- Visibility sampling
- Bandwidth and time average smearing
- Aperture synthesis

Astronomical Techniques II : **Lecture 6**

Ruta Kale

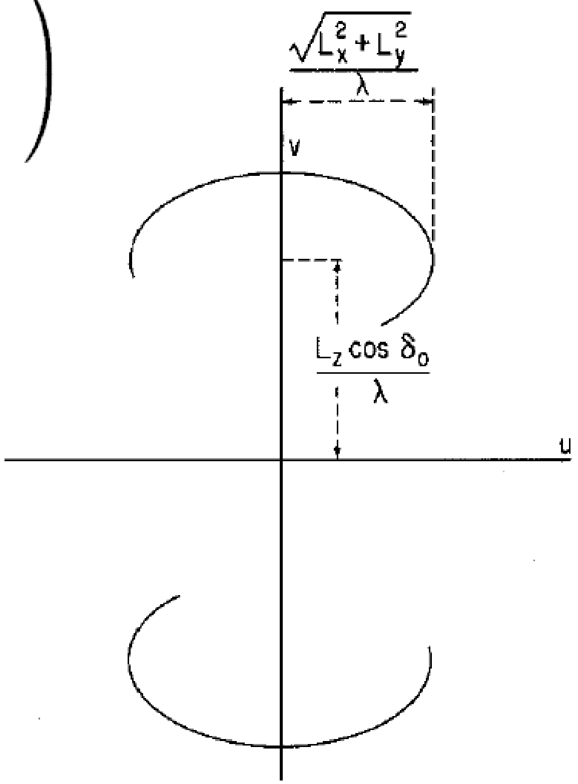
Low Frequency Radio Astronomy (Chp. 4)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy

Synthesis imaging in radio astronomy II, Chp 2

Interferometry and synthesis in radio astronomy (Chp 2)

Coordinate system

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$



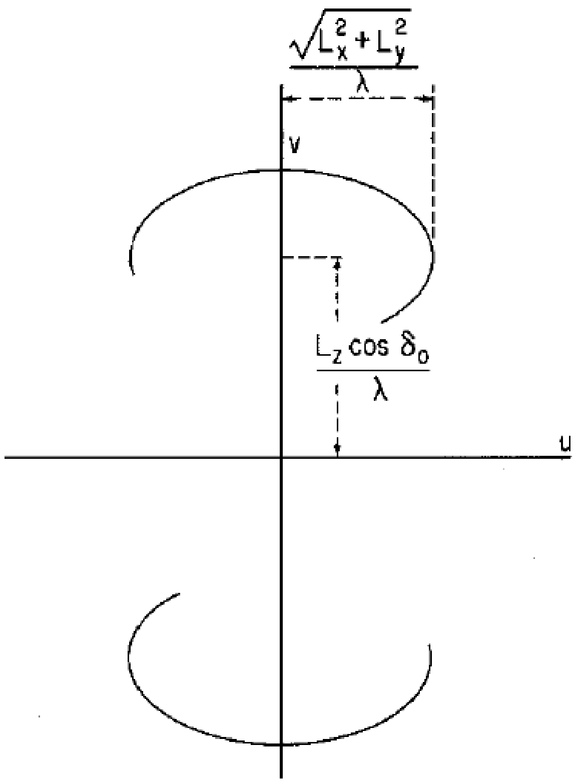
What is the locus of a track in the uv-plane ?
Eliminating H_0 from the equations for u and v:

$$u^2 + \left(\frac{v - (L_Z/\lambda)\cos\delta_0}{\sin\delta_0} \right)^2 = \frac{L_X^2 + L_Y^2}{\lambda^2}$$

$$V(-u, -v) = V^*(u, v)$$

Sampling in the uv-plane

$$u^2 + \left(\frac{v - (L_Z/\lambda)\cos\delta_0}{\sin\delta_0} \right)^2 = \frac{L_X^2 + L_Y^2}{\lambda^2}$$



Visibilities are sampled: the footprint in the uv-plane – *uv-coverage* is the sampling function.

For a point source at the phase centre the visibility is a constant as a function of u and v.

The FT of the sampling function is then the response to a point source – *the synthesized beam.*

The sampling in the uv-plane decides the shape of the synthesized beam.

Coordinate system

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$

?

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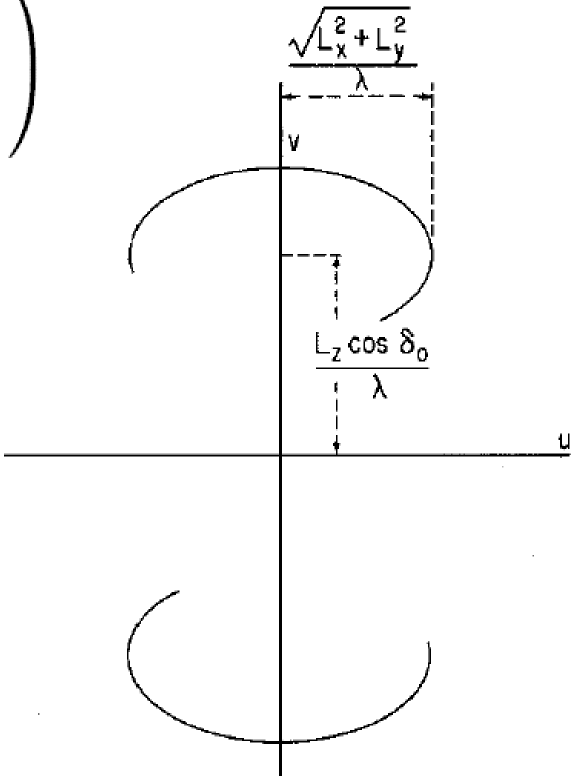
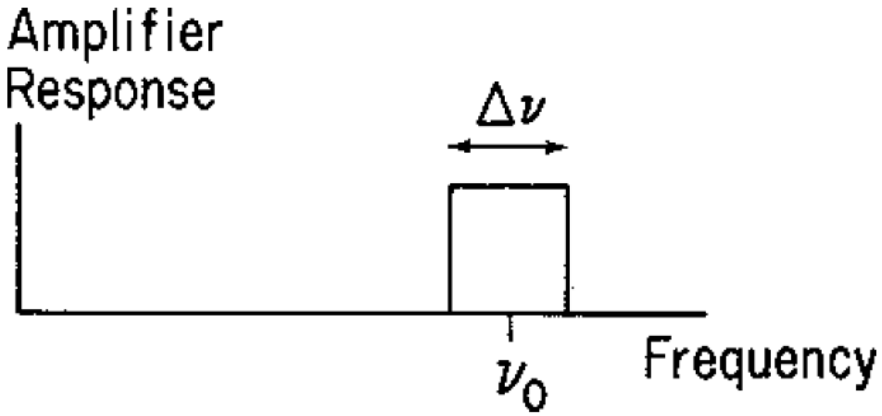


