- Deconvolution
- Imaging in practice

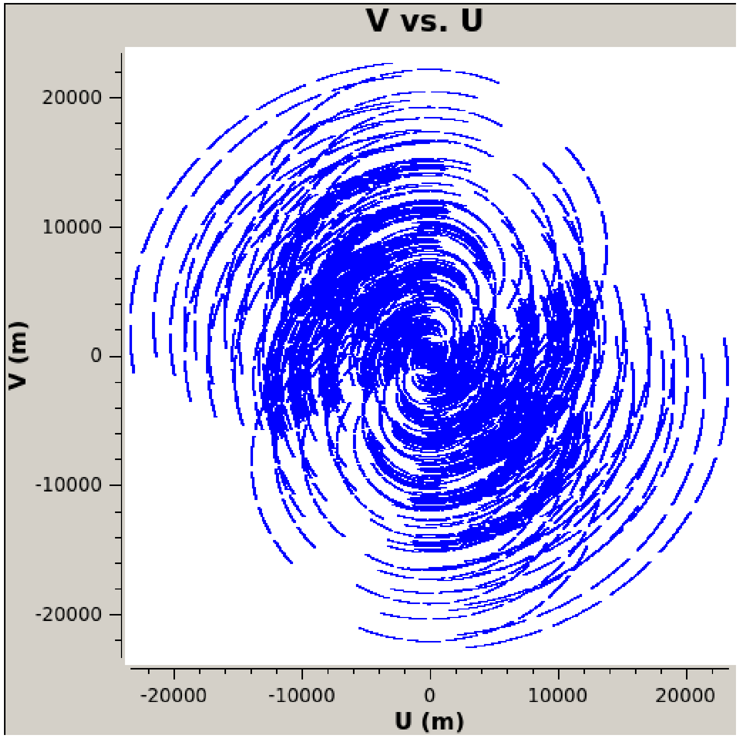
Astronomical Techniques II : **Lecture 9**

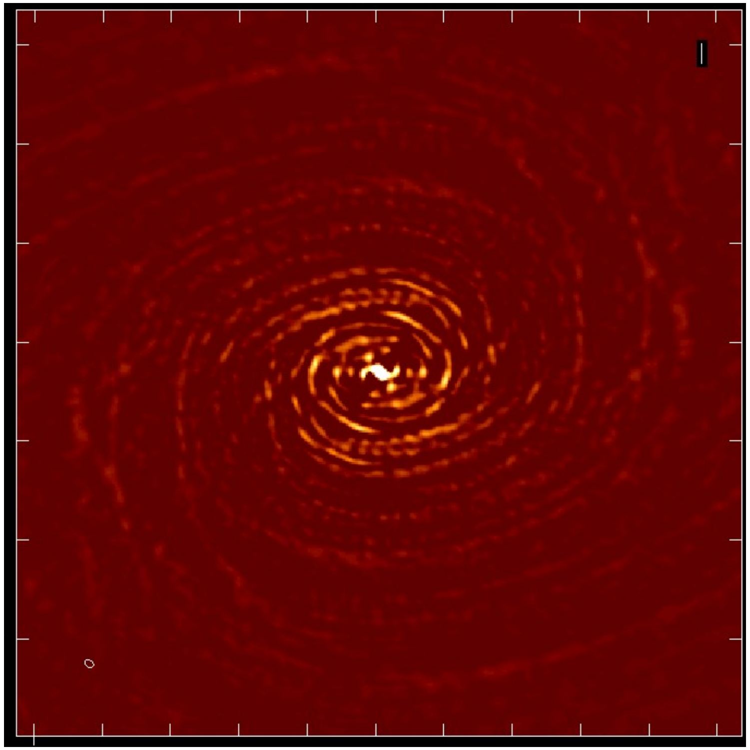
Ruta Kale

Low Frequency Radio Astronomy (Chp. 12)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy

