Assignment I

Ex.1 - From,

$$dU = \left(\frac{\partial U}{\partial P}\right)_V dP + \left(\frac{\partial U}{\partial V}\right)_P dV \tag{1}$$

show that

$$\left(\frac{\partial}{\partial P}\left(\frac{\partial U}{\partial P}\right)_V\right)_P = \left(\frac{\partial}{\partial P}\left(\frac{\partial U}{\partial V}\right)_P\right)_V, \tag{2}$$

and

$$dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\left(\frac{\partial U}{\partial V}\right)_T + P\right) dV, \tag{3}$$

$$dQ = \left(\left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P\right) dT + \left(\left(\frac{\partial U}{\partial P}\right)_T + P\left(\frac{\partial V}{\partial P}\right)_T\right) dP. \tag{4}$$

Ex.2 - Prove the equivalence of the Kelvin and the Clausius statements.

Ex.3 - Prove that no engine operating between two temperatures is more efficient than a Carnot engine.

Ex.4 - Prove Clausius' theorem.

Ex.5 - Prove the following properties of entropy.

1. For any arbitrary transformation $\int_A^B dQ/T \leq S(B) - S(A)$, equality valid for reversible transformations.
2. Entropy of a thermally isolated system never decreases.

Ex.6 - From the fact that dS is an exact differential, show that

1. $\left(\frac{\partial U}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V - P$,
2. for an ideal gas, U is a function of T only and $C_P - C_V = NK_B$ (K_B - Boltzmann's constant).

Ex.7 - Using the above definitions, show that

$$TdS = C_V dT + \frac{\alpha T}{\kappa_T} dV, \tag{5}$$

$$TdS = C_P dT + \alpha T V dP. \tag{6}$$

And,

$$C_P - C_v = \frac{TV\alpha^2}{\kappa_T}, \tag{7}$$

$$C_P/C_V = \gamma = \frac{\partial V/\partial P|_T}{\partial V/\partial P|_S} \text{ for adiabatic processes.} \tag{8}$$

Find C_P, C_V for an ideal gas using the above expressions.

Ex.8 - Using equation (10) of *Section 1* show that $\left(\frac{\partial P}{\partial T}\right)_{V(T\to 0)} \to 0$.