Effect of bandwidth



$\tau_g = b \sin(\theta)/c$

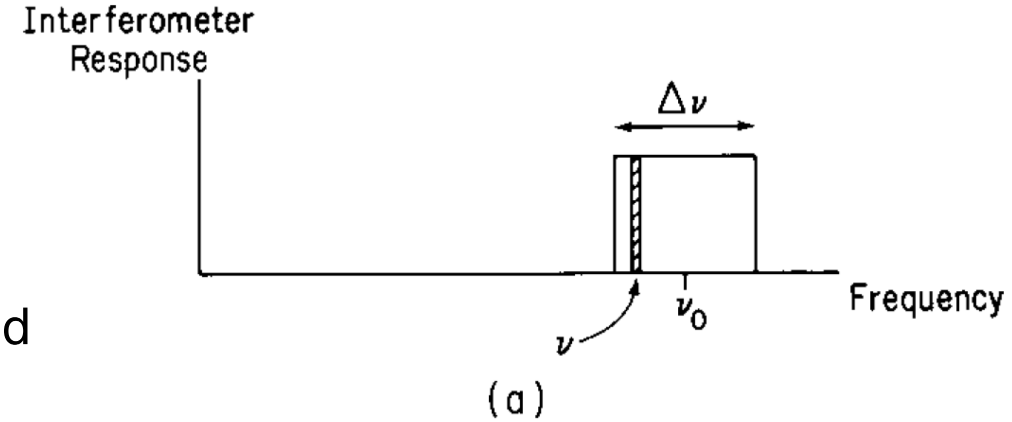
$$r = A_0|V| \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

$$= A_0|V|\Delta\nu \frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

- Bandwidth leads to a modulation of the fringe with a sinc function.
- Introduction of delay tracking to remove this effect: however it is only valid for the delay tracking centre.

Bandwidth smearing

The bandwidth over which the signal that is delay tracked only at the central frequency is averaged and this lead to blurring in the image.



u_0,v_0 for the central frequency and u and v for another frequency.

$$(u_0, v_0) = \left(\frac{\nu_0}{\nu}u, \frac{\nu_0}{\nu}v\right)$$

$$V(u,v) \rightleftharpoons I(l,m)$$

Similarity theorem of FT

$$V\left(\frac{\nu_0}{\nu}u, \frac{\nu_0}{\nu}v\right) \rightleftharpoons \left(\frac{\nu}{\nu_0}\right)^2 I\left(\frac{\nu}{\nu_0}l, \frac{\nu}{\nu_0}m\right)$$

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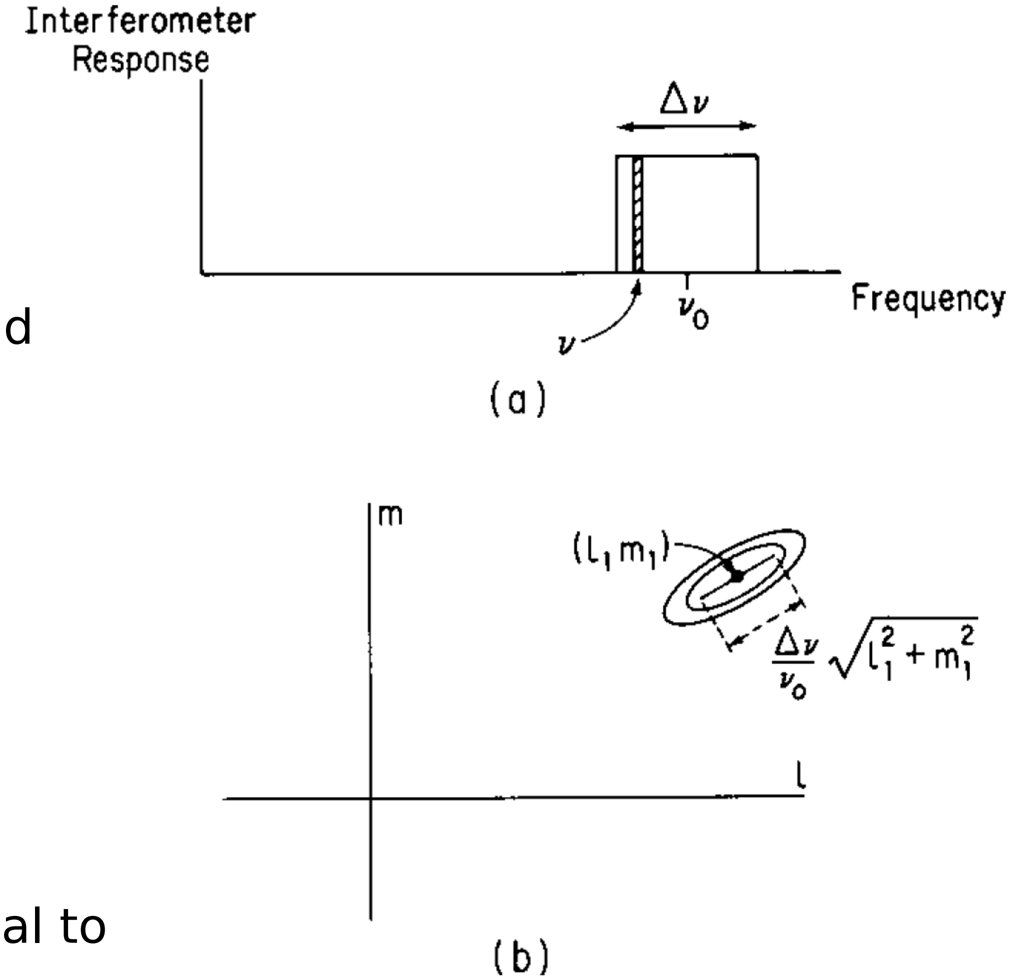
Range of variation in the coordinates decided by ν/ν_0

Introduces a *radial smearing* proportional to their distance from the tracking centre.



Bandwidth smearing

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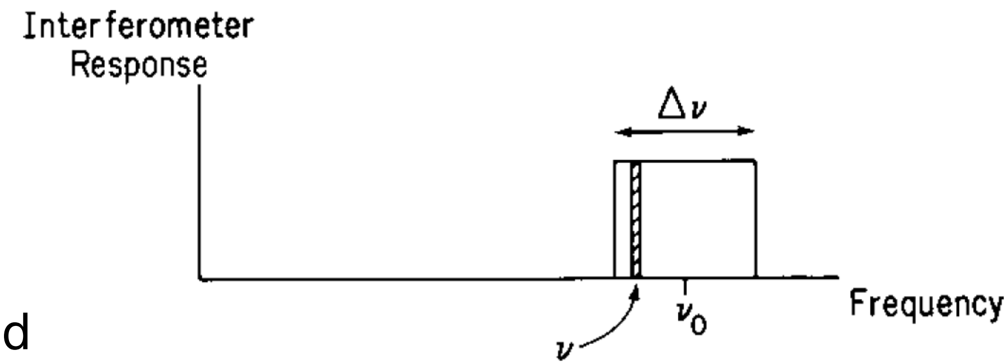
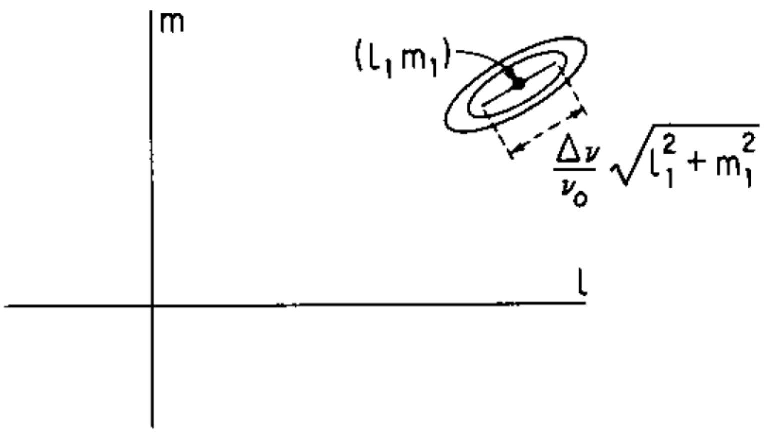