Synthesis imaging in radio astronomy II, Chp 8

Synthesized beam



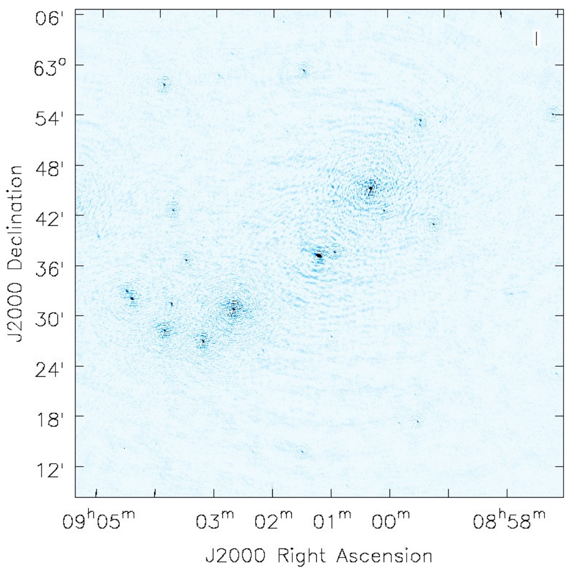


Synthesized beam:

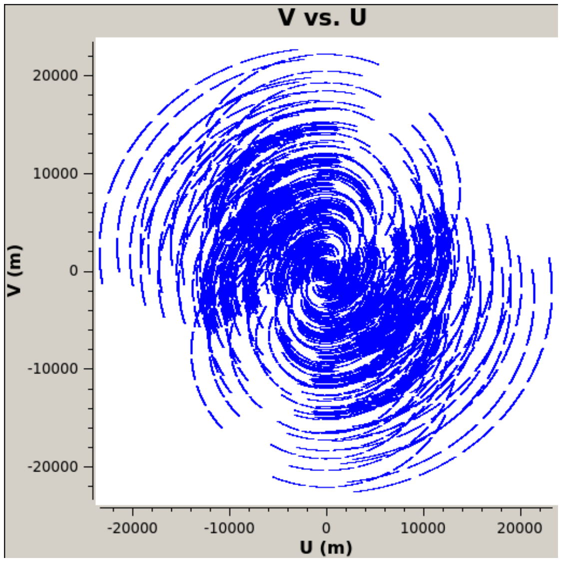
$$B = \mathfrak{F} S$$

Imaging

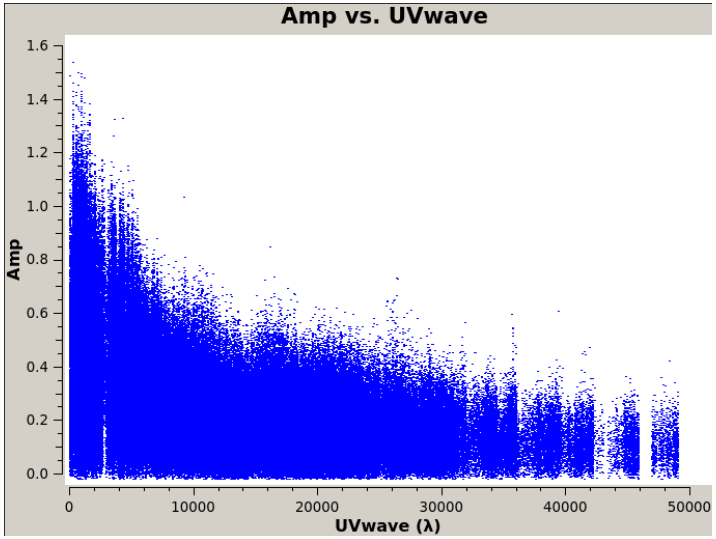
$$I^D(l,m) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(u,v)V'(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$



Image



Sampling



Visibilities (complex numbers)
Only amp. shown here

Imaging

$$V(u,v) = \int\int_S I(l,m) e^{-2\pi i(ul+vm)}\, dl\, dm$$

Only finite measurements of the visibilities are available; thus recovering *I(l,m)* has limitations.

