Statistical Mechanics : Take-Home Examination

1st November - 30th November, 2004

• Total Marks - 50 •

• Report - 25, Seminar - 5, Answers - 15, Question - 5 •

Notes :
A. The report should not exceed **5 printed** pages.
B. The last date of submission (electronic) of the report is **25th November**.
C. A single person group would not be allowed.

Reading Projects

Topic A - Probability & Statistics
Discuss the *Central Limit Theorem* and its significance in Statistical Mechanics.
Basic Reading : S. K. Ma (Statistical Mechanics)

Topic B - Solid-Vapor Equilibrium
Study the problem of the *solid-vapor equilibrium* by setting up both the canonical partition function and the grand partition function of the system (use quantum statistics).
Basic Reading : R. K. Pathria (Statistical Mechanics), J. K. Bhattacharjee (Statistical Mechanics)

Topic C - Magnetism of Electron Gas
Discuss the magnetism of an electron gas in both a) weak field, and b) strong field limit.
Basic Reading : Landau & Lifshitz (Statistical Physics, Part I), J. K. Bhattacharya (Statistical Mechanics)
Abhijit Mardana, Phoolchand Mahata, Rajratan Basu

Topic D - Ising Model
Discuss Ising model using mean-field theory.
Basic Reading : S. K. Ma (Statistical Mechanics), M. Plischke & B. Bergersen (Equilibrium Statistical Physics)

Topic E - Phase Transitions
Discuss the Landau theory of phase transitions.
Basic Reading : M. Plischke & B. Bergersen (Equilibrium Statistical Physics)
Animesh Parashar, Puneet Kala, Rajeev Ranjan Singh, Sunil Kumar Mishra

Topic F - NS Superfluidity
Discuss the nature of neutron superfluidity and proton superconductivity in neutron stars.
Anirban Das, Arnab Banerjee, Chandrajit Basu, Vaibhav Janve

Assignments

Topic G - Fermion Gas
Most of our universe consists of collection of atoms or molecules interacting via short range (*collisional*) forces. Such systems can be studied in the ideal gas approximation, using statistical mechanics. For an electron gas, describe and justify the following regimes (range of densities/temperatures, dependence of pressure on density - for each case) -

a) non-degenerate ideal gas,
b) fully degenerate non-relativistic gas, and
c) fully degenerate ultra-relativistic gas.

Make a $T - \rho$ plot in which delineate the above regions and also indicate the location of the following objects (The plot should be computer generated with units clearly marked.) - a) the central region of Jupiter, b) Sirius B (companion of Sirius, a white dwarf), c) a normal metallic solid at room temperature & pressure.

Topic H - Takahashi Gas
Consider a one-dimensional gas of hard rods (of length a) with interactions only between nearest-neighbour rods. The interaction potential $\phi(r)$ satisfies the following conditions,

$$\begin{aligned} \phi(r) &= \infty, \text{ if } 0 \leq r \leq a \quad (1) \\ &= 0, \text{ if } r > 2a \quad (2) \end{aligned}$$

and is, in principle, arbitrary (but bounded) in the range $a \leq r \leq 2a$. Assuming this interaction potential to be a linear function of r in the intermediate range discuss the physics of this gas.
Arup Ratan Jana, Mrinal Kanti Mandal, Samaresh Das

Topic I - Gas of Diatomic Molecules
Consider a gaseous system of N noninteracting, diatomic molecules, each having an electric dipole moment μ, placed in an external electric field $\mathbf{E}$. The energy of a molecule will be given by the kinetic energy of rotation as well as of translation plus the potential energy of orientation in the applied field. Study the thermodynamics of this system, including electrical polarization and the dielectric constant. Assume the system to be classical. The partition function of this system can be written as

$$Q = \int_0^{N_\mu} e^{\beta EM} g(M) dM, \quad (3)$$

where M is the net electric moment of the system (in the direction **E**) and $g(M)$ is the density of states of the system around a particular value of the variable M. Make a plot of $g(M)$ vs. M and also find,

a) $g(M)$ in the large N limit,

b) $g(M)$ in the limit $M << N\mu$ to notice that this implies $\Delta M_{rms} = \sqrt{N\mu^2/3}$.

