- A two element interferometer

Astronomical Techniques II : **Lecture 6**

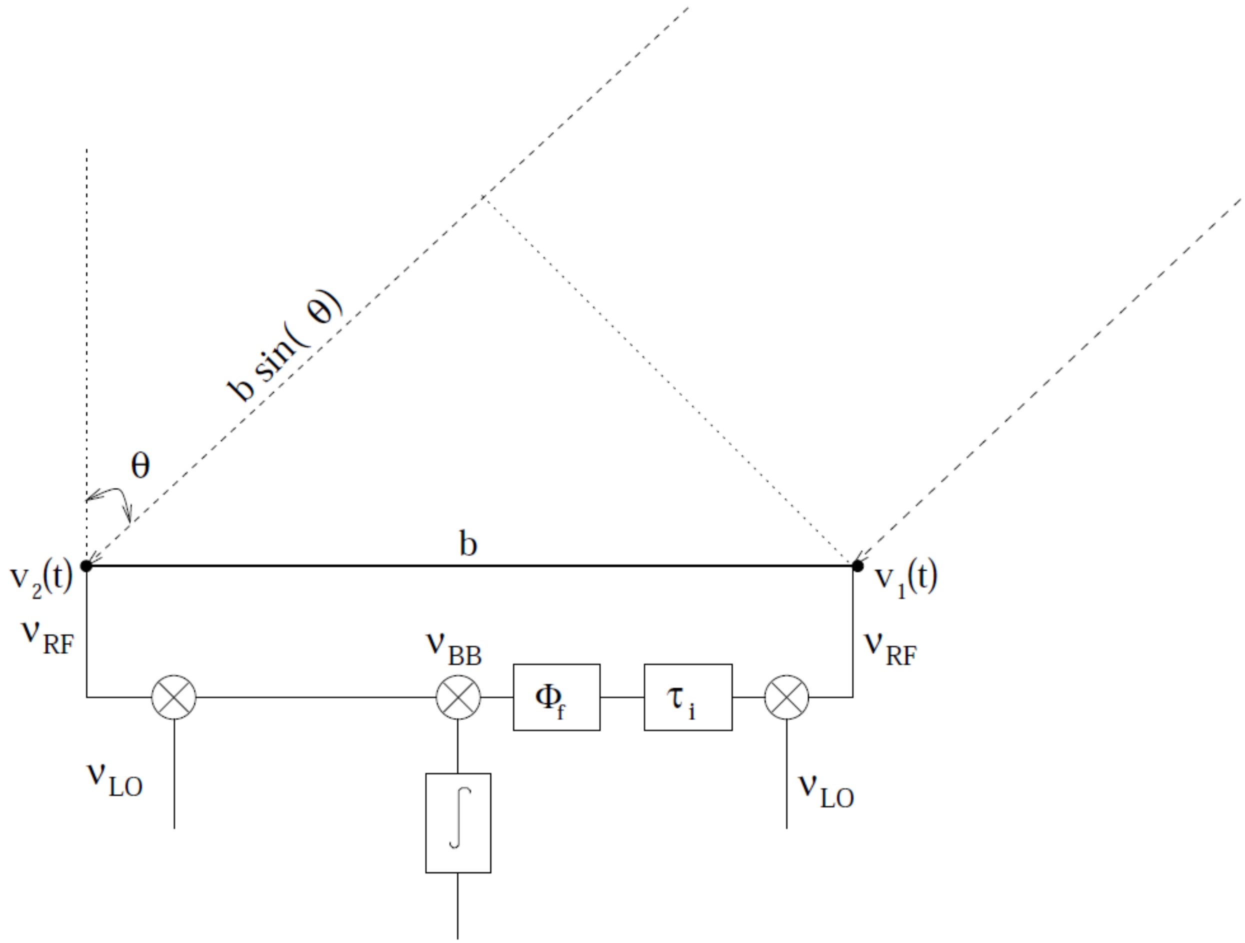
Ruta Kale

Low Frequency Radio Astronomy (Chp. 4)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy

Synthesis imaging in radio astronomy II, Chp 2

Interferometry and synthesis in radio astronomy (Chp 2)

Two element interferometer in practice



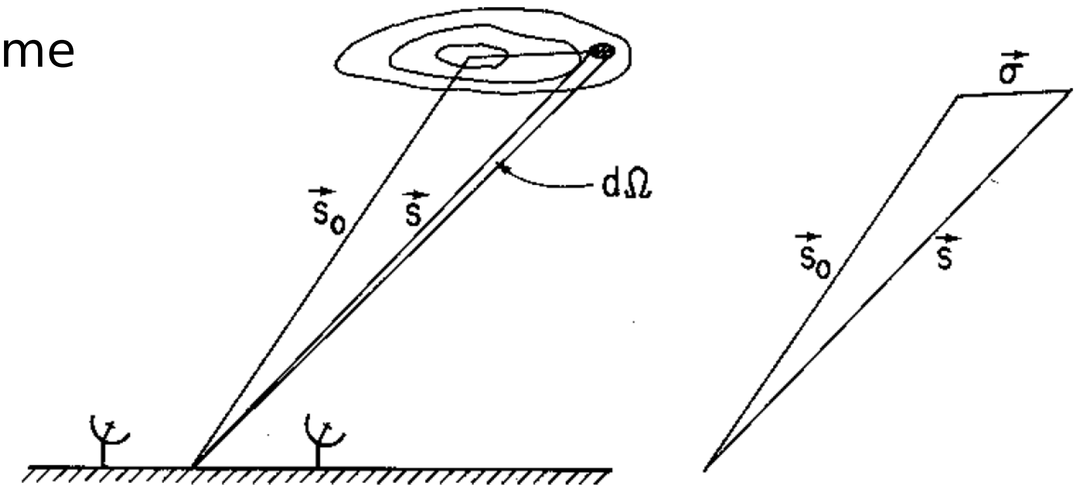
Extended source

Consider a source at s_0 with some small extent. Any point on the source can be written as

$s = s_0 + \sigma$

$s_0 . \sigma = 0$

$\tau_g = s_0.b$



From van Cittert Zernike theorem:

$$r(\tau_g) = Re\left[\int I(\mathbf{s}) e^{\frac{-i2\pi \mathbf{s.b}}{\lambda}} d\mathbf{s}\right]$$

$s.b = s_0.b + \sigma.b$

$$= Re\left[e^{\frac{-i2\pi \mathbf{s_0.b}}{\lambda}} \int I(\mathbf{s}) e^{\frac{-i2\pi \sigma.\mathbf{b}}{\lambda}} d\mathbf{s}\right]$$

$$= |\mathcal{V}| \cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}})$$

where $\mathcal{V} = |\mathcal{V}| e^{-i\Phi_{\mathcal{V}}}$

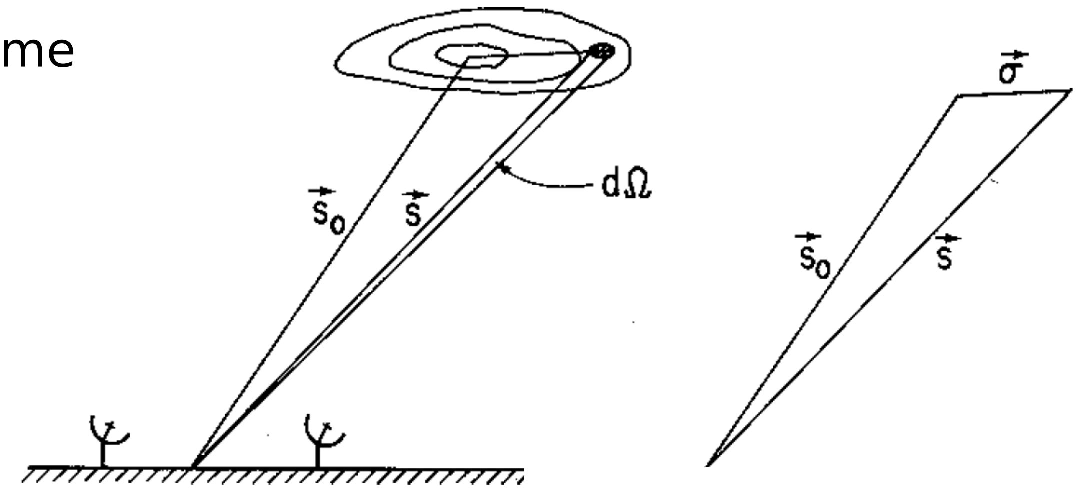
Extended source

Consider a source at s_0 with some small extent. Any point on the source can be written as

$s = s_0 + \sigma$

$s_0 . \sigma = 0$

$\tau_g = s_0 . b$



$$r(\tau_g) = |\mathcal{V}| \cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}})$$ where $$\mathcal{V} = |\mathcal{V}| e^{-i\Phi_{\mathcal{V}}}$$

Only contains the variation of the fringe as a function of earth's rotation or source rise-set. If an equal delay is introduced in the signals' path we will have:

$$r(\tau_g) = |\mathcal{V}| \cos(\Phi_{\mathcal{V}})$$

This instrumental delay has to change continuously as τ_g changes: *delay tracking*

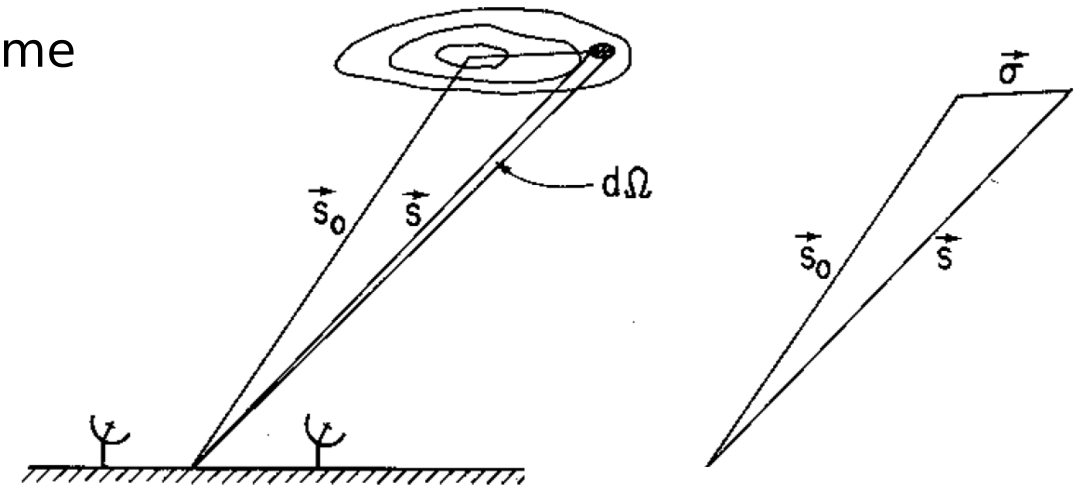
Extended source

Consider a source at s_0 with some small extent. Any point on the source can be written as

$s = s_0 + \sigma$

$s_0 . \sigma = 0$

$\tau_g = s_0 . b$



$$r(\tau_g) \;=\; |\mathcal{V}| \cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}})$$

where

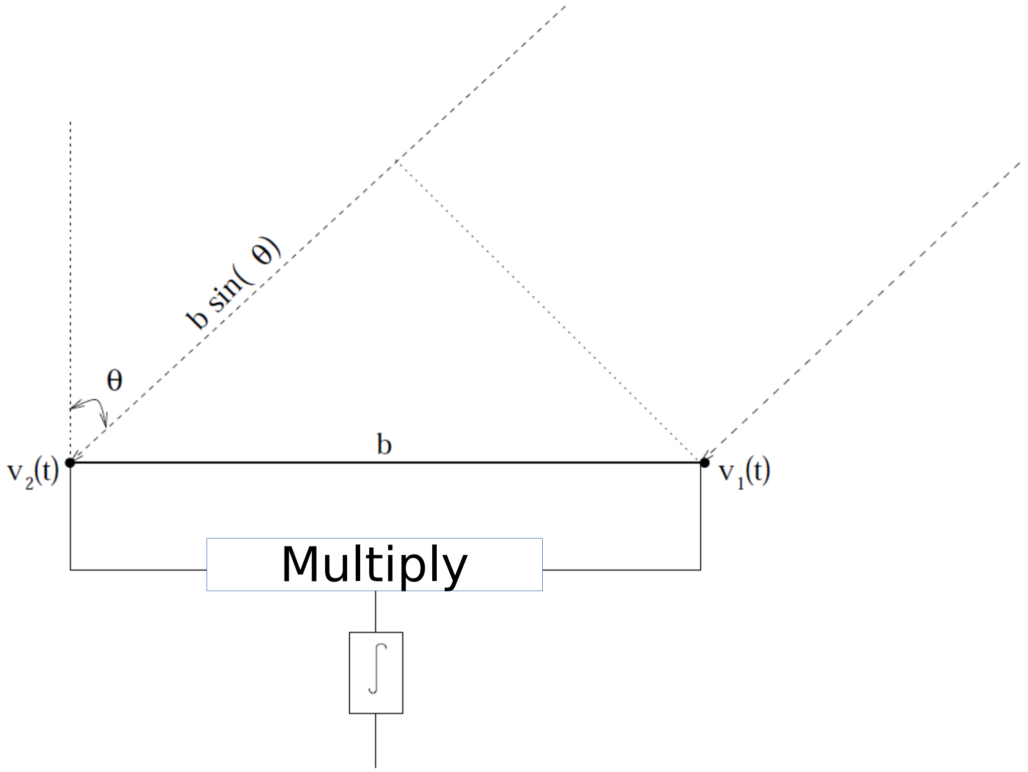
$$\mathcal{V} = |\mathcal{V}| e^{-i\Phi_{\mathcal{V}}}$$

Only contains the variation of the fringe as a function of earth's rotation or source rise-set. If an equal delay is introduced in the signals' path we will have:

$$r(\tau_g) = |\mathcal{V}| \cos(\Phi_{\mathcal{V}})$$

While τ_g is in RF the delay tracking is in baseband and thus needs to be properly accounted.

