Cosmology: Assignment 1

IUCAA-NCRA Graduate School
January - February 2018

9 January 2018
To be returned in the class on 17 January 2018

- The deadline for the submission of the solutions of this assignment will be strictly enforced. No marks will be given if the assignment is not returned in time.
- You are free to discuss the solutions with friends, seniors and consult any books. However, you should understand and be clear about every step in the answers. Marks may be reduced if you have not understood what you have written even though the answer is correct.
- Let me know if you find anything to be unclear or if you think that something is wrong in any of the questions.

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1. **Order of magnitude numerical estimates:**

   (a) Suppose we (erroneously) estimate the value of H_0 to be 10 km s^{-1} Mpc^{-1}. What would be the corresponding characteristic age (to the order of magnitude) of the universe inferred? Give your answer in years.

   (b) Calculate the size of the horizon (i.e., the distance travelled by a photon) for our universe. Give your answer in units of h^{-1} Mpc. An order of magnitude estimate is sufficient.

   (c) A galaxy is found to be at a redshift $z = 0.01$. Calculate its velocity arising because of Hubble expansion. Give your answer in units of km s^{-1}. What is the distance to the galaxy (in Mpc) if $h = 0.7$?

   (d) If our universe is spatially flat and consists only of hydrogen atoms, what will be the total number density of atoms in units of m^{-3}? Assume $h = 0.7$.

   [1+1+2+2]

2. **Einstein-de Sitter universe:** Consider a cosmological model consisting only of non-relativistic matter with $\Omega_m = 1$. Suppose there is a galaxy at $z = 0.481$. Let us assume $h = 0.7$ for this model.

   (a) Derive expressions for the angular diameter and luminosity distances as functions of redshift for this model.

   (b) Find the angular diameter distance to the galaxy (in Mpc).

   (c) If the galaxy has a proper size of 50 kpc, find out its angular size (in arc seconds).

   (d) Find the luminosity distance to the galaxy (in Mpc).

   (e) If the luminosity of the galaxy is 10^{44} erg s^{-1}, what will be the flux received at earth? Give your answer in units of erg s^{-1} cm^{-2}.

   [4+2+1+2+1]

3. **Mattig's formula:** For a Friedmann model with only non-relativistic matter (dust), show that the luminosity distance relation varies with redshift as

$$d_L(z) = \frac{2c}{H_0\ \Omega_{m,0}^2}\left[\Omega_{m,0}\ z + (\Omega_{m,0} - 2)\ \left(\sqrt{\Omega_{m,0}\ z + 1} - 1\right)\right].$$

   [9]

4. **Scale factor for the ΛCDM model:** For a spatially flat Friedmann model consisting of matter component with density $\Omega_{m,0}$ and a cosmological constant with $\Omega_\Lambda = 1 - \Omega_{m,0}$, show that

$$a(t) = \left(\frac{\Omega_{m,0}}{1 - \Omega_{m,0}}\right)^{1/3}\left[\sinh\left(\frac{3}{2}\sqrt{1 - \Omega_{m,0}}\ H_0 t\right)\right]^{2/3}.$$

   [5]