Imaging

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A model with a finite number of parameters is needed.
A general purpose model of the sky is that of a 2-D grid of delta functions with strengths $\widehat{I}(p\Delta l, q\Delta m)$ can be considered.

Imaging

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The visibility predicted by this model is given by:

$$\widehat{V}(u,v) = \sum_{p=1}^{N_l} \sum_{q=1}^{N_m} \widehat{I}(p\Delta l, q\Delta m) e^{-2\pi i(pu\Delta l + qu\Delta m)}$$

Imaging

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N_l and N_m are pixels on each side. And the range of the uv points sampled are required to be:

$$\Delta l \leq \frac{1}{2u_{\max}},\ \Delta m \leq \frac{1}{2v_{\max}}$$

~Pixel size in the image

$$N_l \Delta l \geq \frac{1}{u_{\min}},\ \text{and}\ N_m \Delta m \geq \frac{1}{v_{\min}}$$

~Size of the image

One can estimate source features with widths in the range:

Minimum= $\mathcal{O}(1/\max(u,v))$ Maximum= $\mathcal{O}(1/\min(u,v))$

$N_l N_m$ free parameters that are the cell flux densities.

Imaging

$$\widehat{V}(u,v) = \sum_{p=1}^{N_l}\sum_{q=1}^{N_m} \widehat{I}(p\Delta l, q\Delta m)e^{-2\pi i(pu\Delta l + qu\Delta m)}$$

The measurements constrain the model such that at the sampled u,v points

$$V(u_k, v_k) = V\widehat{(u_k, v_k)} + \epsilon(u_k, v_k)$$

$\epsilon(u_k, v_k)$ is a complex, normally distributed random error due to receiver noise.

At the points in the plane where no sample was taken the model is free to take on any value.

Imaging

$V(u_k, v_k) = \widehat{V(u_k, v_k)} + \epsilon(u_k, v_k)$ can be expressed as a multiplicative relation

$$V(u,v) = W(u,v)(\widehat{V}(u,v) + \epsilon(u,v))$$

W is the weighted sampling function;

$$W(u,v) = \sum_k W_k \delta(u - u_k, v - v_k)$$

Non-zero only for the sampled points.

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In the image plane this translates to a convolution relation:

$$I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \widehat{I}_{p',q'} + E_{p,q}$$

Noise image

$$I^D_{p,q} = \sum_k W(u_k, v_k)\ \mathrm{Re}\left(V(u_k, v_k) e^{2\pi i (p u_k \Delta l + q v_k \Delta m)}\right)$$

Dirty image

$$B_{p,q} = \sum_k W(u_k, v_k)\ \mathrm{Re}\left(e^{2\pi i (p u_k \Delta l + q v_k \Delta m)}\right)$$

Dirty beam

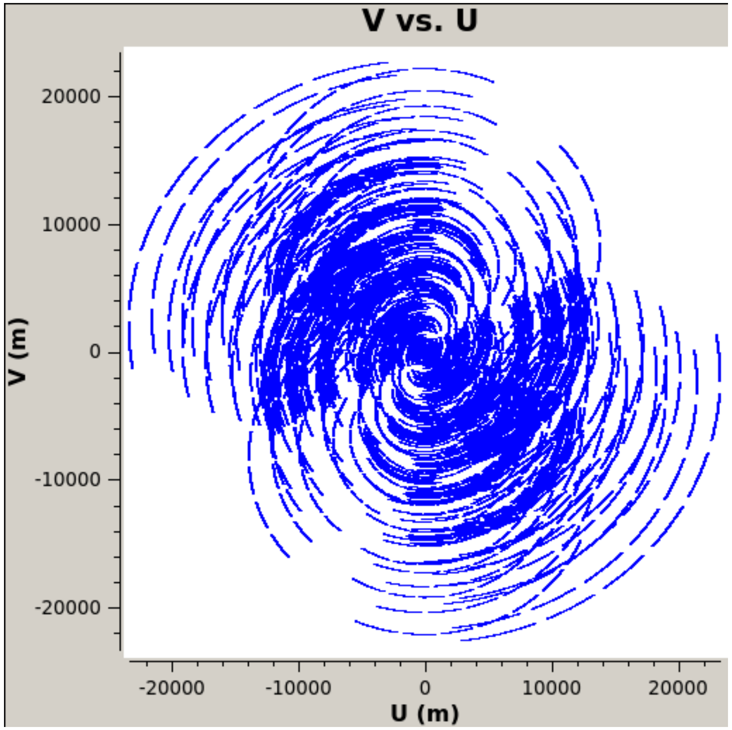
Principal solution and invisible distributions