Two element interferometer: multiplying

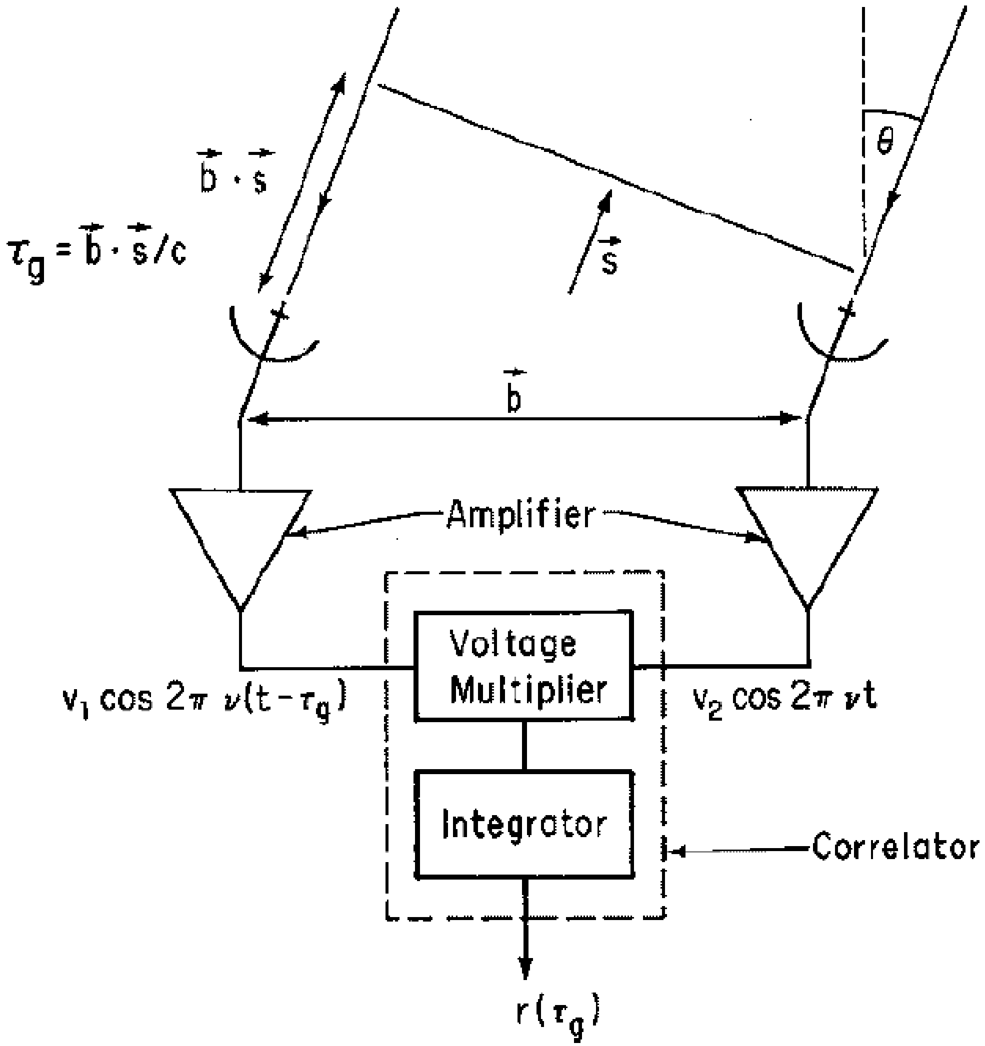


Geometric delay

$\tau_g = b \sin(\theta)/c$

$$r(\tau_g) \;=\; |\mathcal{V}|\cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}})$$ where $$\mathcal{V} = |\mathcal{V}|e^{-i\Phi_{\mathcal{V}}}$$

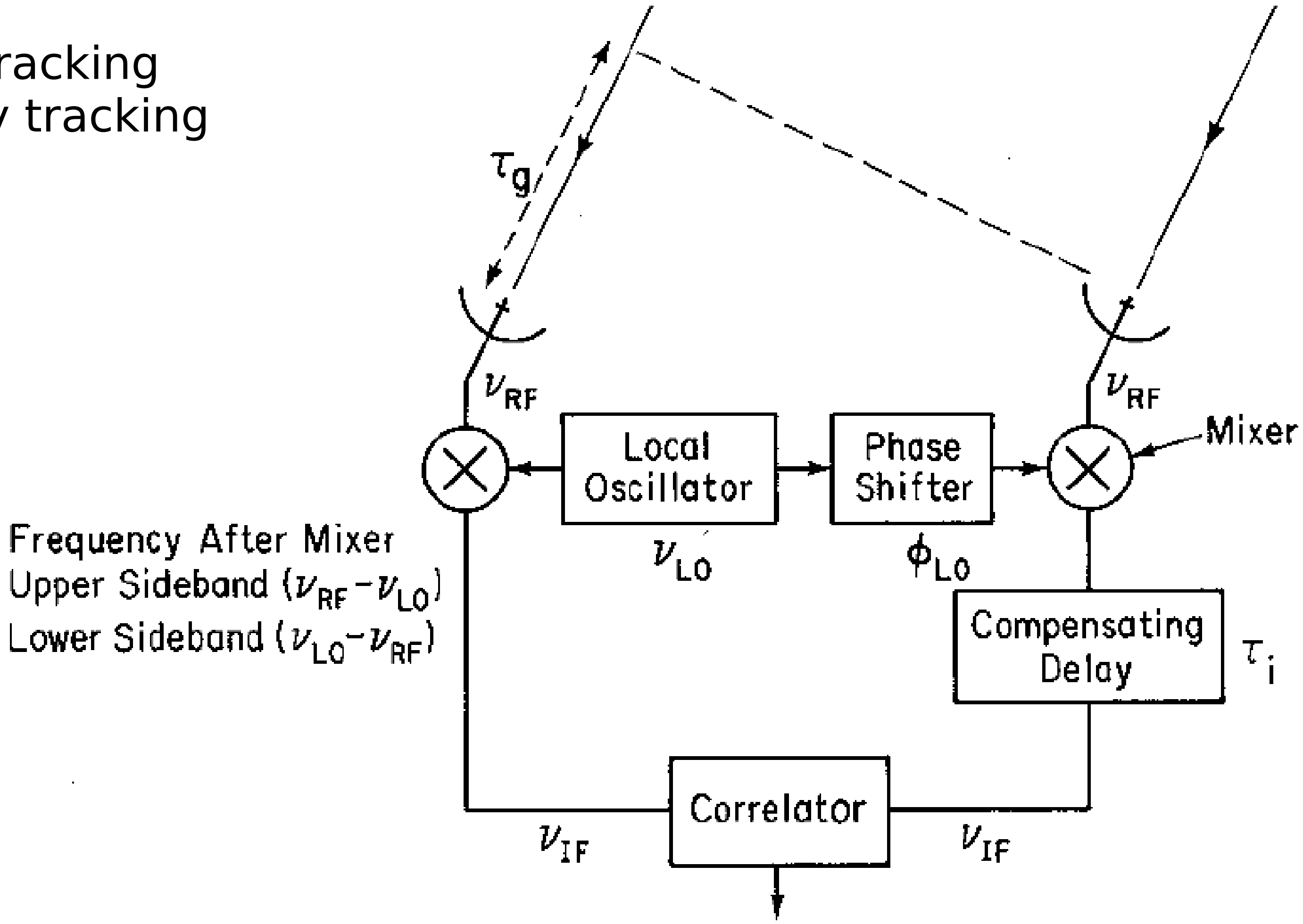
Two element interferometer



Two element interferometer

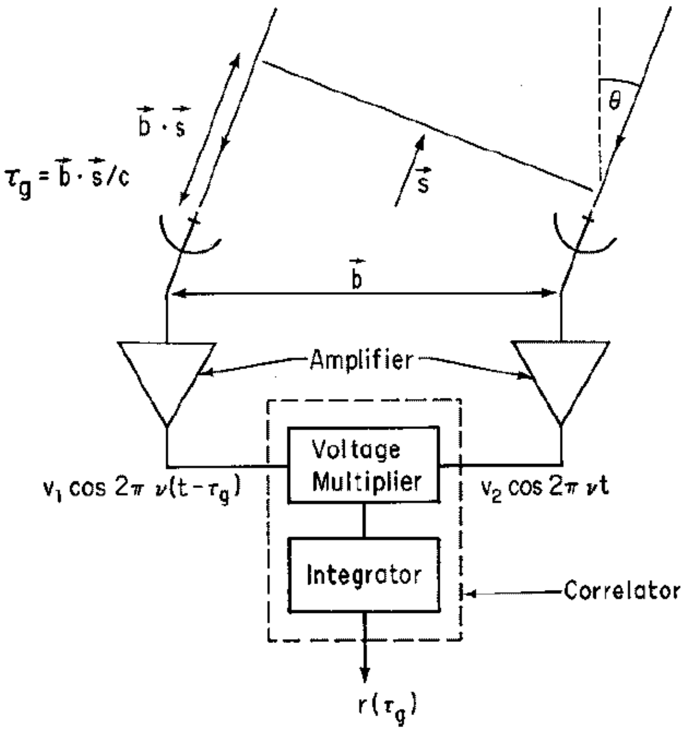
Interferometer in practice including compensation for the geometric delay: delay correction

Phase tracking
or delay tracking
centre

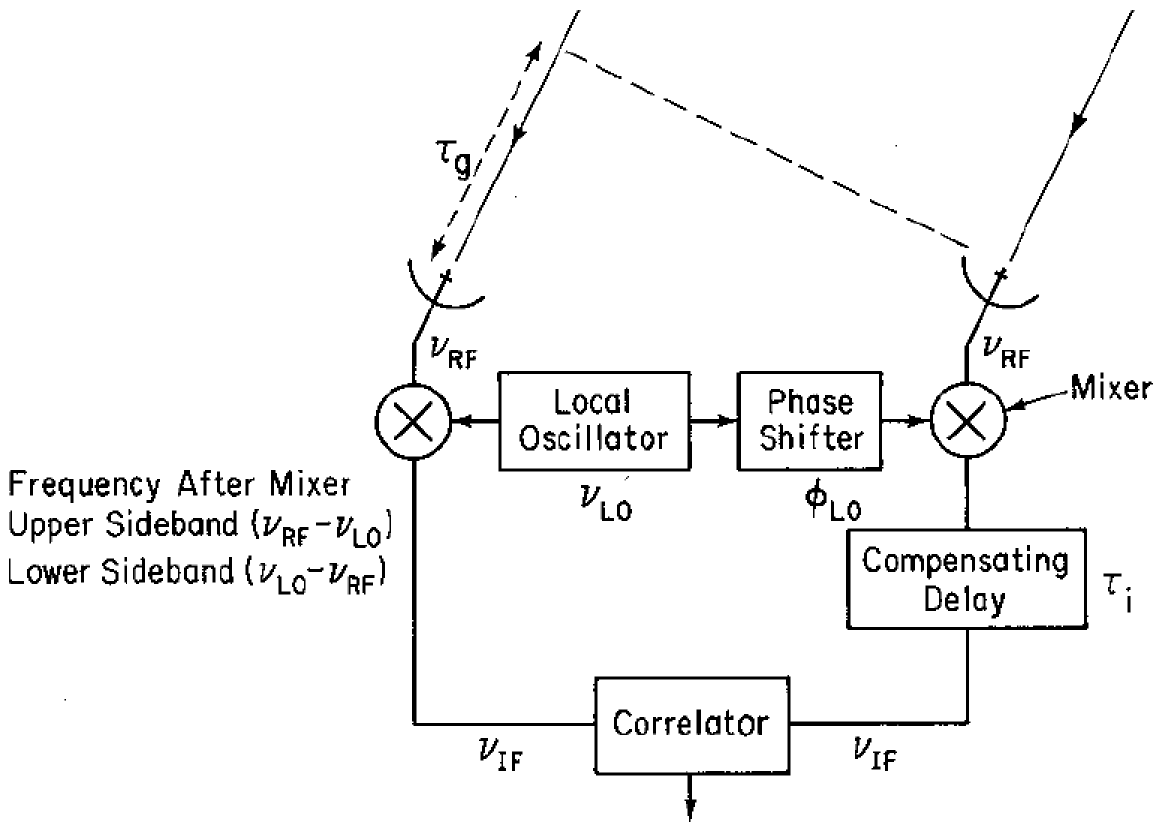


Two element interferometer

Basic interferometer



Interferometer in practice including compensation for the geometric delay: delay correction