(a)

u_0,v_0 for the central frequency and u and v for another frequency.

$$V\left(\frac{\nu_0}{\nu}u, \frac{\nu_0}{\nu}v\right) \rightleftharpoons \left(\frac{\nu}{\nu_0}\right)^2 I\left(\frac{\nu}{\nu_0}l, \frac{\nu}{\nu_0}m\right)$$



(b)

Range of variation in the coordinates decided by ν/ν_0

Introduces a *radial smearing* proportional to their distance from the tracking centre.

Will become significant when it becomes of the order of the synthesized beam.

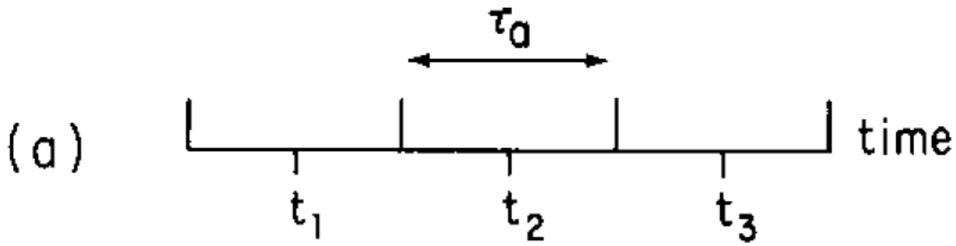
Time average smearing

Data are separated into time intervals τ_a

Data within a time interval $\pm\tau_a/2$ are all clubbed into one.

Easily visualised for a source at the pole.
uv-tracks are concentric circles.
Rotating at the angular velocity of the Earth ω_e

The time offset of assigning the coordinates will be $\omega_e \tau$



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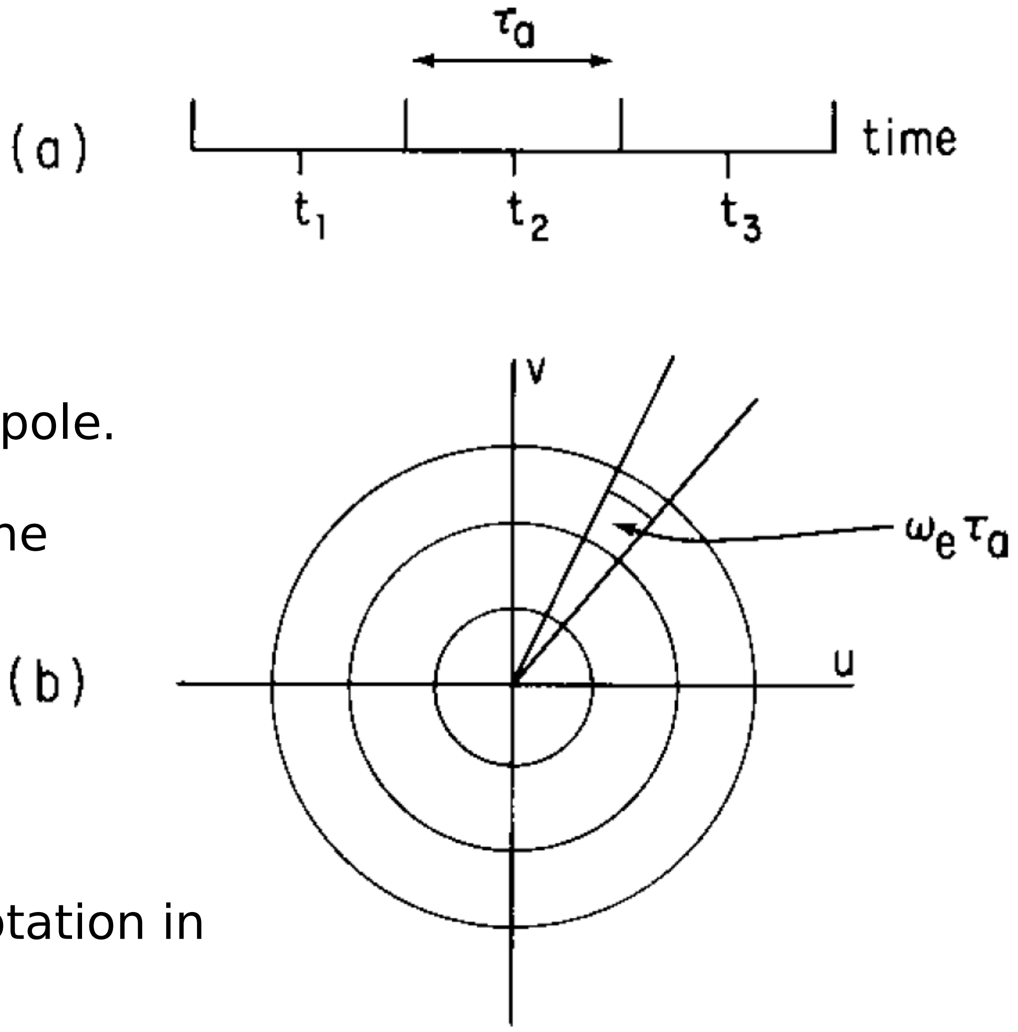
The time offset of assigning the coordinates will be $\omega_e\tau$

Rotation in one domain results in rotation in another for the FT.



