1+z_{\rm rec}} \int_0^{z_{\rm rec}} \frac{{\rm d}z'}{H(z')} \approx \frac{1}{1+z_{\rm rec}} \int_0^{z_{\rm rec}} \frac{{\rm d}z'}{H_0 \sqrt{\Omega_{m,0}}\,(1+z')^{3/2}} = \frac{2\left(\sqrt{1+z_{\rm rec}}-1\right)}{H_0\sqrt{\Omega_{m,0}}\,(1+z_{\rm rec})^{3/2}}$$

  which gives $d_A(z_{\rm rec}) \approx 14$ Mpc. Hence

$$\theta_{\rm hor}(t_{\rm rec}) \approx \frac{0.25}{14} \approx 1^\circ.$$

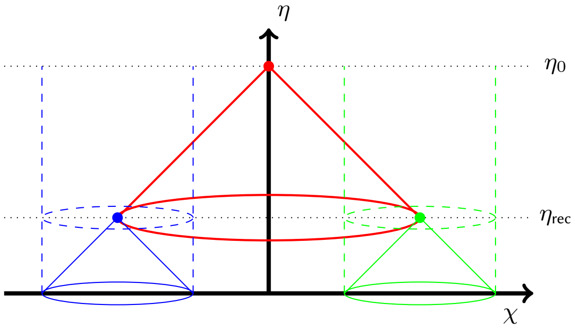
- Thus, points separated by angle larger than 1° on the CMB sky never came into causal contact before the CMB was formed. Then, why does the CMB look so isotropic over the whole sky? This is known as the **horizon problem**.
- In fact, as per this calculation, we expect the CMB to be consisting of $\sim 50,000$ causally disjoint regions.

Visual representation of the problem

- To understand the horizon problem visually, let us define a time coordinate

$$\eta = \int \frac{\mathrm{d}t}{a(t)},$$

  so that the metric becomes $\mathrm{d}s^2 = a^2(\eta)\left[\mathrm{d}\eta^2 - \mathrm{d}\chi^2 - \chi^2\mathrm{d}\Omega^2\right]$ (assuming $k = 0$).
- This new time coordinate is known as the **conformal time**.
- In terms of η, the radial light rays move along the 45° sloped lines $\eta = \pm\chi$ in the spacetime diagrams.



Evolution of curvature

- Imagine for a moment that the universe has a curvature. The Hubble parameter is then

$$H^2(a) = H_0^2 \left[\frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{\Omega_{k,0}}{a^2} \right], \quad \Omega_{k,0} \equiv 1 - \Omega_{\text{tot},0} = 1 - \Omega_{r,0} - \Omega_{m,0} - \Omega_{\Lambda,0}.$$

  Present observations, of course, indicate $\Omega_{\text{tot}} \approx 1$.

- At an earlier epoch a, the matter density parameter is

$$\Omega_m(a) \equiv \frac{\rho_m(a)}{\rho_c(a)} = \frac{\rho_{m,0}\, a^{-3}}{3H^2(a)/8\pi G} = \frac{\rho_{m,0}}{3H_0^2/8\pi G} \frac{a^{-3}}{H^2(a)/H_0^2} = \Omega_{m,0} \frac{a^{-3}}{H^2(a)/H_0^2}.$$

  Similarly

$$\Omega_r(a) = \Omega_{r,0} \frac{a^{-4}}{H^2(a)/H_0^2}, \quad \Omega_\Lambda(a) = \Omega_{\Lambda,0} \frac{1}{H^2(a)/H_0^2}.$$

- Then, the curvature part evolves as

$$\Omega_k(a) = 1 - \Omega_r(a) - \Omega_m(a) - \Omega_\Lambda(a) = 1 - \frac{\Omega_{m,0} a^{-3} + \Omega_{r,0} a^{-4} + \Omega_{\Lambda,0}}{\Omega_{m,0} a^{-3} + \Omega_{r,0} a^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0} a^{-2}}$$

$$= \frac{\Omega_{k,0} a^{-2}}{\Omega_{m,0} a^{-3} + \Omega_{r,0} a^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0} a^{-2}} = \frac{H_0^2\, \Omega_{k,0}}{H^2(a)\, a^2}.$$

Fine-tuned flat universe at early times

- Consider a very early epoch $a \ll 1$. Then only the radiation term contributes in the denominator

$$\Omega_k(a) \approx \frac{\Omega_{k,0} a^2}{\Omega_{r,0}}.$$

- Suppose $\Omega_{k,0} = 0.5$ today, then at, say, $a = 10^{-8}$, we have

$$\Omega_k(a) \approx 6 \times 10^{-13} \implies \Omega_{\text{tot}}(a) \approx 0.9999\,9999\,9999\,4$$

- Suppose, instead we had at $a = 10^{-8}$ the value

$$\Omega_{\text{tot}}(a) \approx 0.9999\,9999\,9999\,9 \implies \Omega_{k,a} \approx 10^{-13}$$

  we would have ended up with $\Omega_{k,0} \approx 0.086$.
- This shows that the initial curvature needs to be extremely fine-tuned close to zero in order to provide the value of $\Omega_{k,0}$ observed today. This is known as the **flatness problem**.
- Most likely the initial conditions were generated much earlier. Smaller values of a would make the problem much worse.