Two element interferometer

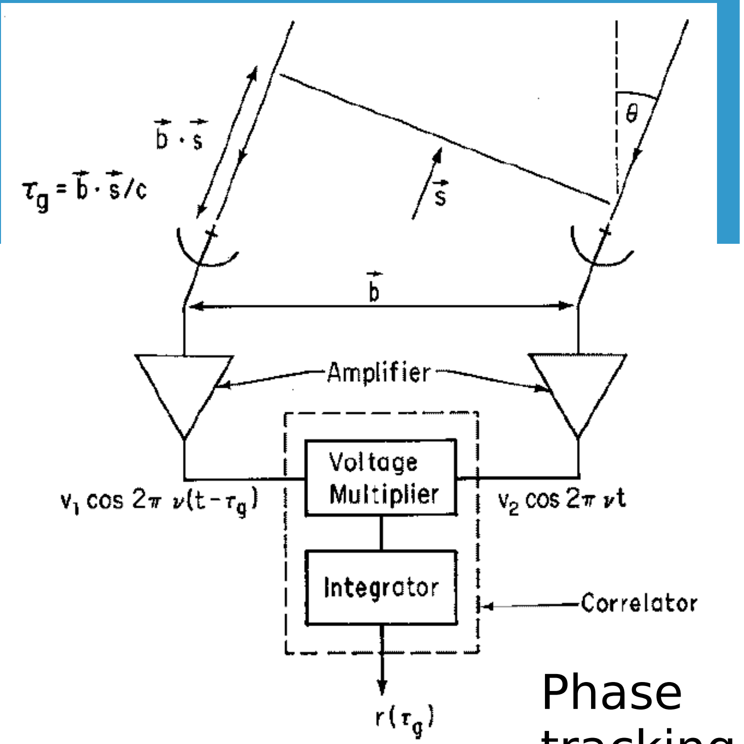
$$\langle V_1(t) V_2(t) \rangle$$

$$V_1(t) = v_1 \cos 2\pi\nu(t - \tau_g)$$

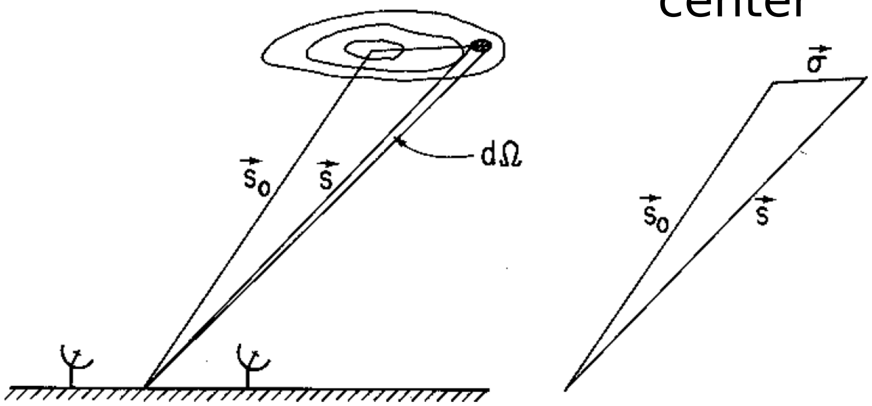
$$V_2(t) = v_2 \cos 2\pi\nu t$$

$$r(\tau_g) = v_1 v_2 \cos 2\pi\nu\tau_g$$

We would like to express the interferometer output in terms of brightness integrated over the sky.

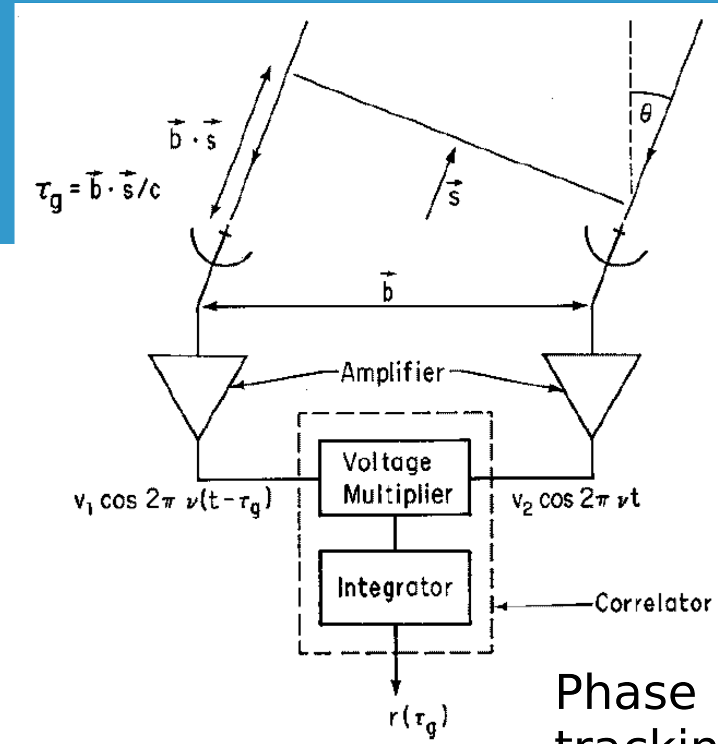


Phase tracking center



$$\mathbf{s} = \mathbf{s}_0 + \boldsymbol{\sigma}$$

Two element interferometer



$$\langle V_1(t) V_2(t) \rangle$$

$$V_1(t) = v_1 \cos 2\pi\nu(t - \tau_g)$$

$$V_2(t) = v_2 \cos 2\pi\nu t$$

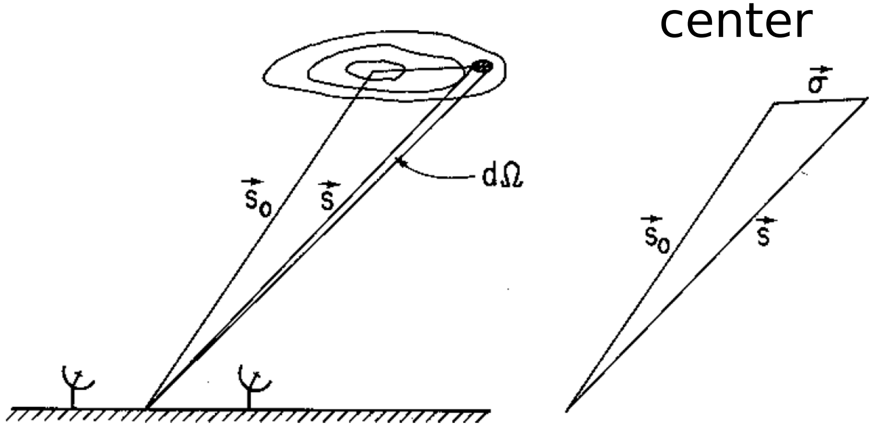
$$r(\tau_g) = v_1 v_2 \cos 2\pi\nu\tau_g \, .$$

Phase tracking center

We would like to express the interferometer output in terms of brightness integrated over the sky.

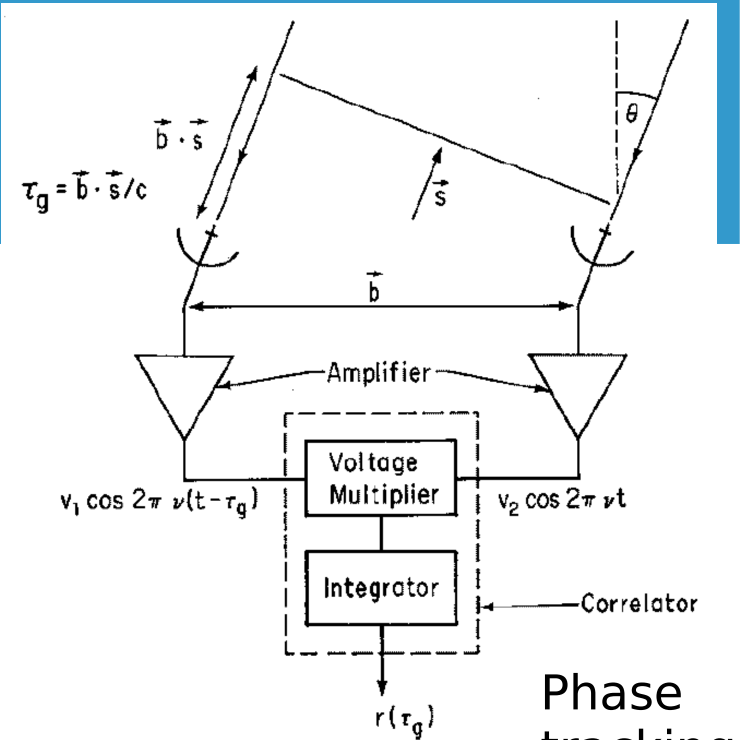
I(s) is the brightness in the direction s at frequency ν

