# PHY-T311 ASTRONOMY AND ASTROPHYSICS-I: Assignment 3

**Department of Physics**
**Savitribai Phule Pune University**
**July – December 2018**

**16 September 2018**
**To be returned in office 236 on 28 September 2018 (11:00–11:30 AM)**

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- The deadline for the submission of the solutions of this assignment will be strictly enforced. No marks will be given if the assignment is not returned in time.
- You are free to discuss the solutions with friends, seniors and consult any books. However, you should understand and be clear about every step in the answers. Marks may be reduced if you have not understood what you have written even though the answer is correct.
- Let me know if you find anything to be unclear or if you think that something is wrong in any of the questions.

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1. **The vorticity equation:**

   (i) Show that the Euler equation can be written as

   $$\frac{\partial \vec{V}}{\partial t} + + \frac{1}{2}\vec{\nabla}V^2 - \vec{V} \times \vec{\Omega} = -\frac{1}{\rho}\vec{\nabla}P + \vec{g},$$

   where $\Omega = \vec{\nabla} \times \vec{V}$ is the **vorticity**.

   (ii) Show, from the above equation, that the vorticity evolves as (assuming the external force $\vec{g}$ to be conservative)

   $$\frac{\partial \vec{\Omega}}{\partial t} = \vec{\nabla} \times (\vec{V} \times \vec{\Omega}) + \frac{1}{\rho^2}\vec{\nabla}\rho \times \vec{\nabla}P.$$

   (iii) For an incompressible fluid, the density $\rho$ does not change. Show that such a fluid obeys the equations

   $$\begin{aligned} \vec{\nabla} \cdot \vec{V} &= 0, \\ \frac{\partial \vec{\Omega}}{\partial t} &= \vec{\nabla} \times (\vec{V} \times \vec{\Omega}). \end{aligned}$$

   [1+3+2]

2. **Isothermal accretion:**

   (i) Show that the equations governing the spherical accretion for the isothermal case can be written in dimensionless form as

   $$x^2 yz = \lambda, \quad \frac{z^2}{2} + \ln y - \frac{1}{x} = 0,$$

   where

   $$x = \frac{r c_s^2(\infty)}{GM}, \quad y = \frac{\rho}{\rho(\infty)}, \quad z = \frac{|V_r|}{c_s(\infty)},$$

   and $\lambda$ is a constant.

   (ii) Manipulate these equations to give

   $$\left(z - \frac{1}{z}\right) \mathrm{d}z = \left(\frac{2}{x} - \frac{1}{x^2}\right) \mathrm{d}x.$$

   (iii) Show that the sonic transition occurs when $\lambda$ has a critical value $\lambda_c = \mathrm{e}^{3/2}/4 \approx 1.12$.

   [5+2+2]

3. **The virial theorem for stars:** Using the equation for hydrostatic equilibrium, show that the virial theorem

$$E_T = -\frac{1}{2} E_G$$

holds within stars. Here $E_G$ is the total gravitational energy of the star and $E_T$ is the total thermal (kinetic) energy.

[5]