Assignment II

Date of Submission - 30.10.03

Classical Statistics

Ex.1 - For the canonical ensemble description of a statistical system, find the following quantities - $<\Delta T \Delta P>, <\Delta V \Delta P>, <\Delta S \Delta V>, <\Delta S \Delta T>$.

Ex.2 - Consider a system placed in an external field, such that its energy $E(p, q)$ can be written as

$$E(p, q) = E_0(p, q) + V(p, q) \tag{1}$$

where $V(p, q)$ is a potential representing the contribution of the external field to the energy. Treating V as a perturbation show that the free energy is given by (to $\mathcal{O}(V^2)$)

$$A = A_0 + <V> - \frac{1}{2T} <(\Delta V)^2>, \tag{2}$$

where A_0 is the free energy in the absence of V.

Ex.3 - Prove that in the general case, when the ideal-gas system is mildly relativistic, the equipartition theorem reduces to

$$<m_0 v^2 (1 - v^2/c^2)^{-1/2}> = 3kT, \tag{3}$$

where m_0 is the rest mass of a particle and v is the speed.

Ex.4 - Consider a system of charged particles obeying classical mechanics and classical statistics. Show that, the magnetic susceptibility of this system is identically zero (Bohr-van Leeuwen theorem).

Ex.5 - Show that, for any magnetizable system, the heat capacities at constant field H and at constant mean moment M are connected by the following relation

$$C_H - C_M = -T \left(\frac{\partial H}{\partial T} \right)_M \left(\frac{\partial M}{\partial T} \right)_H. \tag{4}$$

In particular, for a paramagnetic material obeying Curie's law

$$C_H - C_M = \frac{CH^2}{T^2}, \tag{5}$$

where C is the *Curie constant* of the given material.

Quantum Statistics

Ex.1 - Prove the following theorem due to Peierls,
``If H is the Hamiltonian of a given physical system and $\{\phi_n\}$ an arbitrary orthonormal set of wave functions satisfying the symmetry requirements and the boundary conditions of the problem then the partition function of the system satisfies the following inequality:

$$Q(\beta) \geq \Sigma_n e^{\{-\beta <\phi_n|H|\phi_n>\}}. \tag{6}$$

The equality holds when $\{\phi_n\}$ is the complete orthonormal set of eigenfunctions of the Hamiltonian itself.''

Ex.2 - Derive the density matrix ρ for a free particle in -

- the co-ordinate representation, and
- the momentum representation

and study its main properties.

Ex.3 - Calculate the grand partition function for a system of N non-interacting quantum mechanical harmonic oscillators, all of which have the same natural frequency ω, for the following cases -

- Boltzmann statistics,

- Bose statistics.

Ex.4 – Find the equations of state for an ideal Fermi gas and an ideal Bose gas in the limit of high temperatures. Include the first correction due to quantum effects. Estimate, for each of the ideal gases, the temperature below which quantum effects would be important : H_2, He, N_2.

Ex.5 – Give numerical estimates for the Fermi energy of

- electrons in a typical metal
- nucleons in a heavy nucleus
- He^3 atoms in liquid He^3 (atomic volume – $46.2A^{O^3}$/atom).

Assume all the above mentioned systems to be composed of free particles.