W Hz^{-1} m^{-2} sr^{-1}



$$\mathbf{s} = \mathbf{s}_0 + \boldsymbol{\sigma}$$

Two element interferometer



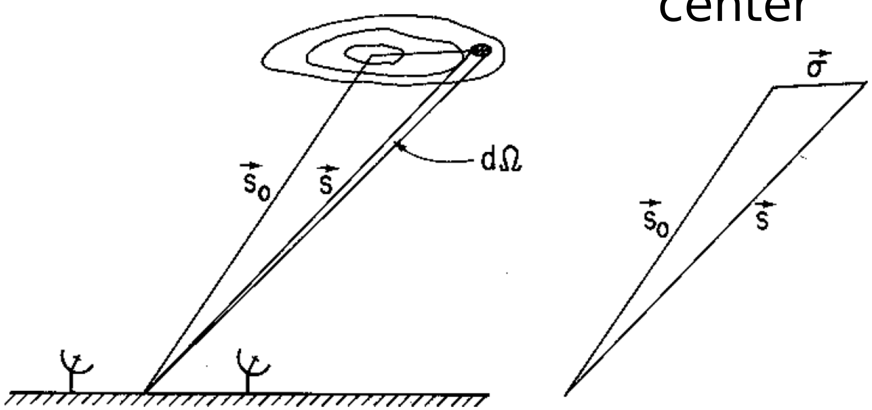
$$\langle V_1(t) V_2(t) \rangle$$

$$V_1(t) = v_1 \cos 2\pi\nu(t - \tau_g)$$

$$V_2(t) = v_2 \cos 2\pi\nu t$$

$$r(\tau_g) = v_1 v_2 \cos 2\pi\nu\tau_g \,.$$

Phase tracking center



I(s) is the brightness in the direction s at frequency ν

W Hz^{-1} m^{-2} sr^{-1}

The total power received in a bandwidth Δν from a source element dΩ is

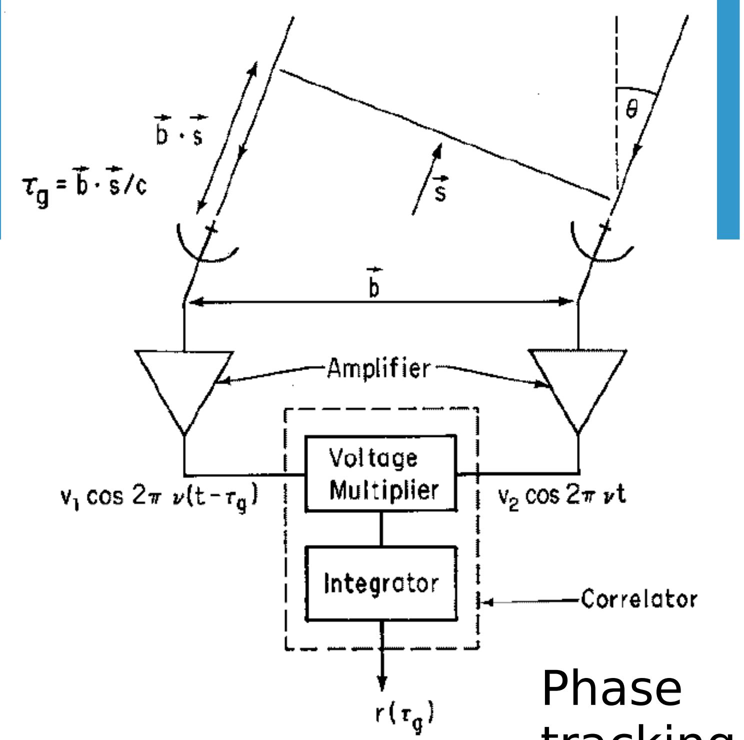
A(s)I(s)ΔνdΩ

A(s) is the effective collecting area of the antennas in the direction s

$$\mathbf{s} = \mathbf{s}_0 + \boldsymbol{\sigma}$$

Assumed to be the same for both the antennas

Two element interferometer



$$\langle V_1(t)V_2(t)\rangle$$

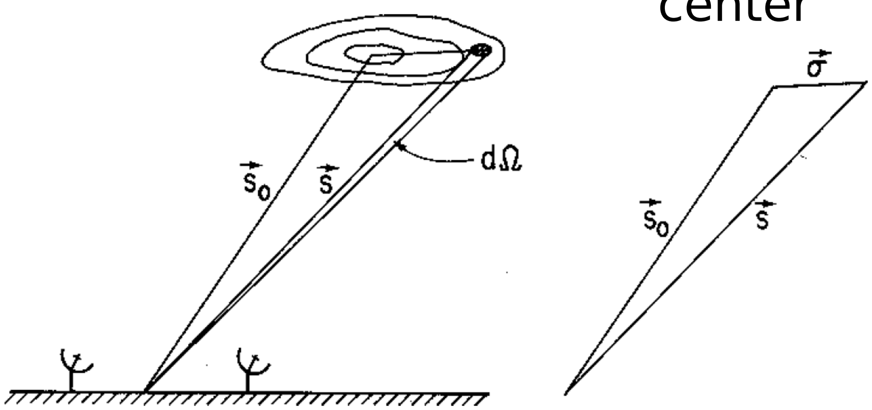
$$V_1(t) = v_1 \cos 2\pi\nu(t-\tau_g)$$

$$V_2(t) = v_2 \cos 2\pi\nu t$$

$$r(\tau_g) = v_1 v_2 \cos 2\pi\nu\tau_g \, .$$

$$dr = A(\mathbf{s})I(\mathbf{s})\Delta\nu\, d\Omega \cos 2\pi\nu\tau_g$$

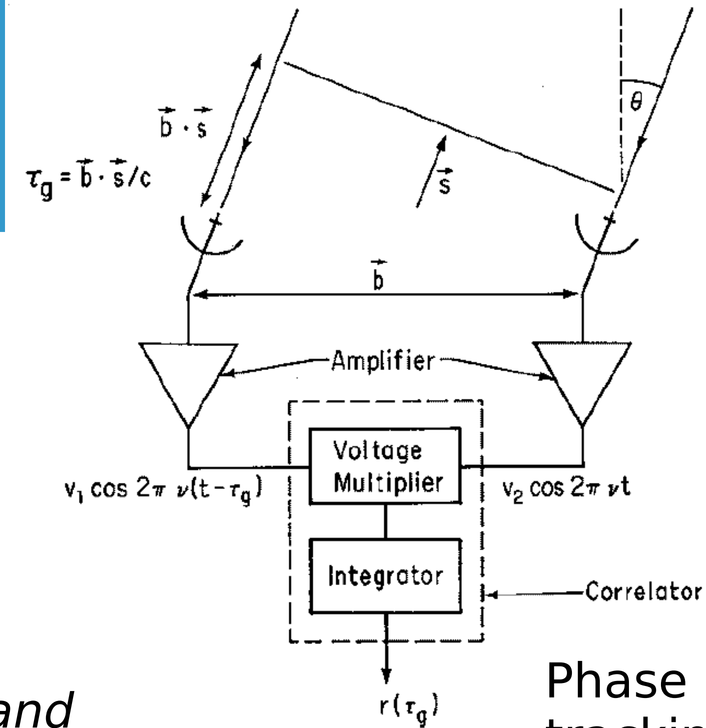
Phase tracking center



$$\mathbf{s} = \mathbf{s}_0 + \boldsymbol{\sigma}$$

Source is in the far field and is spatially incoherent.

Two element interferometer



$$\langle V_1(t)V_2(t)\rangle$$

$$V_1(t) = v_1 \cos 2\pi\nu(t - \tau_g)$$

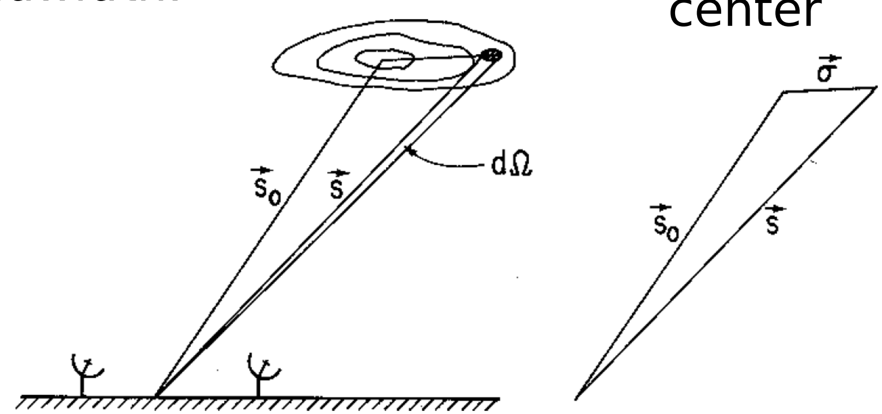
$$V_2(t) = v_2 \cos 2\pi\nu t$$

$$r(\tau_g) = v_1 v_2 \cos 2\pi\nu\tau_g \,.$$

Ignored the variation of A and I with bandwidth.

Phase tracking center

$$dr = A(\mathbf{s})I(\mathbf{s})\Delta\nu\, d\Omega \cos 2\pi\nu\tau_g$$

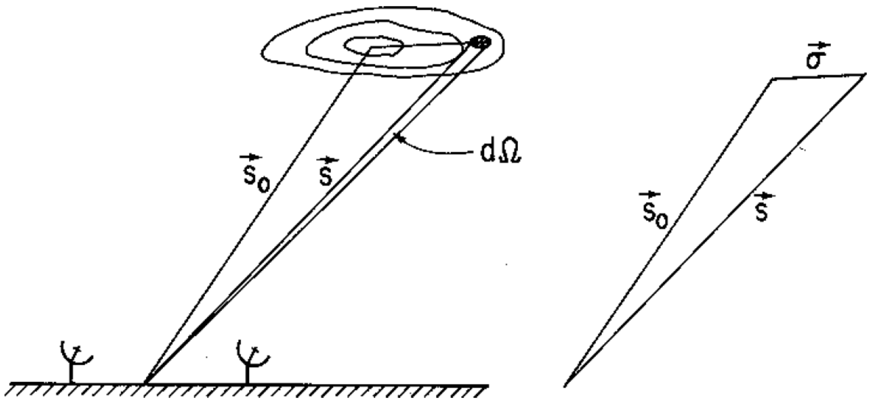


$$r = \Delta\nu \int_S A(\mathbf{s})I(\mathbf{s}) \cos \frac{2\pi\nu\, \mathbf{b}\cdot\mathbf{s}}{c}\, d\Omega$$

Integral taken over the entire sphere – however field of view, source structure restrict this to a small region.

$$\mathbf{s} = \mathbf{s}_0 + \boldsymbol{\sigma}$$

$$r = \Delta\nu \int_S A(\mathbf{s}) I(\mathbf{s}) \cos \frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}}{c}\, d\Omega$$



s_0 is the phase tracking centre

$$\mathbf{s} = \mathbf{s}_0 + \boldsymbol{\sigma}$$

$$\begin{aligned} r &= \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right) \int_S A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) \cos \frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\, d\Omega \\ &- \Delta\nu \sin\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right) \int_S A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) \sin \frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\, d\Omega \end{aligned}$$

Visibility

$$
\begin{aligned}
r &= \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega \\
&- \Delta\nu \sin\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega
\end{aligned}
$$

Introducing visibility:

$$
V \equiv |V|e^{i\phi_V} = \int_S \mathcal{A}(\boldsymbol{\sigma})I(\boldsymbol{\sigma})e^{-2\pi i\nu\mathbf{b}\cdot\boldsymbol{\sigma}/c}\,d\Omega
$$

Visibility

$$r = \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$
$$- \Delta\nu \sin\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$

Introducing visibility:

$$V \equiv |V|e^{i\phi_V} = \int_S \mathcal{A}(\boldsymbol{\sigma})I(\boldsymbol{\sigma})e^{-2\pi i\nu\mathbf{b}\cdot\boldsymbol{\sigma}/c}\,d\Omega$$

$$\mathcal{A}(\boldsymbol{\sigma}) \equiv A(\boldsymbol{\sigma})/A_0$$

Normalized antenna reception pattern. A_0 is the reception pattern at the beam centre

Writing real and imaginary parts separately gives:

Visibility

$$r = \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$
$$- \Delta\nu \sin\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$

$\mathcal{A}(\boldsymbol{\sigma}) \equiv A(\boldsymbol{\sigma})/A_0$ Normalized antenna reception pattern.

$$V \equiv |V|e^{i\phi_V} = \int_S \mathcal{A}(\boldsymbol{\sigma})I(\boldsymbol{\sigma})e^{-2\pi i\nu\mathbf{b}\cdot\boldsymbol{\sigma}/c}\,d\Omega$$

$$A_0|V|\cos\phi_V = \int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$ Real

$$A_0|V|\sin\phi_V = -\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$ Imaginary

Visibility

$$r = \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$

$$- \Delta\nu \sin\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$

$\mathcal{A}(\boldsymbol{\sigma}) \equiv A(\boldsymbol{\sigma})/A_0$ Normalized antenna reception pattern.

$$V \equiv |V|e^{i\phi_V} = \int_S \mathcal{A}(\boldsymbol{\sigma})I(\boldsymbol{\sigma})e^{-2\pi i\nu\mathbf{b}\cdot\boldsymbol{\sigma}/c}\,d\Omega$$

$$A_0|V|\cos\phi_V = \int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$ Real

$$A_0|V|\sin\phi_V = -\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$ Imaginary

$$
\begin{aligned}
r &= \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega \\
&- \Delta\nu \sin\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega
\end{aligned}
$$

$$\mathcal{A}(\boldsymbol{\sigma}) \equiv A(\boldsymbol{\sigma})/A_0$$

$$V \equiv |V|e^{i\phi_V} = \int_S \mathcal{A}(\boldsymbol{\sigma})I(\boldsymbol{\sigma})e^{-2\pi i\nu\mathbf{b}\cdot\boldsymbol{\sigma}/c}\,d\Omega$$

$$r = A_0\Delta\nu|V|\cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c} - \phi_V\right)$$

$$
\begin{aligned}
r &= \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega \\
&- \Delta\nu \sin\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega
\end{aligned}
$$

$$\mathcal{A}(\boldsymbol{\sigma}) \equiv A(\boldsymbol{\sigma})/A_0$$

$$V \equiv |V|e^{i\phi_V} = \int_S \mathcal{A}(\boldsymbol{\sigma})I(\boldsymbol{\sigma})e^{-2\pi i\nu\mathbf{b}\cdot\boldsymbol{\sigma}/c}\,d\Omega$$

$$r = A_0\Delta\nu|V|\cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c} - \phi_V\right)$$

Amplitude and phase of the fringe is measured and then amplitude and phase of V are derived after calibration. Source brightness then derived by inversion of V.

$$r = \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$

$$- \Delta\nu \sin\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$

$$\mathcal{A}(\boldsymbol{\sigma}) \equiv A(\boldsymbol{\sigma})/A_0$$

$$V \equiv |V|e^{i\phi_V} = \int_S \mathcal{A}(\boldsymbol{\sigma})I(\boldsymbol{\sigma})e^{-2\pi i\nu\mathbf{b}\cdot\boldsymbol{\sigma}/c}\,d\Omega$$

$$r = A_0\Delta\nu|V|\cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c} - \phi_V\right)$$

V needs to be measured at sufficiently wide range of $\nu\mathbf{b}\cdot\boldsymbol{\sigma}/c$