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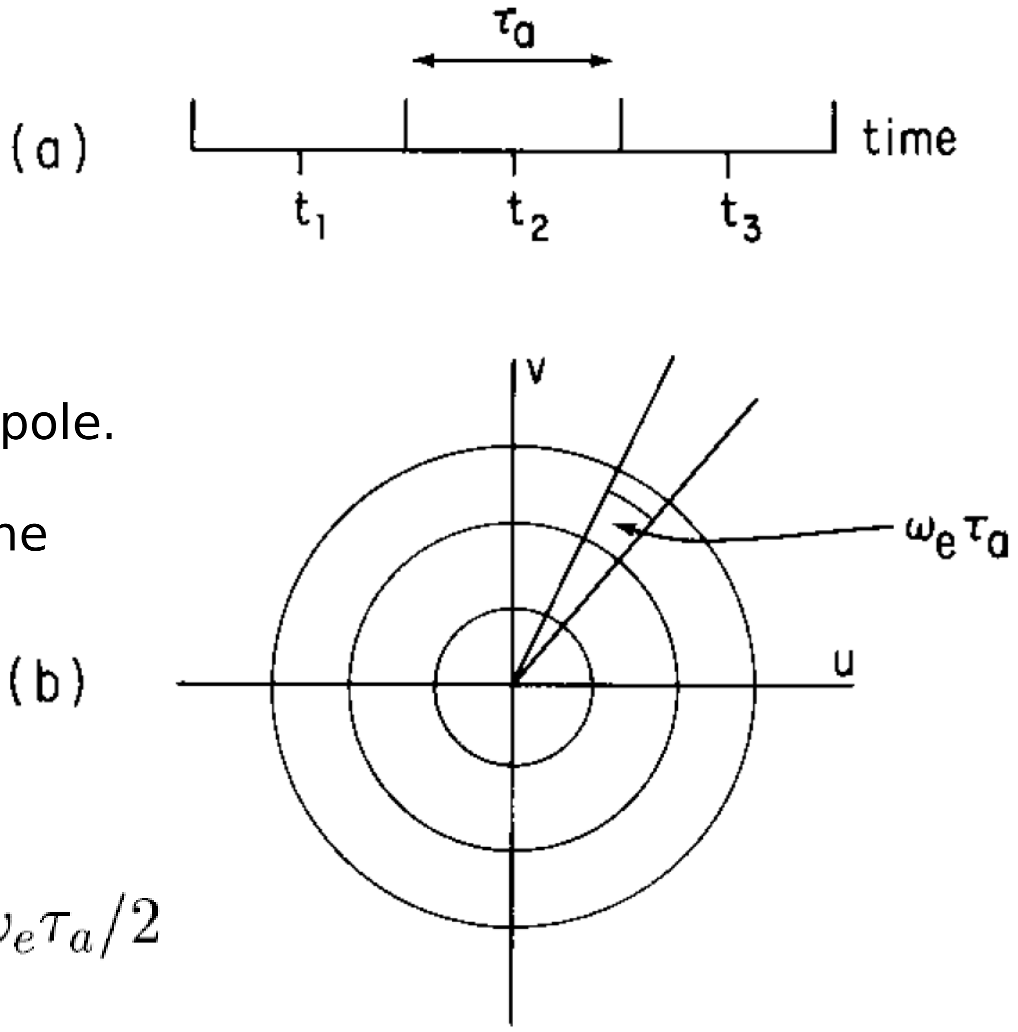
The time offset of assigning the coordinates will be $\omega_e\tau_a$

Images offsets distributed over $\pm\omega_e\tau_a/2$

$$\omega_e\tau_a\sqrt{l^2+m^2}$$



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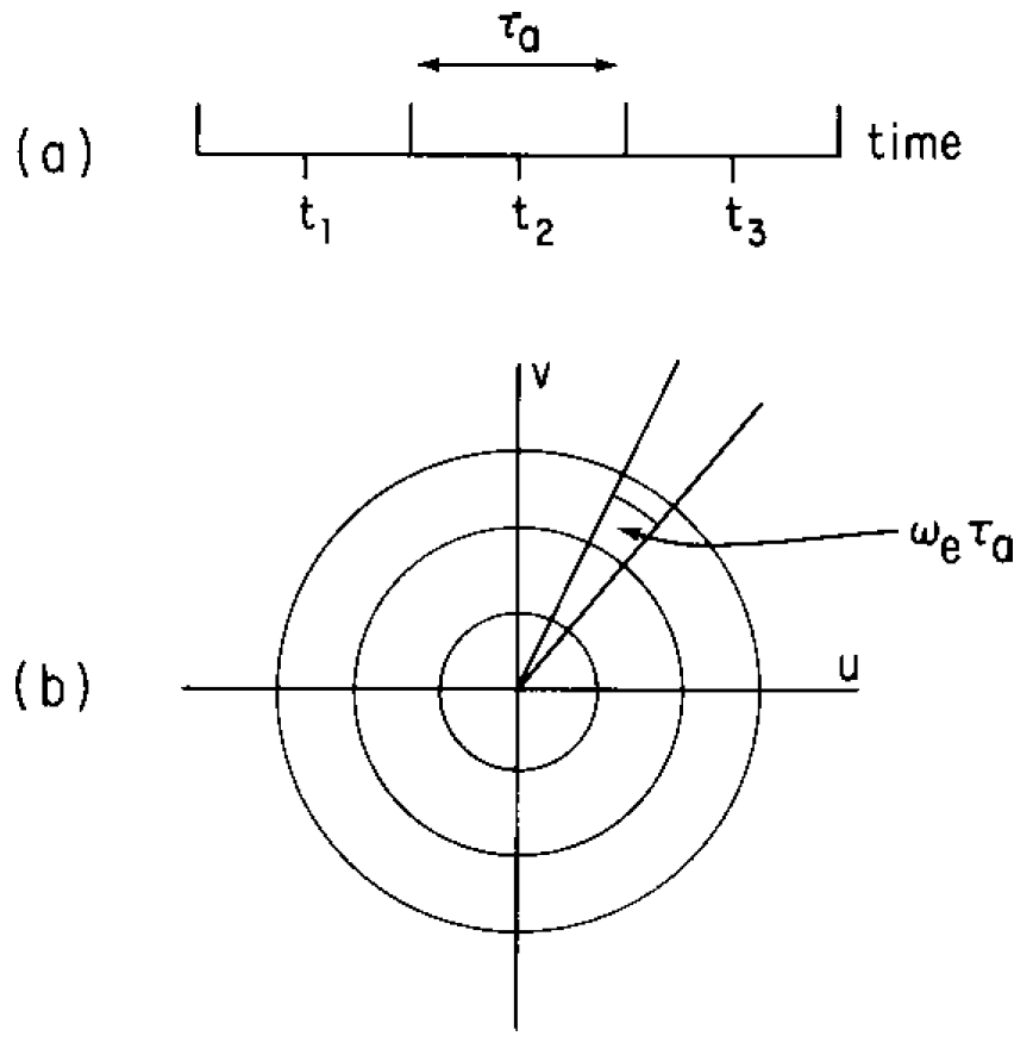


In general for a non-polar source and a non E-W baseline the effect is not a simple rotation.

Full treatment of BW and time average smearing in:
Chapter 18, Synthesis Imaging in Radio Astronomy

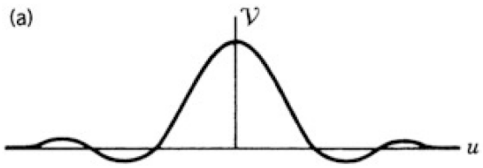
Short summary

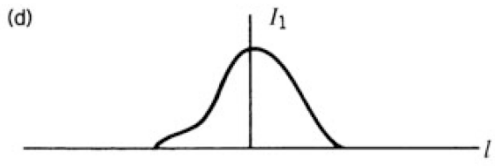
- FT of the aperture gives the antenna pattern.
- What we measure is the convolution of antenna pattern with the sky.

- For an interferometer we have the “synthesized” aperture: uv-coverage.
- FT of the uv-coverage gives the synthesized beam.
- And we measure visibilities of the sky convolved with the synthesized beam and attenuated by the response of the individual antenna called the primary beam.
- To obtain a good image, one needs a well sampled aperture. And the design of antenna configurations is aimed towards obtaining the best uv-coverage with minimum number of elements.