Interpret the last result in view of the fluctuation of energy in a canonical ensemble.

Nilanjan Maity, Shivanand, Vikrant Chauhan

Topic j - The Magnetic Drunk

a. - Calculate the probability of a drunk who is walking in one-dimension and having a step of length L to be found at a distance x from his initial location after N steps. Show that this has the same mathematical structure as the probability for a paramagnet of N spins to have magnetization M.

b. - Show that when $N \gg 1$ but $x/L \ll N$ the probability calculated in **(a)** goes over to a Gaussian distribution.

c. - Calculate the width (standard deviation) of the distribution δx, and its relative width, $\delta x/NL$.

Hasibur Rahman, Jeebak Tewary, Prasun Dutta, Sajal Dhara

Topic K - Transport Properties

Consider a degenerate Fermi gas. How does the viscosity vary with the mass of the Fermions? If the gas is at a finite temperature T then how does the viscosity vary with the density? Estimate the viscosity of the electron gas in a typical metal at room temperature. Also estimate the electrical conductivity of this electron gas (neglect the electron-phonon interaction).

Topic L - Self-Gravitating Systems

There are self-gravitating systems supported by degeneracy pressure. In the non-relativistic limit the equation of state has the form $P \propto \rho^{5/3}$ and, in the relativistic limit, we have $P \propto \rho^{4/3}$. Obtain the equation of state $P = P(\rho)$ for a fully degenerate system without making non-relativistic or ultra-relativistic approximation. Integrate the equation of hydrostatic pressure balance in the non-relativistic and ultra-relativistic limit and obtain the relation between the mass and the radius of the system.

Suggested Reading : S. Chandrasekhar (Stellar Structure)

Amit K. Das, Goutam Chadra, Prakash Sarkar, Sanjit Das

Topic M - Classical Unpredictability

Consider a large number N of molecules kept inside a cubical box. At $t = 0$, all the particles are arranged to move parallel to the x-axis. We approximate the molecules to be hard spheres of radius r, mean separation l and mean velocity v. The collisions are elastic and occur head-on when two particles, which are moving along the same line in the x-direction, meet. The collisions with the walls are also elastic. If may seem that, in this situation, the initial conditions can be preserved for arbitrarily long periods of time and no relaxation will ensue. Prove that the above conclusion is erroneous. Consider the gravitational force on this system due to an electron located at the edge of the universe at a distance $D \simeq 6000 Mpc$. Show that this perturbation is enough to destroy the orderly state within ~ 60 collisions if the system is made up of molecules with $r \simeq 10^{-8}$ cm and $m \simeq 10^{-23}$ gm at room temperature and pressure.

Topic N - Temporal Coherence and Thermal Radiation

The thermal spectrum can be thought of as arising because of a randomly fluctuating electromagnetic field with specific stochastic properties. Let any one component of the electric field be denote by a Gaussian random variable $E(t)$, with $\langle E \rangle = 0$ and $\langle E(t)E'(t) \rangle = C(t - t')$ where $c(t - t')$ is the temporal correlation function. Relate C to a Fourier transform of the Planck spectrum and obtain its explicit form.

Suggested Reading : T. Padmanabhan (Theoretical Astrophysics, Vol. I)

Topic O - Neutron Stars

The interior of a neutron star is supposed to be made up mainly of neutrons with a small admixture ($\sim 10\%$) of protons (and equal number of electrons to ensure charge neutrality). The neutrons and protons are expected to be in a superfluid and a superconducting state respectively. Assuming the neutrons to be non-interacting, estimate the transition temperature for the Bose-Einstein condensation. Justify the claim that the protons would form a type-II superconductor in the core of a neutron star.

Suggested Reading : S. L. Shapiro & S. A. Teukolsky (Black Holes, White Dwarfs and Neutron Stars)

Manimala Mitra, Rumpa Choudhury