Effect of bandwidth

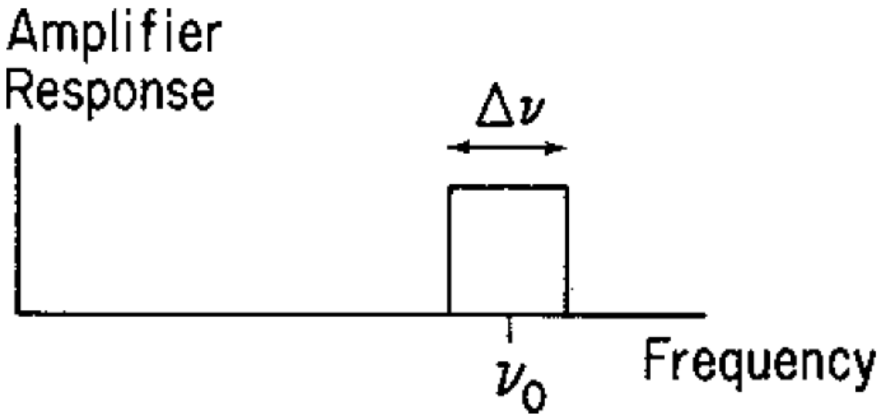
$$r = A_0 \Delta\nu |V| \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c} - \phi_V\right)$$

Response in an infinitesimal bandwidth dν

$$dr = A_0 |V| \cos\left(2\pi\nu\tau_g - \phi_V\right)\ d\nu$$

Effect of bandwidth

$$r = A_0 \Delta\nu |V| \cos\left(\frac{2\pi\nu\, \mathbf{b}\cdot\mathbf{s}_0}{c} - \phi_V\right)$$

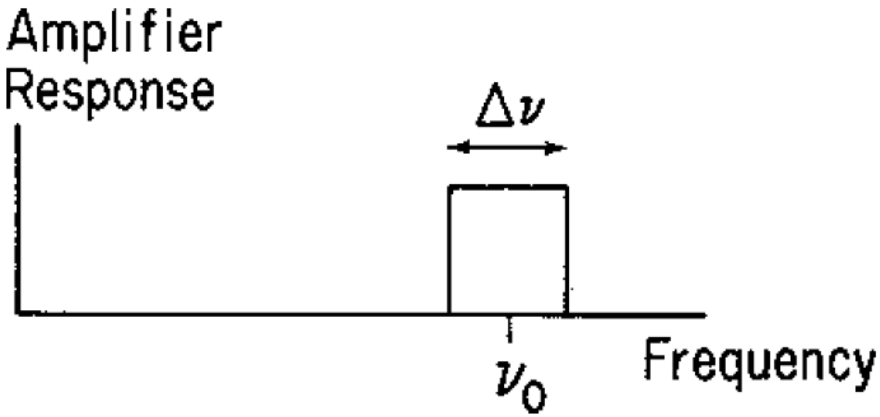


Response in an infinitesimal bandwidth dν

$$dr = A_0 |V| \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

Effect of bandwidth

$$r = A_0 \Delta\nu |V| \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c} - \phi_V\right)$$



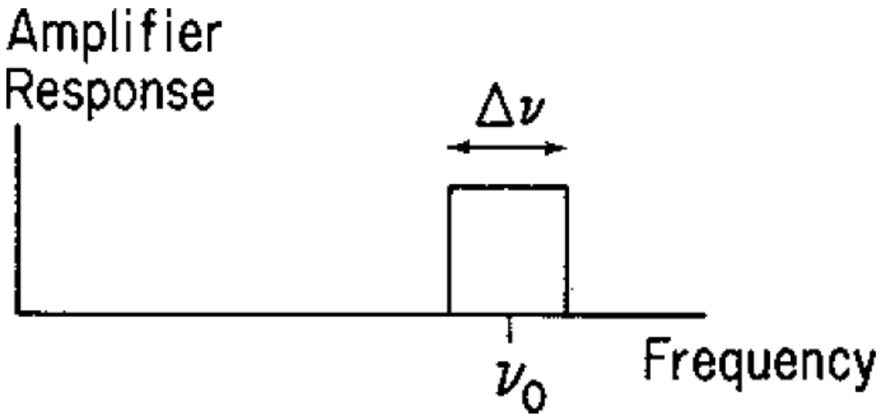
Response in an infinitesimal bandwidth dν

$$dr = A_0 |V| \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

$$r = A_0 |V| \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

Effect of bandwidth

$$r = A_0 \Delta\nu |V| \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c} - \phi_V\right)$$



Response in an infinitesimal bandwidth dν

$$dr = A_0 |V| \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

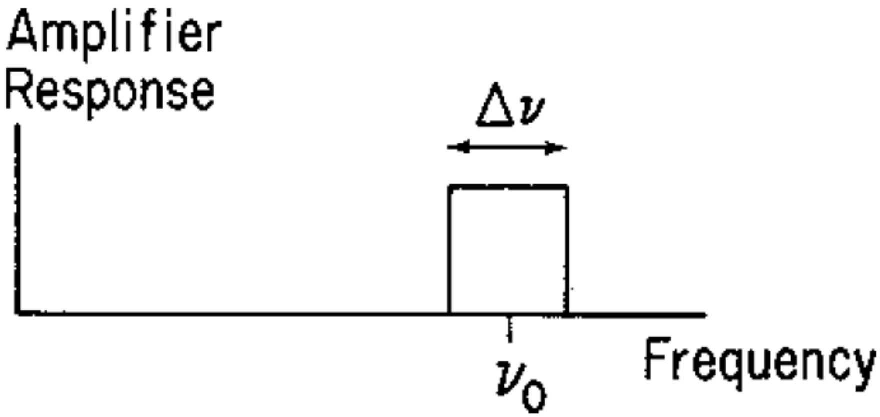
$$r = A_0 |V| \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

Work this out.

sin(a+b) = sin(a) cos(b) + cos(a) sin(b)

Effect of bandwidth

$$r = A_0 \Delta\nu |V| \cos\left(\frac{2\pi\nu\, \mathbf{b}\cdot\mathbf{s}_0}{c} - \phi_V\right)$$



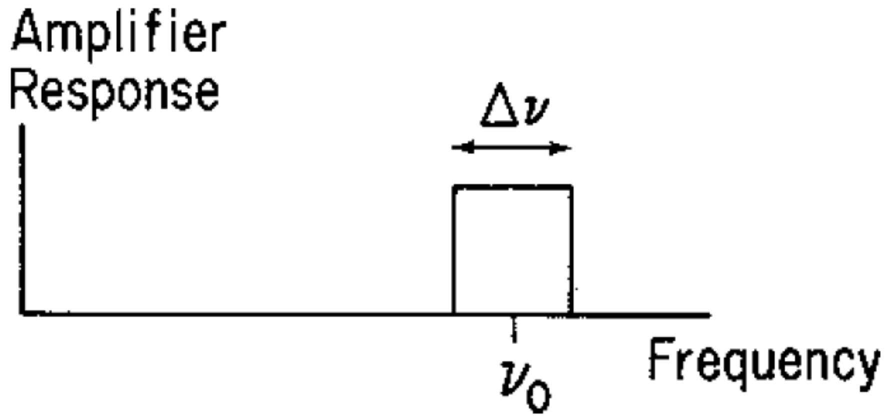
Response in an infinitesimal bandwidth dν

$$dr = A_0 |V| \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

$$r = A_0 |V| \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

$$= A_0 |V| \Delta\nu \frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

Effect of bandwidth



Modifies the fringe amplitude – maximum only when geometric delay is zero.

$$r = A_0|V|\Delta\nu \frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

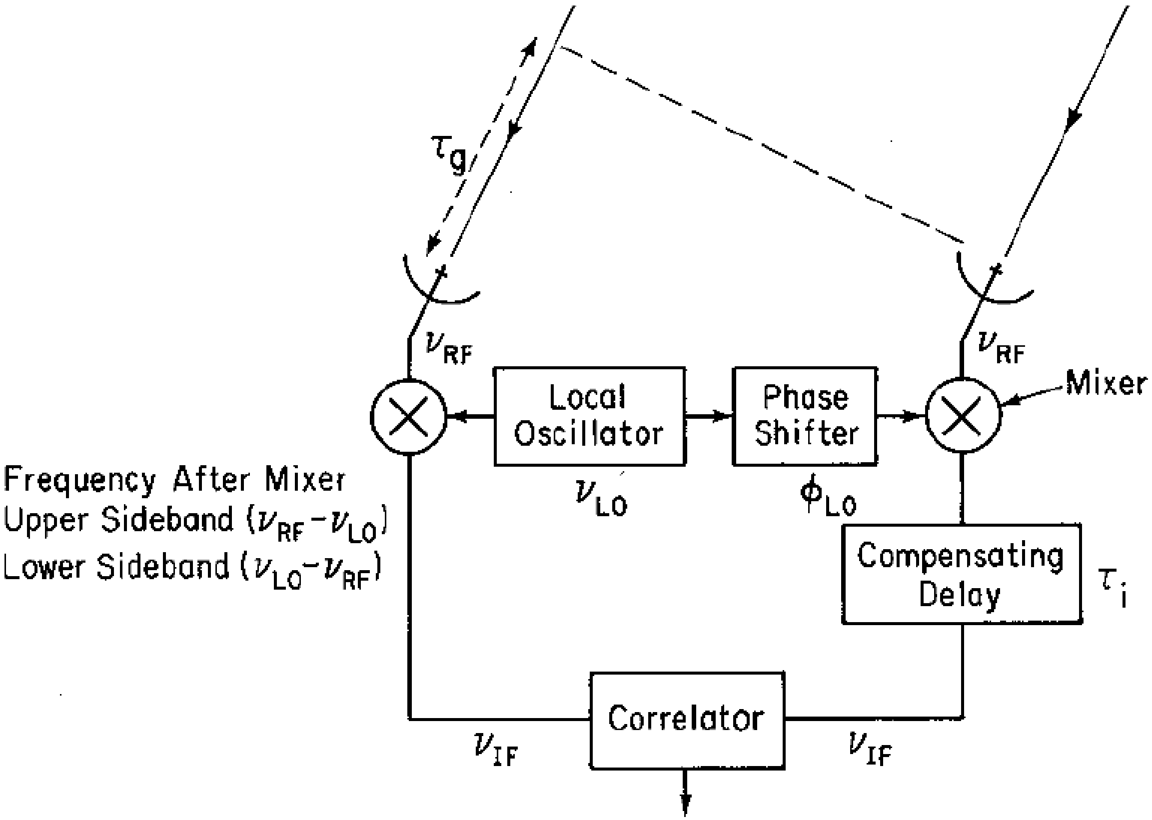
Instructive to calculate at what angular offset the fringe amplitude fall for e. g. to 1% of its maximum. Use of the following approximation:

$$|\pi\Delta\nu\tau_g| \ll 1 \qquad \frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \approx 1 - \frac{(\pi\Delta\nu\tau_g)^2}{6}$$

Delay tracking and frequency conversion

The geometric delay needs to be compensated in order to observe a source from rise to set.

RF converted to IF in a mixer and in one of the signals a delay to compensate the geometric delay is introduced.