$$I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \widehat{I}_{p',q'} + E_{p,q}$$

Solution not unique in the absence of boundary conditions.
Existence of “homogenous” solutions: called invisible distributions in radio astronomy.

Invisible distribution is that which has non-zero amplitude in only the unsampled spatial frequencies.

The solution having zero amplitude in all the unsampled spatial frequencies is called the *principal solution*.

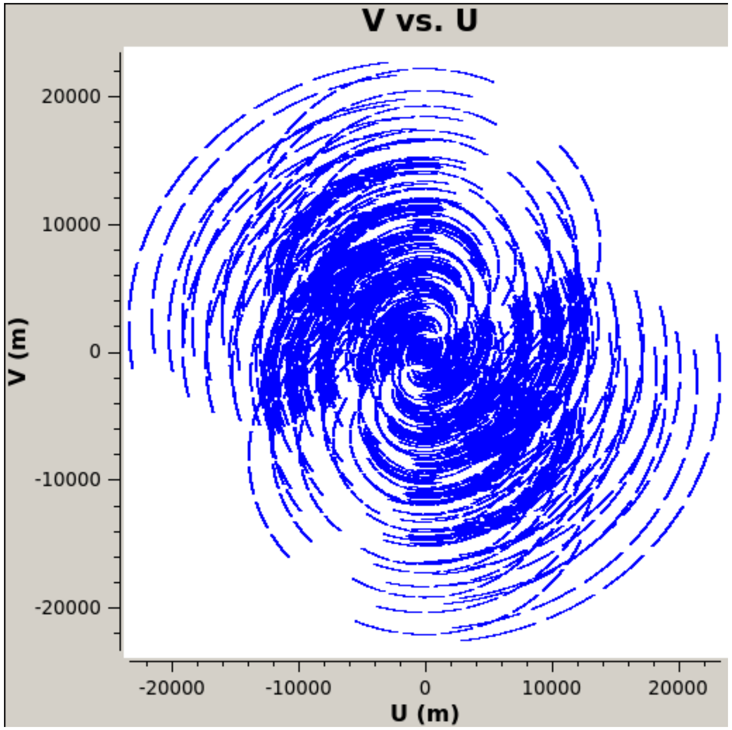


Principal solution and invisible distributions

$$I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \widehat{I}_{p',q'} + E_{p,q}$$

Solution not unique in the absence of boundary conditions.
Existence of “homogenous” solutions: called invisible distributions in radio astronomy.

Invisible distributions arise due to :
- Limit on the extent of u,v coverage.
- Holes in the u,v coverage



Problems with the principal solution

For a data on a regular grid, choose a weighting function that corrects for the sampling biases. And further one can normalize the total weights to unity. This is the same as “uniform” weigthing.
Principal solution here is the convolution of the true brightness with the dirty beam.
However this image is not enough as we cannot make out the if the source is a point source or is shaped like the beam.
Also it will change as we change the visibilities. *We need a method to estimate the visibilities in the unsampled range.*

We can use the information at the total intensity of the source must be positive.

Use of a priori information is the key to making an image of the sky.

