

# Extra-Galactic Astronomy I (Cosmology)

## Assignment 2

**Total marks: 10**

All questions are compulsory. Please hand in your answer sheets no later than **5pm Monday 6 February 2017**.

### 1

#### 1.1 Epochs of equality (*4 marks = 2+2*)

Consider a spatially flat cosmology with non-relativistic matter, radiation and a cosmological constant, i.e.,  $\Omega_{m0} + \Omega_{R0} + \Omega_{\Lambda0} = 1$ ,  $\Omega_{k0} = 0$ . Find analytical expressions for the redshifts of the following epochs:

1. Equality of matter and radiation energy densities.
2. Equality of matter energy density and energy density associated with the cosmological constant.

For the standard cosmological parameters discussed in class, find numerical values for these two redshifts.

#### 1.2 Decoupling when non-relativistic (*6 marks*)

If a species  $A$  of mass  $m_A$  decouples at temperature  $T_{\text{dec}}$  during radiation domination when it has become highly non-relativistic, i.e.,  $m_A \gg T_{\text{dec}}$ , then prove that its distribution function after decoupling maintains the non-relativistic *equilibrium* form, with temperature  $T_A(t) \propto a(t)^{-2}$  and chemical potential  $\mu_A(t) = m_A + (\mu_{\text{dec}} - m_A)T_A(t)/T_{\text{dec}}$ , where  $\mu_{\text{dec}}$  is its chemical potential at decoupling. [*Hint*: Use the fact that the number density  $n_A$  and particle momentum  $p_A$  of a decoupled species have well-defined behaviours with scale factor  $a(t)$ .]