Sampled visibility

Consider observation of a source.
Measurements of visibilities with various baseline lengths are made.
What we have is a sampled visibility





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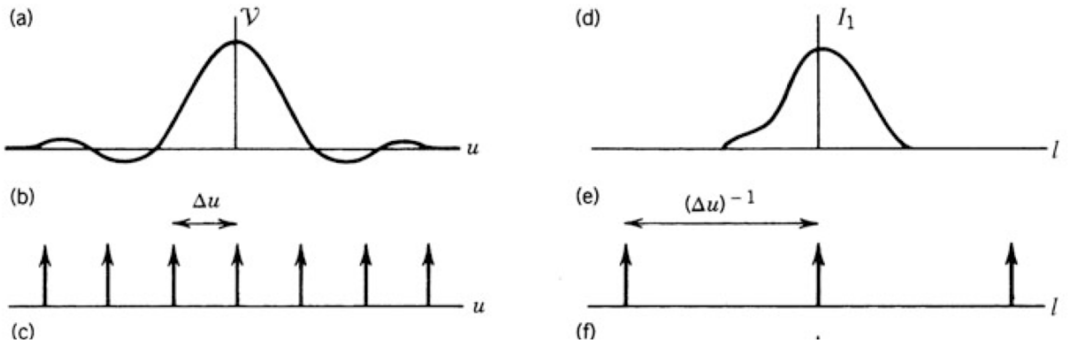

$$\left[\frac{1}{\Delta u}\right] \mathrm{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

Shah function

Sampled visibility

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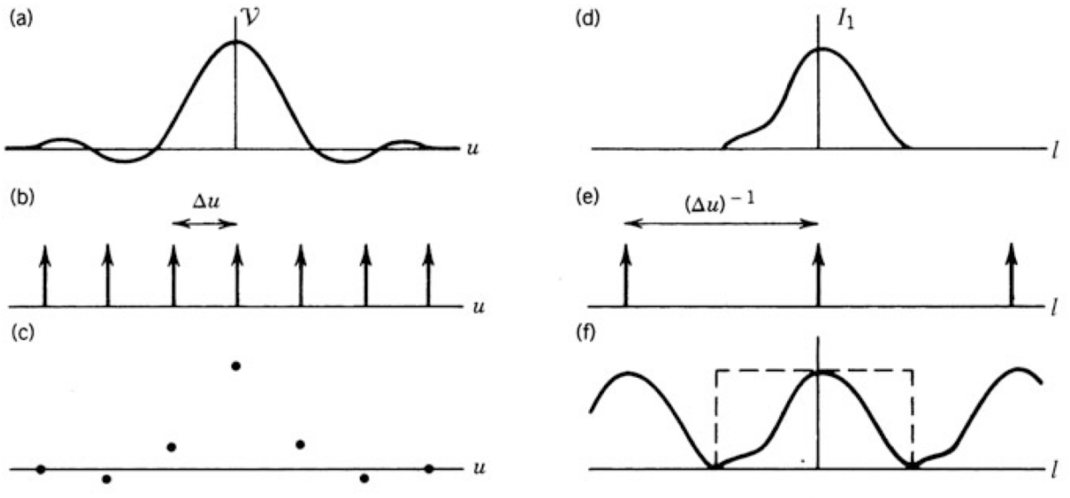

$$\left[\frac{1}{\Delta u}\right] \mathrm{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

Shah function

FT of sampling:

$$\mathrm{III}(l\Delta u) = \frac{1}{\Delta u} \sum_{p=-\infty}^{\infty} \delta\left(l - \frac{p}{\Delta u}\right)$$

Sampled visibility

Consider observation of a source.
Measurements of visibilities with various baseline lengths are made.
What we have is a sampled visibility

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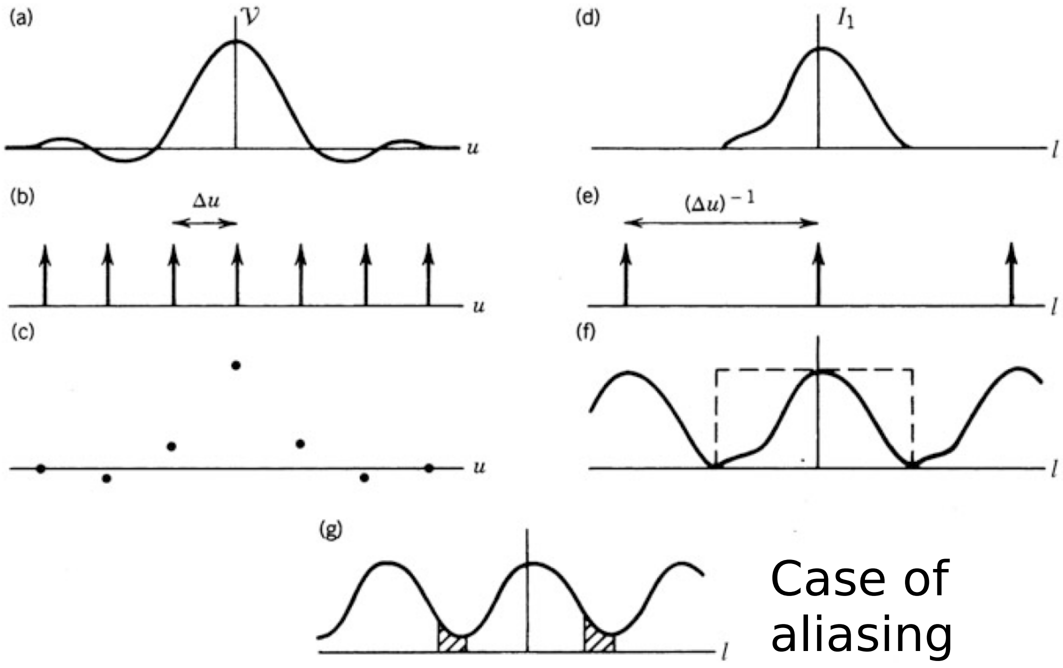


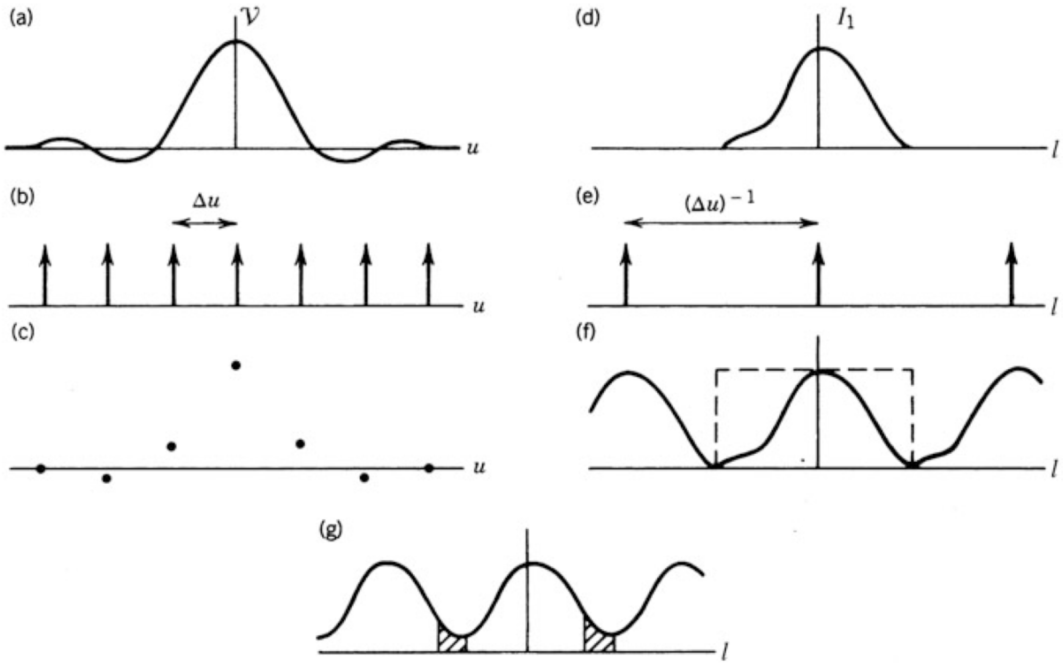
Case of aliasing

Sampled visibility

Consider observation of a source. Measurements of visibilities with various baseline lengths are made.
What we have is a sampled visibility

$$\left[\frac{1}{\Delta u}\right] \mathrm{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