Deconvolution algorithms use this to obtain better estimates of the sky than given by the principal solution.

Deconvolution: non-linear, iterative image re-construction

CLEAN algorithm : Hogbom 1974

The CLEAN algorithm (Högbom 1974)

- Provides a solution to the convolution equation by representing any source as a collection of point sources. An iterative approach is used to find the positions and strengths of the point sources.

- The final “deconvolved” image is called CLEAN image – it is the sum of the point source components convolved with the CLEAN beam – chosen usually to be a Gaussian.

Högbom’s CLEAN algorithm

1. Find the position and strength of the brightest point in the dirty image, $I^D_{p,q}$.

2. Multiply the peak with the dirty beam B and a “damping factor” (loop gain) and subtract from the dirty image.

3. Save the position and strength of the peak in a “model image”.

4. Go to (1) and repeat for the next peak until there is no peak above a user specified level.

Finally one will have “residual” image.

5. Convolve the model image with an idealized CLEAN beam (Gaussian fitter to the central peak of the dirty beam) to form a CLEAN image.

6. Add the residuals and the CLEAN image.

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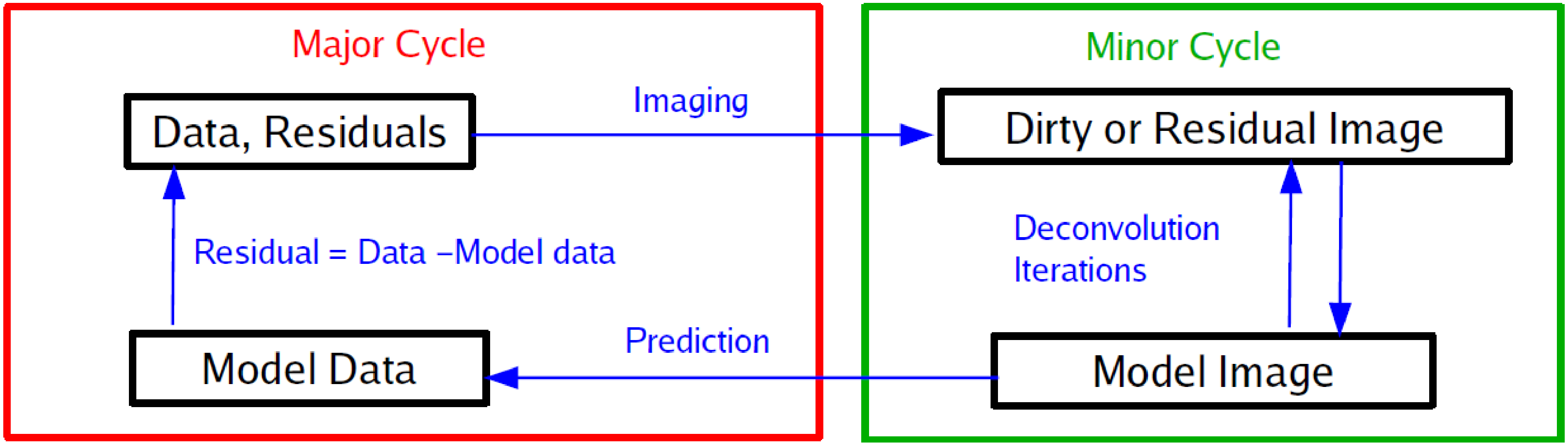
Clark CLEAN: use of psf patches
Cotton-Schwab CLEAN: Periodically predict model-visibilities, calculate residual visibilities and re-grid – major and minor cycles

