Quantum & Statistical Mechanics II

Class Test II : 30.11.2017

Total Marks - 25, Time - 90 Minutes

Special Instruction : Calculators are not to be used.

Short Questions : 2.5 x 4 = 10.0

1. A 100-watt heating coil is placed in a vessel containing water and is switched on. After a while the water attains a steady temperature. If the heating coil is now removed how long would it take for the water to cool by 1°C? (Assume, specific heat of water = 4.5 KJ/Kg.°C)
2. In a suitable parameter-plane, draw the phase diagram of water, marking all the phases, the triple point and the critical point.
3. For a thermodynamic system of N particles in 3-D, prove Liouville's theorem -

$$\frac{d\rho(p,q)}{dt} = 0\,,$$

   where $\rho(p,q)$ is the phase-space density of a thermodynamic system.
4. Find the Chandrasekhar mass (appropriate for a White Dwarf) in terms of the relevant fundamental constants.

Medium Questions : 5.0 x 3 = 15.0

1. Find the critical point (V_c, T_c) for a van der Waals gas and discuss the behaviour of the system below the critical point.
2. Discuss the behaviour of the chemical potential (μ) for a non-interacting gas of - a) fermions and b) bosons. Consider both non-relativistic and ultra-relativistic cases.
3. Consider a He^2 White Dwarf, with interior density ranging from 10^6 g cm^{-3}on the surface to 10^{10} g cm^{-3}at the centre. Find whether the electrons are relativistic (or not) across this density range. What would be the Fermi temperature of the electrons at the centre?

Physical & Astrophysical Data :

$$\begin{aligned} c &= 3\times10^{10} cms^{-1} \\ G &= 6.6732\times10^{-8} cgs \\ \hbar &= 1.0546\times10^{-27} cgs \\ m_p &= 1.6726\times10^{-24} gm \\ m_e &= 9.1095\times10^{-28} gm \\ \mathrm{M}_\odot &= 1.989\times10^{33} gm \\ e &= 4.8032\times10^{-10} cgs \end{aligned}$$