Shah function



Interpolation in u-domain will correspond to removing the replications in the l domain.
This corresponds to a sinc function in the u-domain.

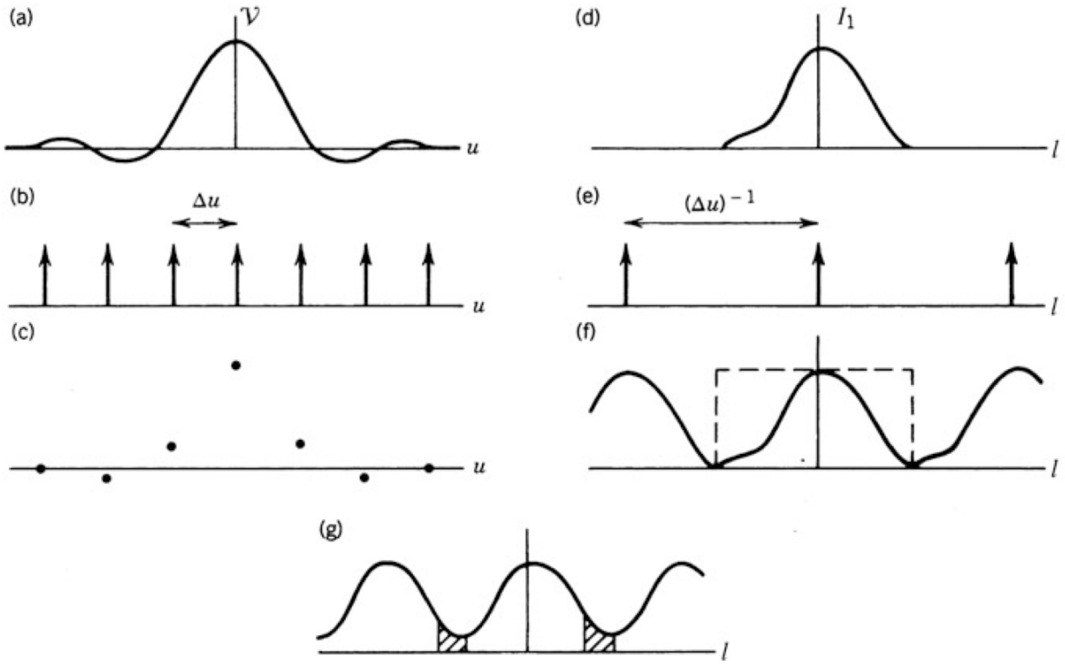
$$\frac{\sin \pi u / \Delta u}{\pi u}$$

Sampled visibility

Consider observation of a source. Measurements of visibilities with various baseline lengths are made.
What we have is a sampled visibility

$$\left[\frac{1}{\Delta u}\right] \mathrm{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

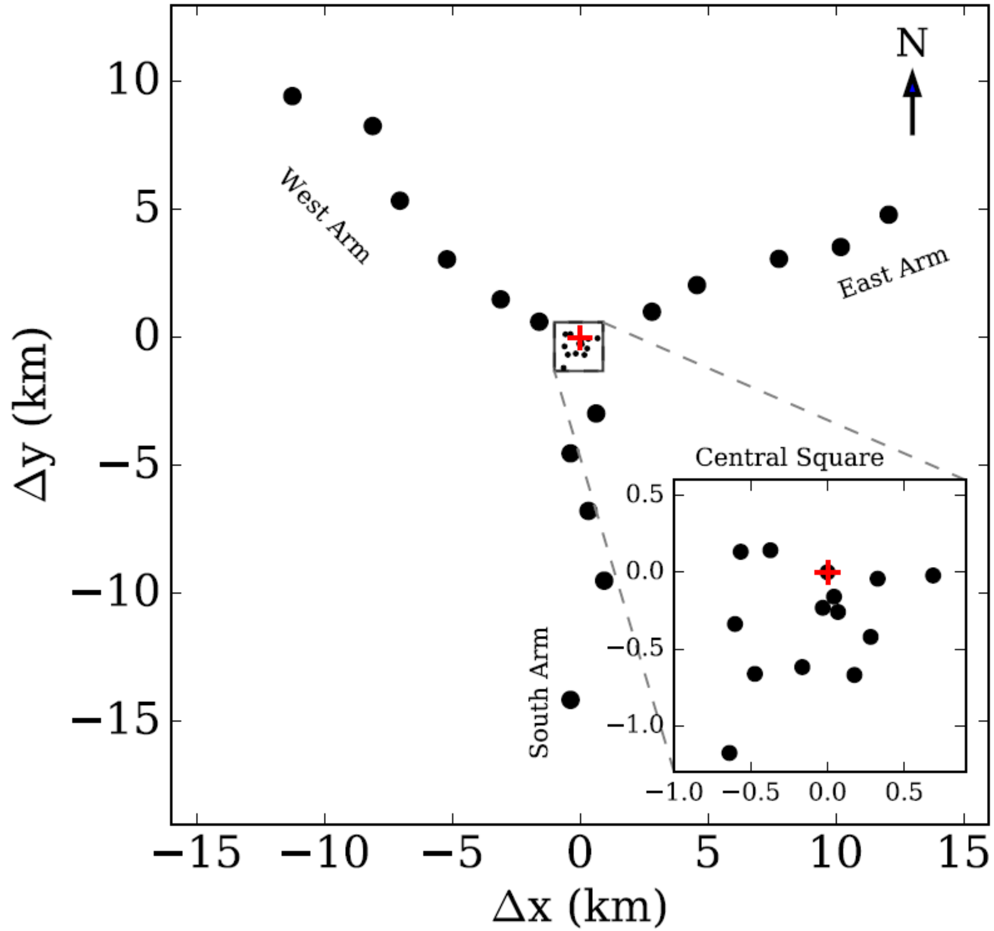
Shah function



If the intensity distribution is nonzero only within an interval of width l_w, $I_1(l)$ *is fully specified by sampling the visibility function at points spaced* $\Delta u = l_w^{-1}$ *in u – the critical sampling interval.*