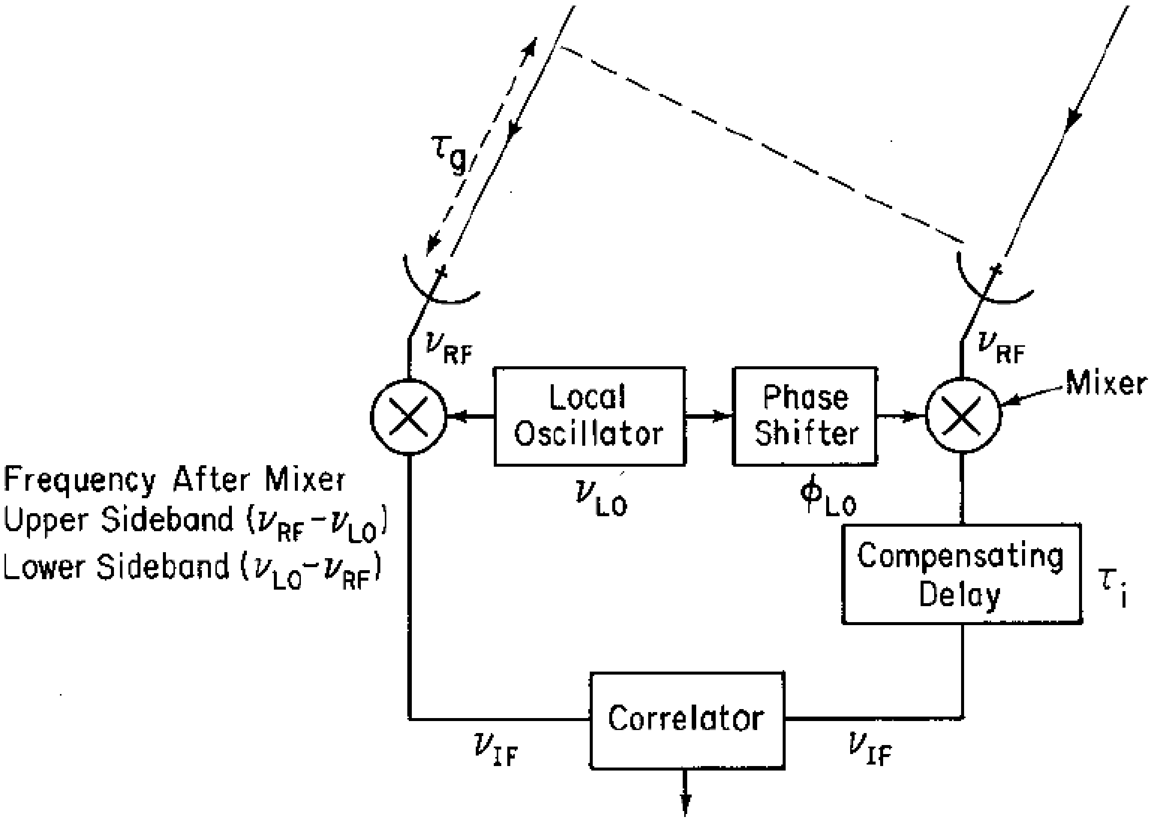


Low noise amplifier – not shown here but is present before the mixer in low frequency systems.

Delay tracking and frequency conversion

The geometric delay needs to be compensated in order to observe a source from rise to set.

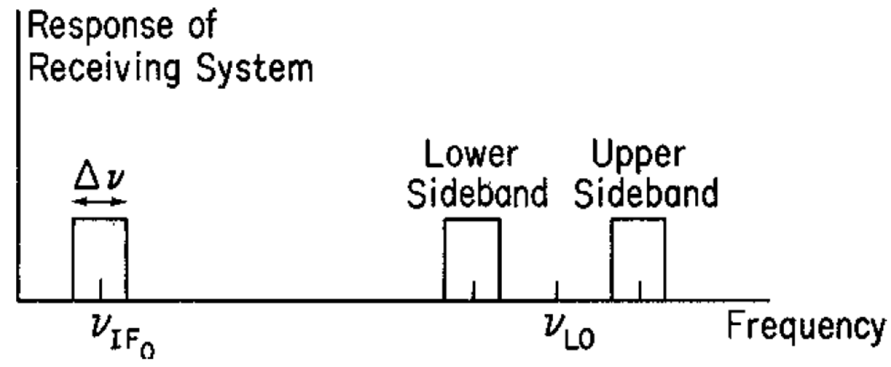
RF converted to IF in a mixer and in one of the signals a delay to compensate the geometric delay is introduced.



Low noise amplifier – not shown here but is present before the mixer in low frequency systems.

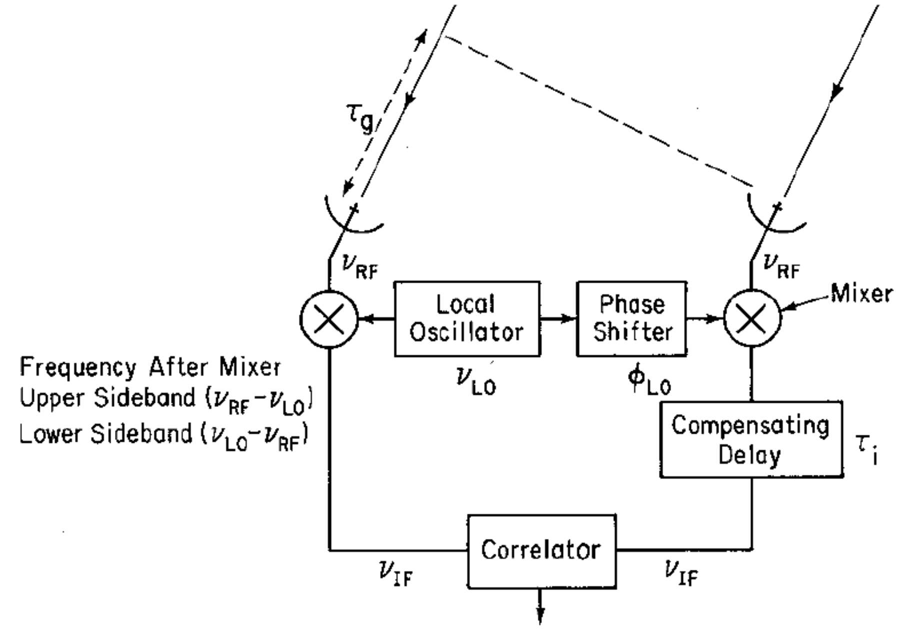
Delay tracking and frequency conversion

$$\nu_{\mathrm{RF}} = \nu_{\mathrm{LO}} \pm \nu_{\mathrm{IF}}$$



Upper side band and lower side band.

Both can be processed further (*double sideband system*) or using filters only one may be taken – called a *single sideband system*.



We need to see the phase changes before reaching the input of the correlator.

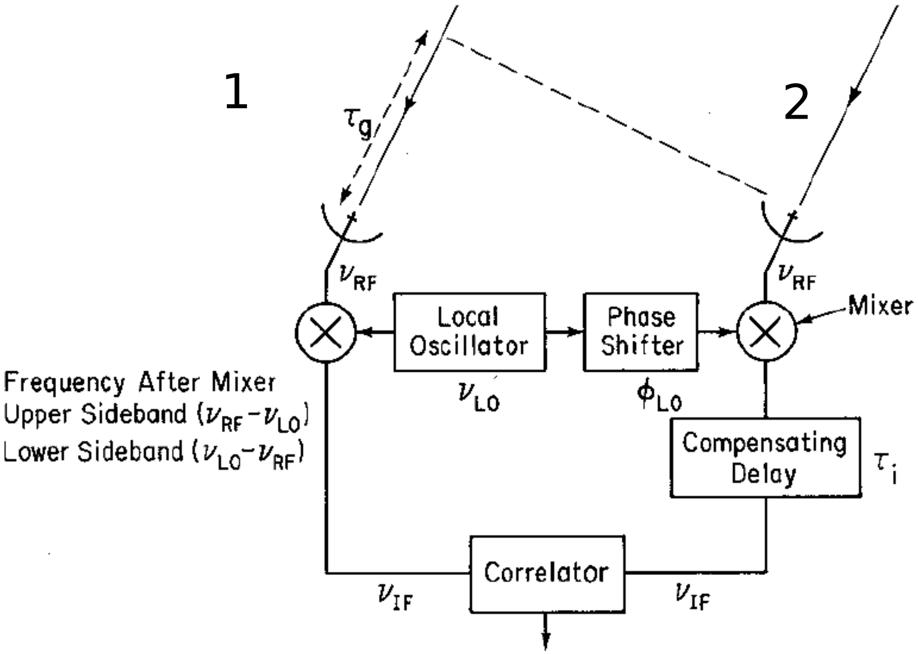
Delay tracking and frequency conversion

$$\nu_{\mathrm{RF}} = \nu_{\mathrm{LO}} \pm \nu_{\mathrm{IF}}$$

For USB (single sideband system):

$$\phi_1 = 2\pi\nu_{\mathrm{RF}}\tau_g = 2\pi(\nu_{\mathrm{LO}} + \nu_{\mathrm{IF}})\tau_g$$

$$\phi_2 = 2\pi\nu_{\mathrm{IF}}\tau_i + \phi_{\mathrm{LO}}$$

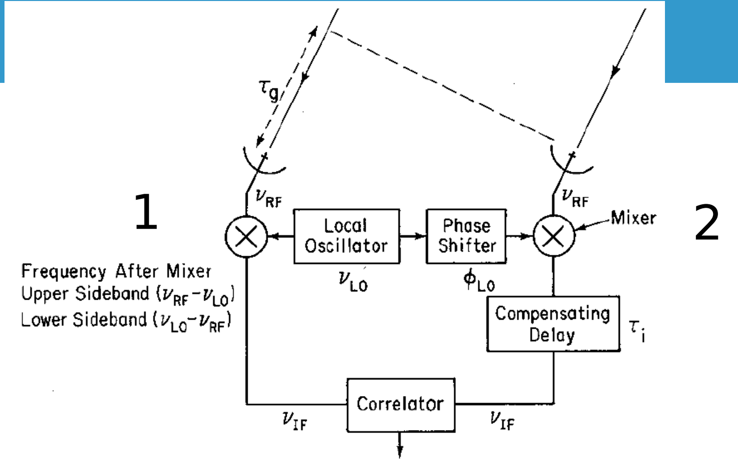


Instrumental delay that is given to compensate for the geometric delay

Is the phase difference between the LO signal at the two mixers

LO provides a single frequency but the RF and IF have a bandwidth; Two sidebands

Delay tracking and frequency conversion



$$\nu_{\rm RF} = \nu_{\rm LO} \pm \nu_{\rm IF}$$

For USB:

$$\phi_1 = 2\pi\nu_{\rm RF}\tau_g = 2\pi(\nu_{\rm LO} + \nu_{\rm IF})\tau_g$$

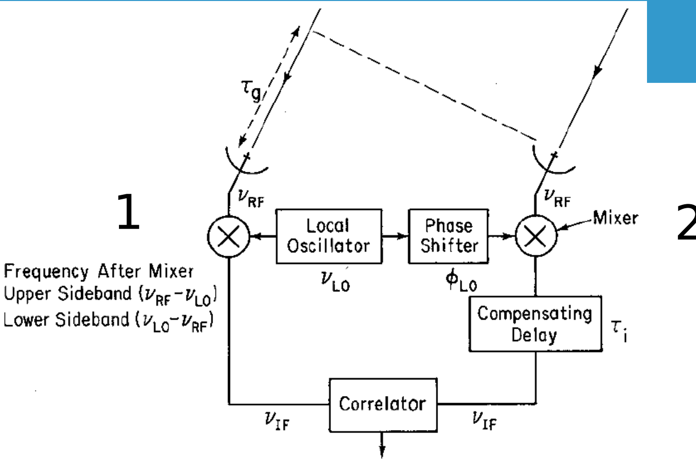
$$\phi_2 = 2\pi\nu_{\rm IF}\tau_i + \phi_{\rm LO}$$

$$r = A_0|V|\Delta\nu\frac{\sin\pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g}\cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

Obtain the response by replacing the argument of cosine function with $\phi_1 - \phi_2 - \phi_V$ and integrating over IF from $\nu_{\rm IF_0} - \Delta\nu/2$ to $\nu_{\rm IF_0} + \Delta\nu/2$

Recall $$r = A_0|V|\int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2}\cos\left(2\pi\nu\tau_g - \phi_V\right)\,d\nu$$

Delay tracking and frequency conversion



$$\nu_{RF} = \nu_{LO} \pm \nu_{IF}$$

For USB:

$$\phi_1 = 2\pi\nu_{RF}\tau_g = 2\pi(\nu_{LO} + \nu_{IF})\tau_g$$

$$\phi_2 = 2\pi\nu_{IF}\tau_i + \phi_{LO}$$

$$r = A_0|V|\Delta\nu \frac{\sin\pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

Obtain the response by replacing the argument of cosine function wit

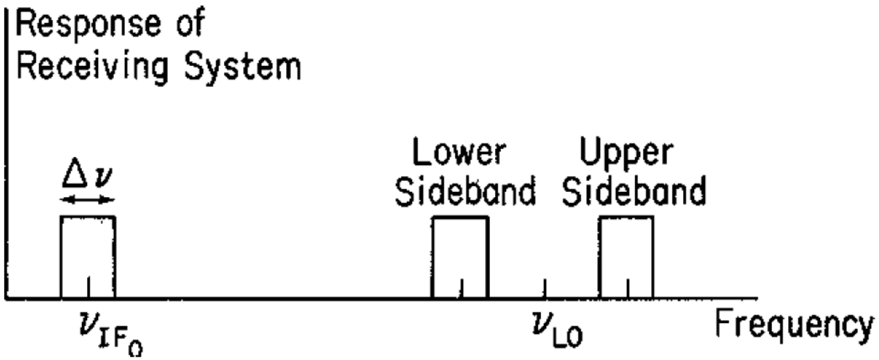
$\phi_1 - \phi_2 - \phi_V$ and integrating over IF from $\nu_{IF_0} - \Delta\nu/2$ to $\nu_{IF_0} + \Delta\nu/2$

$$r_u = A_0\Delta\nu|V| \frac{\sin\pi\Delta\nu\Delta\tau}{\pi\Delta\nu\Delta\tau} \cos[2\pi(\nu_{LO}\tau_g + \nu_{IF_0}\Delta\tau) - \phi_V - \phi_{LO}]$$

