Statistical Mechanics : Mid-Semester Examination

24th September, 2004

• Total Marks - 30 • Total Time - 2 Hours •

N.B. - The full marks is 30. But the question paper is of 35 marks. You can attempt everything. If you get less than 30 then that would be your marks. However, if you get more than 30, your marks would be 30.

Thermodynamics

1. - The entropy of a system of N non-inetracting particles is given by,

$$S = Nk_{\rm B} \ln \frac{aVE^{3/2}}{N^{5/2}} \tag{1}$$

where a is a constant.

a). - Obtain all the thermodynamic information about the system (i.e, P, T etc.) and find the equation of state. [2]

b). - Show that the chemical potential of a monatomic ideal gas is given by

$$\mu = -k_{\rm B}T \ln \frac{bVT^{3/2}}{N} \tag{2}$$

where b is a constant. [3]

Ensemble Formulation

2. - Find the expressions for Gibb's free energy and enthalpy in terms of the appropriate derivatives of the canonical partition function. [5]

3. - Assume that the partition function of a system is given by,

$$Q = (V - Nb)^N (2\pi m k_{\rm B} T)^{3N/2} e^{aN^2/Vk_{\rm B}T} \tag{3}$$

where a, b, m are constants and the other symbols have their usual meaning. Calculate the equation of state and try to identify the system. [5]

4. - Write down the expression for the grand canonical partition function for a system confined to a region of volume V and described by the Hamiltonian,

$$\mathcal{H} = \Sigma_1^N \left(\frac{p_i^2}{2m} + u(\mathbf{r}_i) \right). \tag{4}$$

Show that the equation of state in the grand-canonical description of this system is always that of the ideal-gas form, regardless of the single particle potential $u(\mathbf{r})$.

Non-Interacting Systems

5. - Consider a collection of N spin-half ($S = 1/2$) particles in an external magnetic field H. Show that the chemical potential μ equals the Helmholtz free energy $A = U - TS$ per particle. Thus determine μ at a given T and H. Make a schematic plot of μ as a function of T and check that your answer makes sense in the limits $T \to 0^+$ and $T \to \infty$. [6]

6. - The hamiltonian of a system of N particles is given by,

$$\mathcal{H} = \Sigma_i (AP_i^x + BQ_i^x) \tag{5}$$

where, A, B, x are constants and P_i, Q_i are the momentum & position co-ordinates of the i-th particle. Derive the equipartition of energy for this system. [4]

Interacting Systems

7. - Describe the general assumptions under which a gas of interacting particles behave according to the van der Waals equation of state. [5]