Aperture synthesis

GMRT array configuration

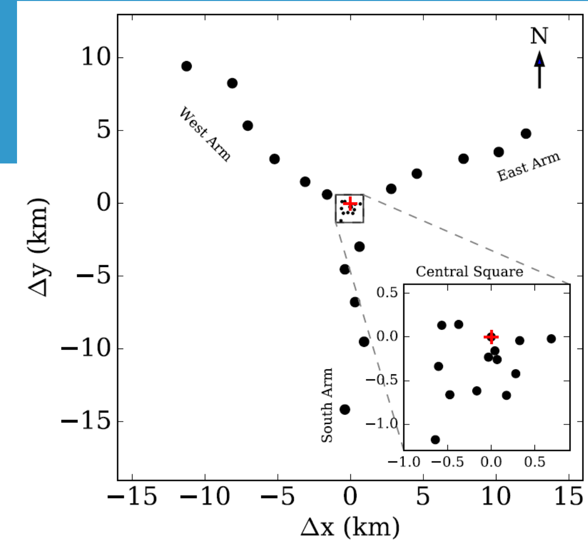


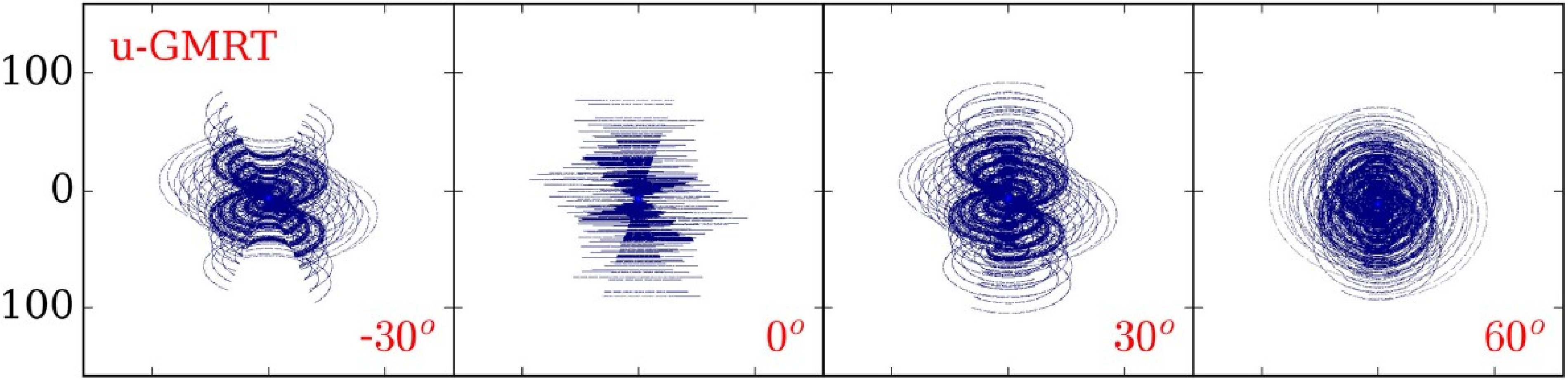
Latitude ~+19 degrees

Patra et al 2019

Aperture synthesis

GMRT array configuration and uv-coverage



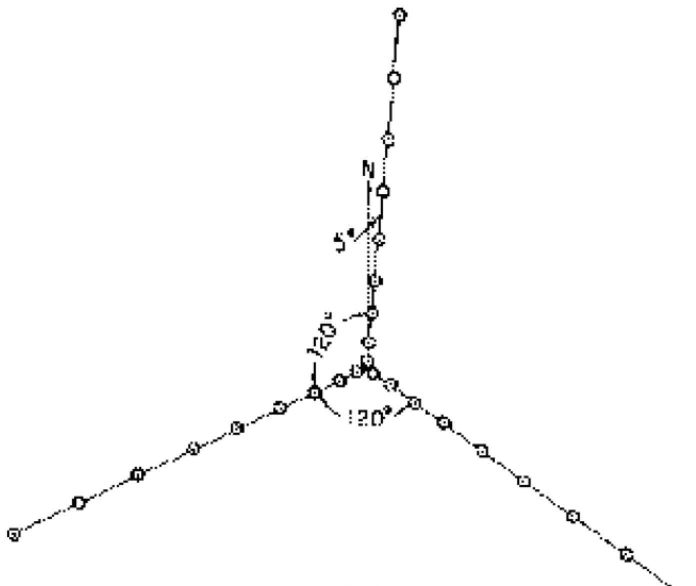


u (kilolambda)

v (kilolambda)