$\Delta\tau = \tau_g - \tau_i$ Tracking error of the compensating delay

Delay tracking and frequency conversion

$$\nu_{\rm RF} = \nu_{\rm LO} \pm \nu_{\rm IF}$$



For LSB

$$\phi_1 = -2\pi(\nu_{\rm LO} - \nu_{\rm IF})\tau_g$$

$$\phi_2 = 2\pi\nu_{\rm IF}\tau_i - \phi_{\rm LO}$$

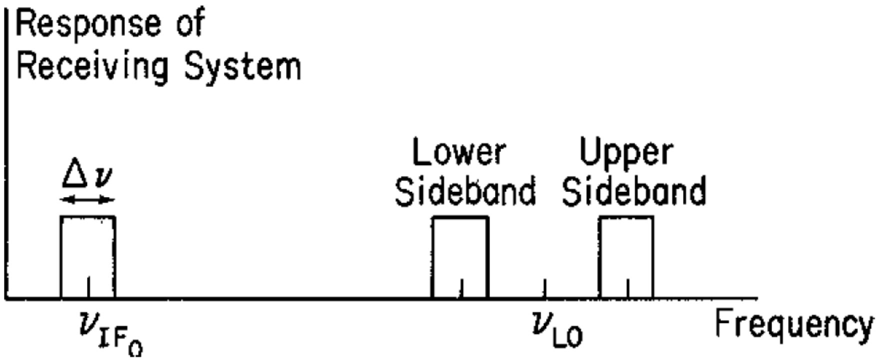
$$r_l = A_0 \Delta\nu |V| \frac{\sin \pi\Delta\nu\Delta\tau}{\pi\Delta\nu\Delta\tau} \cos[2\pi(\nu_{\rm LO}\tau_g - \nu_{{\rm IF}_0}\Delta\tau) - \phi_V - \phi_{\rm LO}]$$

$$\Delta\tau = \tau_g - \tau_i$$ Tracking error of the compensating delay

Delay tracking and frequency conversion

$$\nu_{\rm RF} = \nu_{\rm LO} \pm \nu_{\rm IF}$$

For a double sideband system:



$$r_u = A_0 \Delta\nu |V| \frac{\sin \pi \Delta\nu \Delta\tau}{\pi \Delta\nu \Delta\tau} \cos[2\pi(\nu_{\rm LO}\tau_g + \nu_{\rm IF_0}\Delta\tau) - \phi_V - \phi_{\rm LO}]$$

$$r_l = A_0 \Delta\nu |V| \frac{\sin \pi \Delta\nu \Delta\tau}{\pi \Delta\nu \Delta\tau} \cos[2\pi(\nu_{\rm LO}\tau_g - \nu_{\rm IF_0}\Delta\tau) - \phi_V - \phi_{\rm LO}]$$

$$\begin{aligned} r_d &= r_u + r_l \\ &= 2\Delta\nu A_0 |V| \frac{\sin(\pi \Delta\nu \Delta\tau)}{\pi \Delta\nu \Delta\tau} \cos(2\pi\nu_{\rm LO}\tau_g - \phi_V - \phi_{\rm LO}) \cos(2\pi\nu_{\rm IF_0}\Delta\tau) \end{aligned}$$

$$\Delta\tau = \tau_g - \tau_i$$ Tracking error of the compensating delay

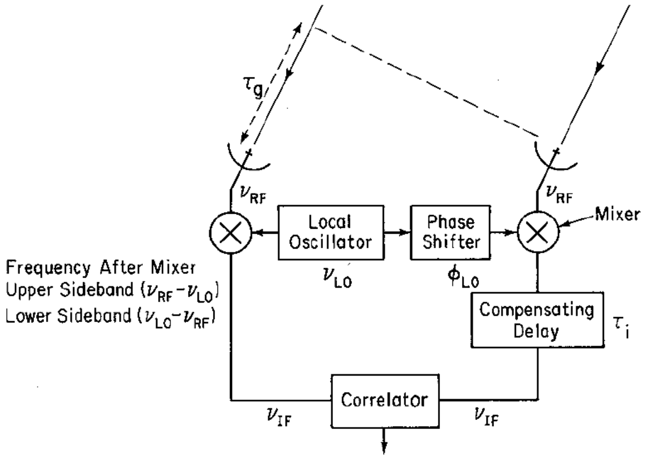
Fringe rotation/stopping

$$r_u = A_0 \Delta\nu |V| \frac{\sin \pi \Delta\nu \Delta\tau}{\pi \Delta\nu \Delta\tau} \cos[2\pi(\nu_{\mathrm{LO}} \tau_g + \nu_{\mathrm{IF}_0} \Delta\tau) - \phi_V - \phi_{\mathrm{LO}}]$$

If the term $(2\pi\nu_{\mathrm{LO}}\tau_g - \phi_{\mathrm{LO}})$ can be kept constant then the output will vary with changes in V and slow drifts in the instrument.

The control of LO phase shift is referred to as fringe stopping or fringe rotation.

The phase shifter allows to have this control and thus is introduced in the system.



Complex correlator: briefly

To measure the complex fringe both the real and imaginary components need to be measured. The imaginary part is just a $\pi/2$ phase shifted copy of the same.

Complex correlator: briefly

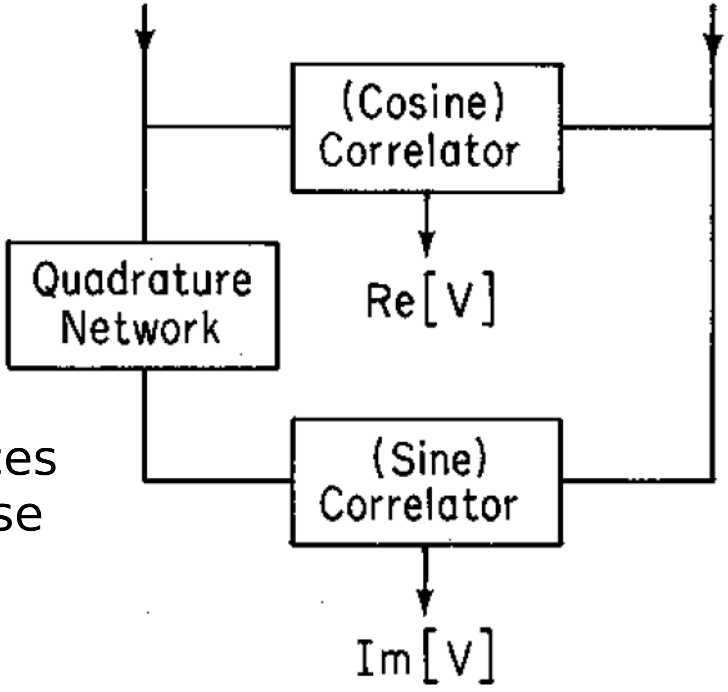
To measure the complex fringe both the real and imaginary components need to be measured. The imaginary part is just a $\pi/2$ phase shifted copy of the same.

For each antenna pair a second correlator with the shift is added in one of the inputs.

This is called complex correlator.

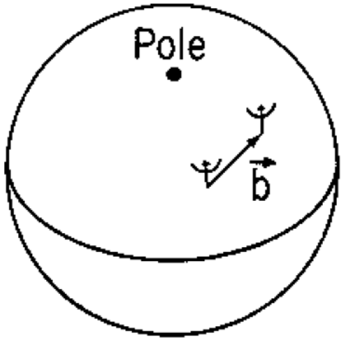
We will come to further details when we will discuss correlators.

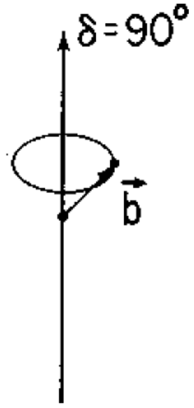
Introduces the phase shift

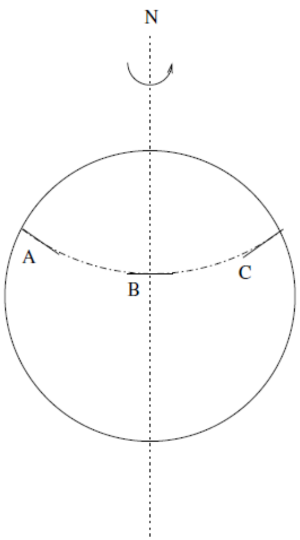


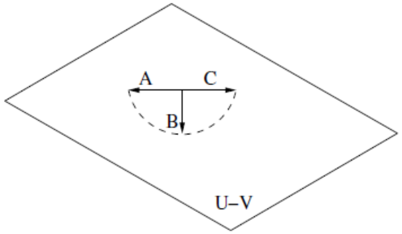
Coordinate systems

Baseline orientation; Track in the uv-plane.



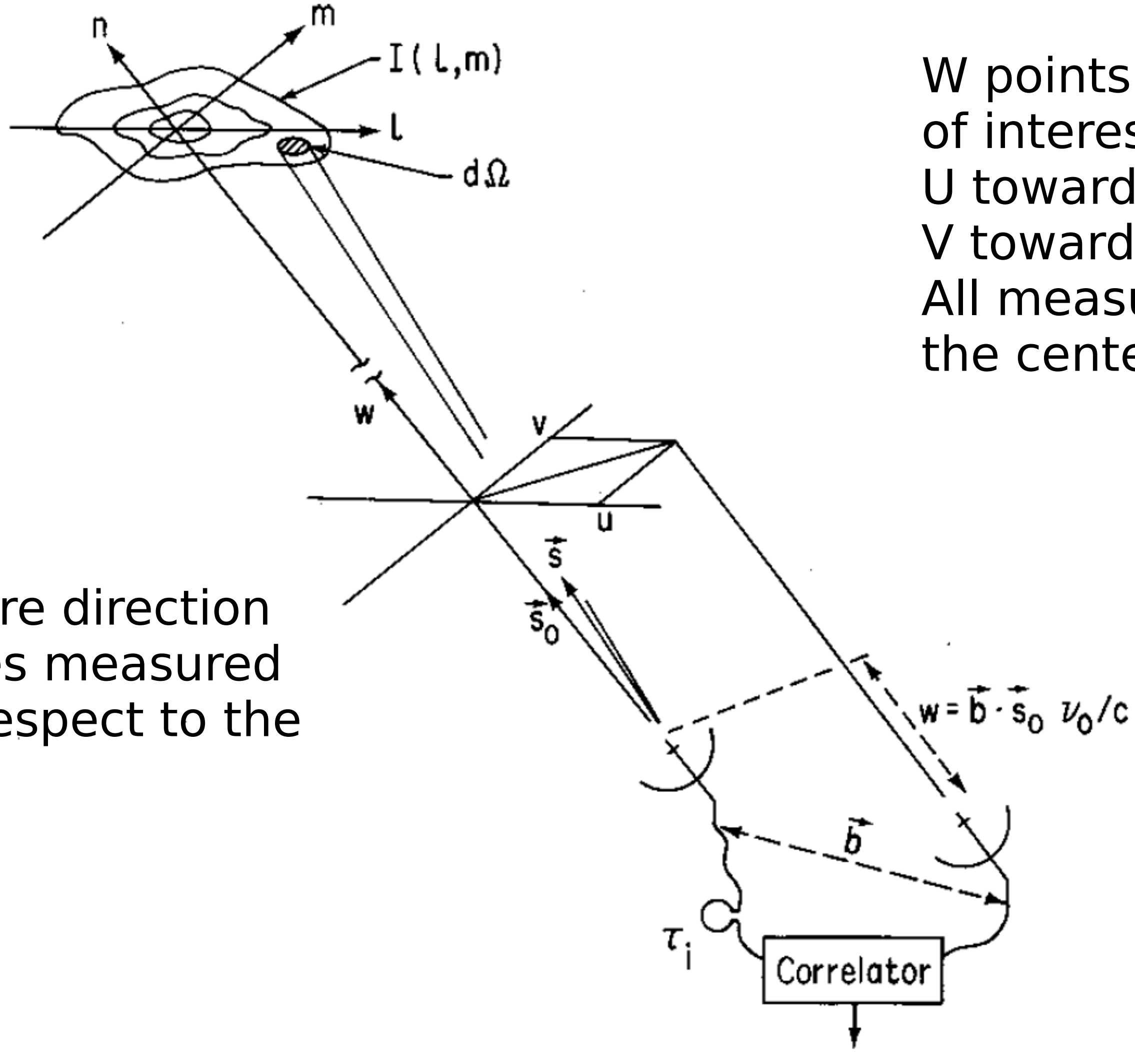






Coordinate systems

W points towards the direction of interest – phase centre.
U towards East
V towards North
All measured in wavelength of the center of the RF.