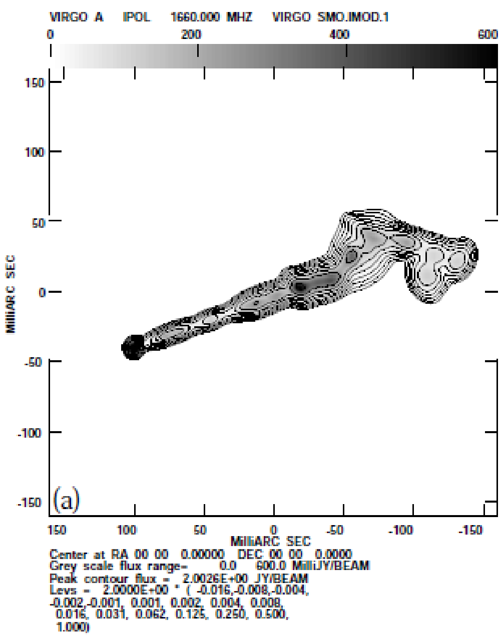
Major and minor cycles



Schematic from U. Rau

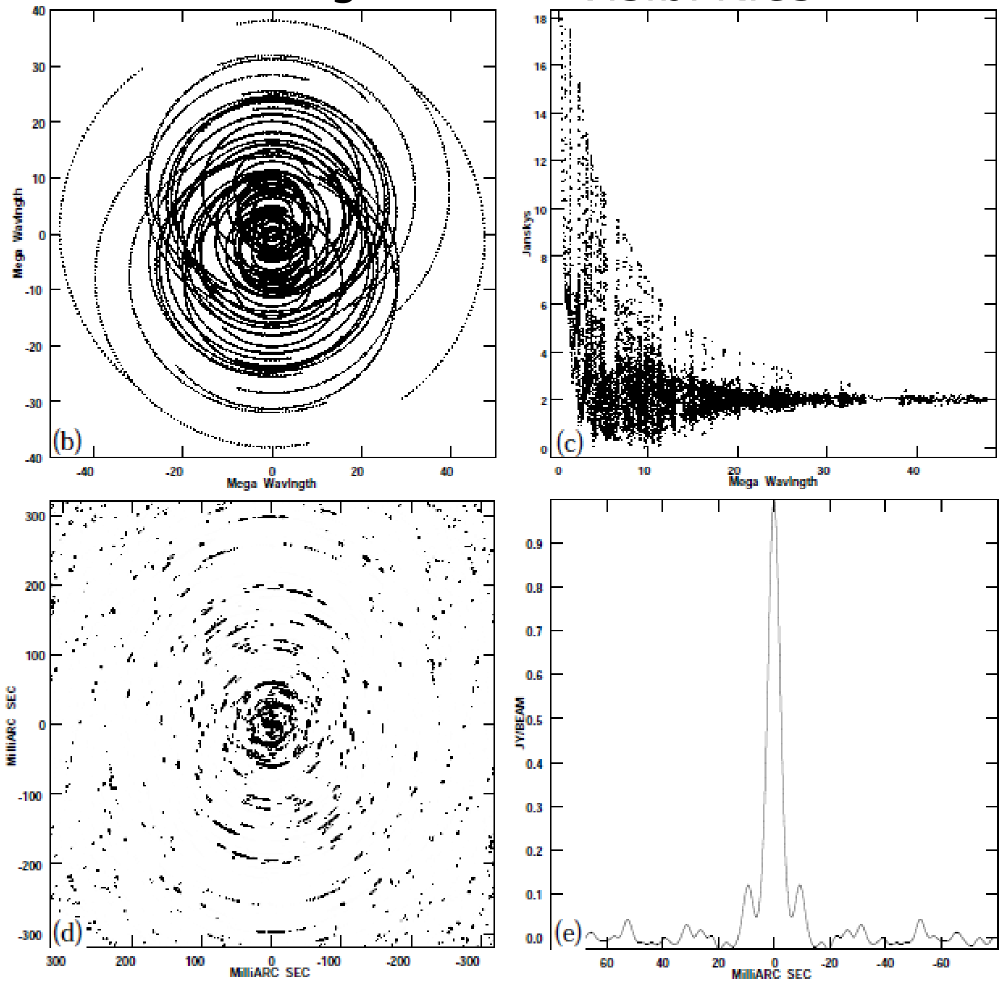
Example

Model source



uv-coverage

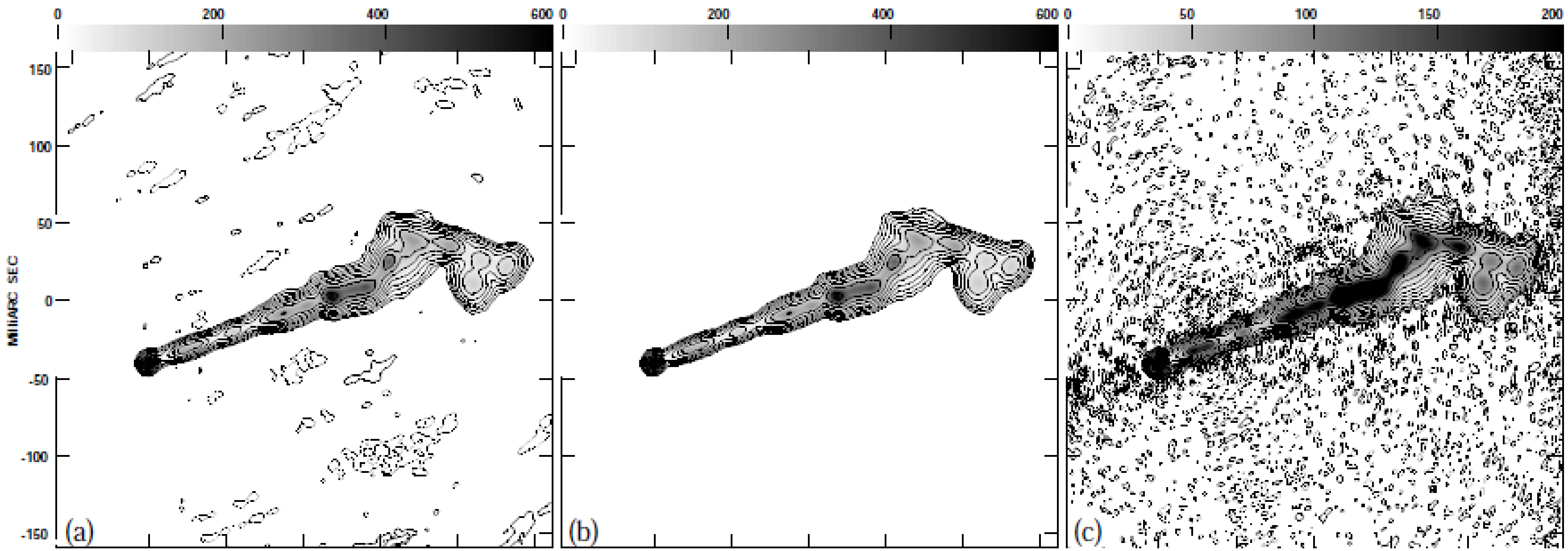
Visibilities



1.6 GHz VLBA source at dec. 50 degrees
Nearly horizon to horizon coverage.

Dirty beam

Dirty beam



Restored CLEAN image;
CLEANing without constraint.

Restored CLEAN image;
CLEANing with a constraint to be within the region of the source.

Same as panel b but with contours drawn starting at 10 times lower level to show the pattern in the rest of the image.

Softwares implementing CLEAN

NRAO CASA: Common Astronomy Software Applications

NRAO AIPS: Astronomical Image Processing System

ATNF MIRIAD

We will use CASA version 5.7.2:

Download it from:

https://casa.nrao.edu/casa_obtaining.shtml

Alternatives to CLEAN

Maximum Entropy Method (MEM)

In the problem of deconvolution we are trying to select one answer from many possible answers – basically one image from the many possible that can fit the visibilities.

MEM uses a statistical approach to find the most likely image.

We will discuss MEM and other more recent approaches in the last lecture in this course.