Patra et al 2019

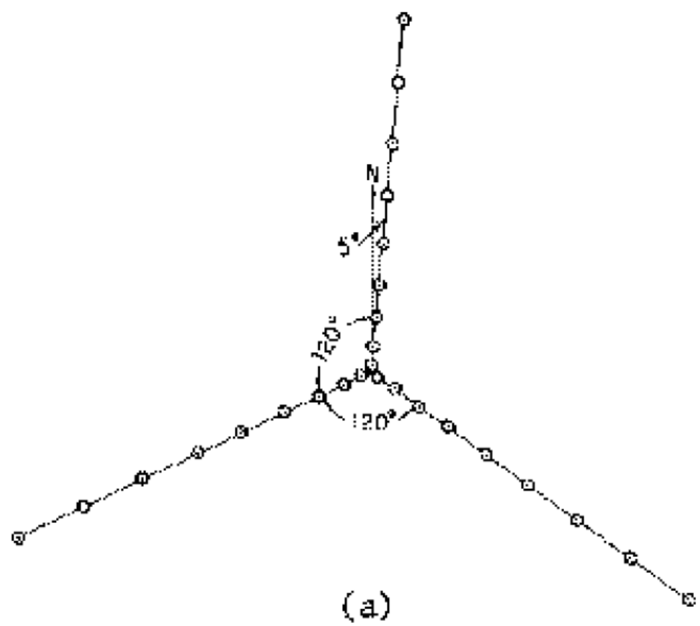
Aperture synthesis

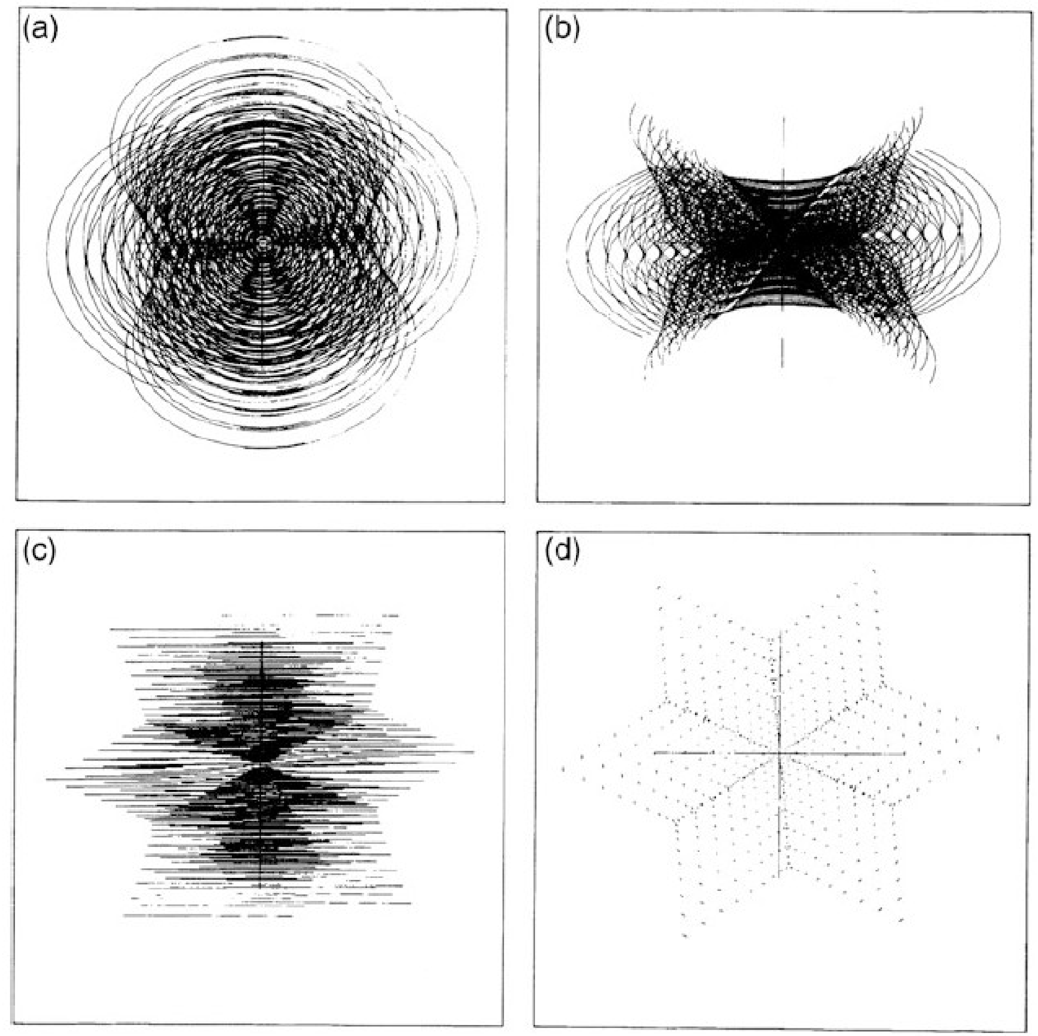


(a)

Jansky Very Large Array configuration
Latitude ~ 34 degrees

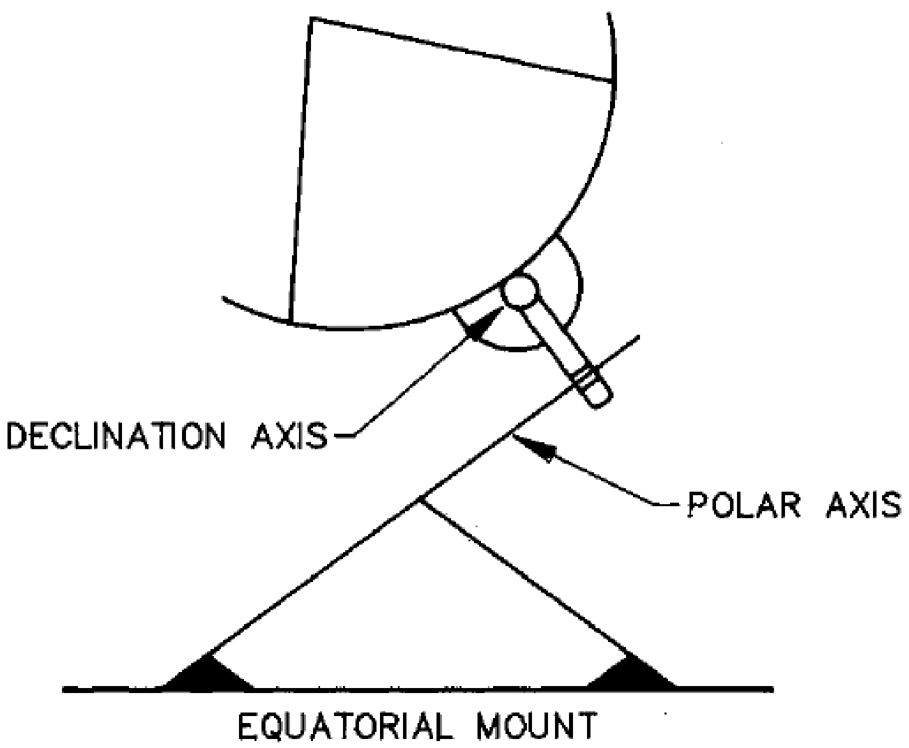
Aperture synthesis

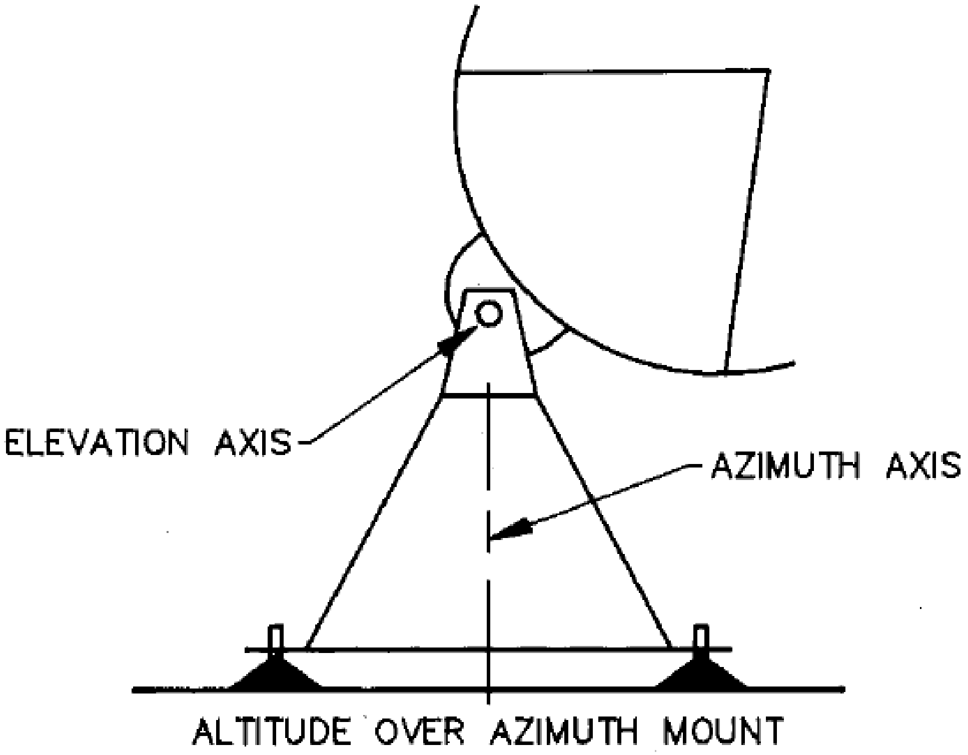




Declinations +45, +30 and 0 degrees shown in a, b and c. d is the snapshot coverage looking at the zenith.

Telescope mounts





Equatorially mounted telescopes

Westerbrok Synthesis Radio Telescope (WSRT), The Netherlands
An East-West Array

Ooty Radio Telescope

Images: Wikipedia

Alt-Az mounted telescopes

The GMRT

JVLA

Alt-Az mount and rotation of beam

Circum polar source observed with a non-circular beam of a alt-az mounted antenna.