L, m are direction cosines measured with respect to the u,v.

$$\frac{\nu\,\mathbf{b}\cdot\mathbf{s}}{c} = ul + vm + wn$$

$$\frac{\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c} = w,$$

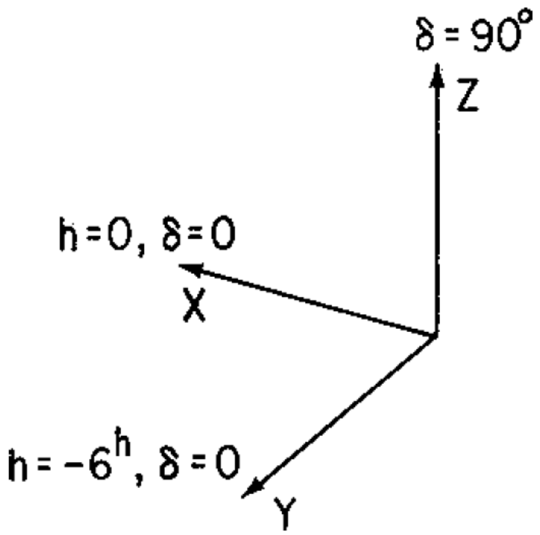
$$d\Omega = \frac{dl\,dm}{n} = \frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

$$V(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{A}(l,m) I(l,m) e^{-2\pi i\left[ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right]} \frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

Integrand taken as zero when $l^2+m^2 \geq 1$

We have been through the conditions under which this is a 2-D Fourier transform.

Antenna spacings and u,v,w



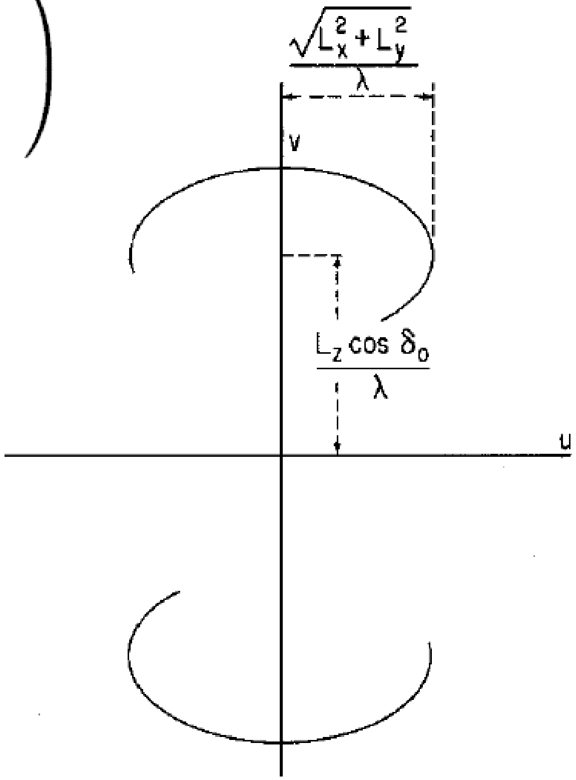
Coordinate system for baseline parameters:
X – direction of the meridian at the celestial equator
Y – towards East
Z- toward the North celestial pole

L_X, L_Y and L_Z are coordinate differences for the baseline, then

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$

H_0 and δ_0 are the hour angle and the declination of the phase reference position.

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin \delta_0 \cos H_0 & \sin \delta_0 \sin H_0 & \cos \delta_0 \\ \cos \delta_0 \cos H_0 & -\cos \delta_0 \sin H_0 & \sin \delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$



What is the locus of a track in the uv-plane ?
Eliminating H_0 from the equations for u and v:

$$u^2 + \left(\frac{v - (L_Z/\lambda) \cos \delta_0}{\sin \delta_0} \right)^2 = \frac{L_X^2 + L_Y^2}{\lambda^2}$$

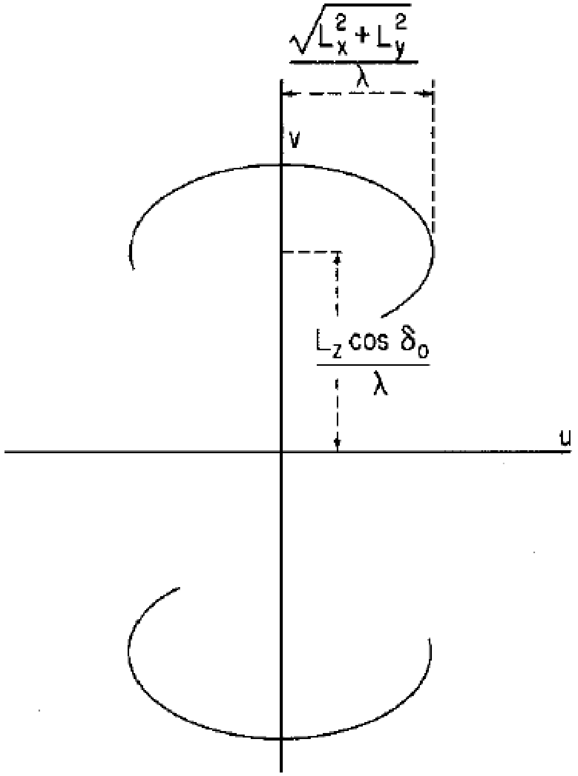
$$V(-u, -v) = V^*(u, v)$$

Sampling in the uv-plane

Visibilities are sampled: the footprint in the uv-plane – *uv-coverage* is the sampling function.

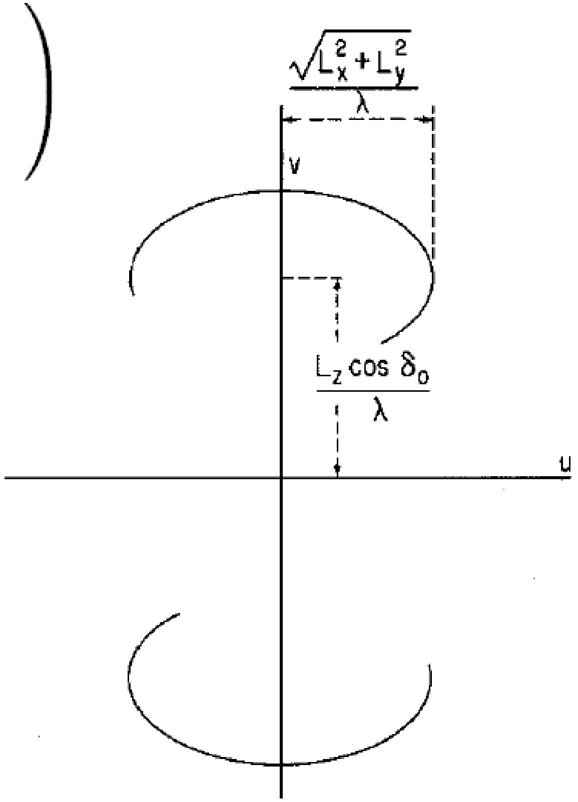
For a point source at the phase centre the visibility is a constant as a function of u and v.

The FT of the sampling function is then the response to a point source – *the synthesized beam.*



Coordinate system

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$



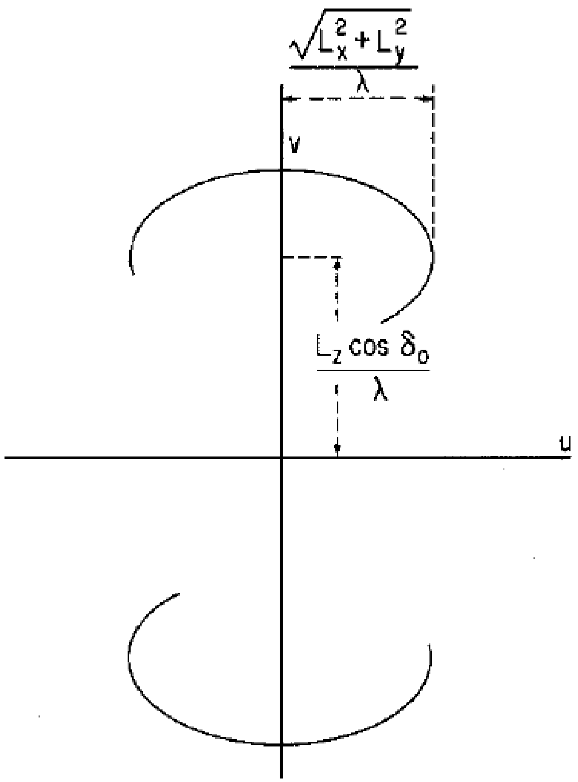
What is the locus of a track in the uv-plane ?
Eliminating H_0 from the equations for u and v:

$$u^2 + \left(\frac{v - (L_Z/\lambda)\cos\delta_0}{\sin\delta_0} \right)^2 = \frac{L_X^2 + L_Y^2}{\lambda^2}$$

$$V(-u, -v) = V^*(u, v)$$

Sampling in the uv-plane

$$u^2 + \left(\frac{v - (L_Z/\lambda)\cos\delta_0}{\sin\delta_0} \right)^2 = \frac{L_X^2 + L_Y^2}{\lambda^2}$$



Visibilities are sampled: the footprint in the uv-plane – *uv-coverage* is the sampling function.

For a point source at the phase centre the visibility is a constant as a function of u and v.

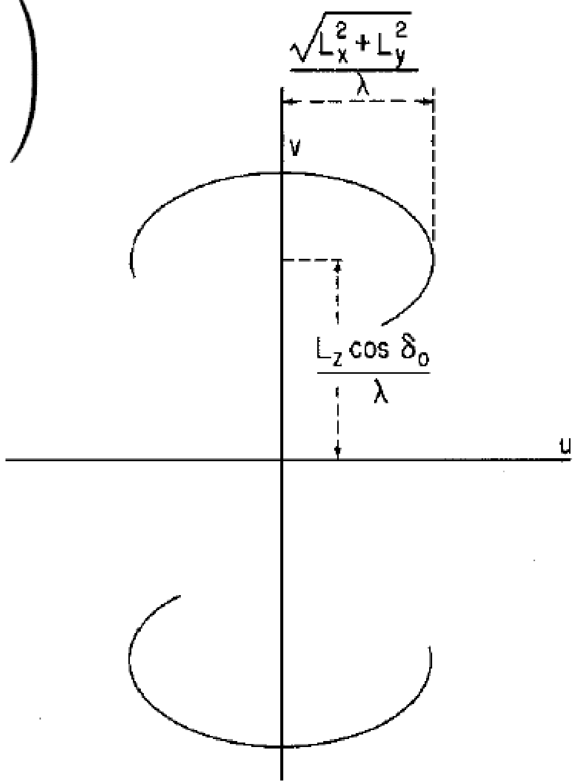
The FT of the sampling function is then the response to a point source – *the synthesized beam.*

The sampling in the uv-plane decides the shape of the synthesized beam.

Coordinate system

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$

?

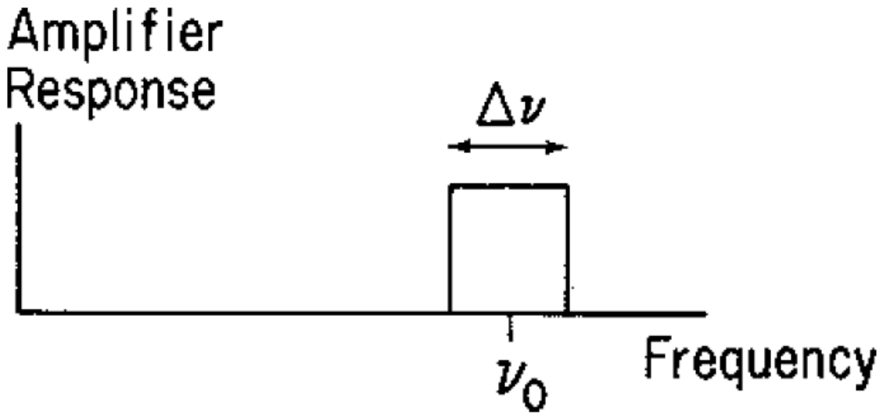


What is the locus of a track in the uv-plane ?
Eliminating H_0 from the equations for u and v:

$$u^2 + \left(\frac{v - (L_Z/\lambda)\cos\delta_0}{\sin\delta_0} \right)^2 = \frac{L_X^2 + L_Y^2}{\lambda^2}$$

$$V(-u, -v) = V^*(u, v)$$

Effect of bandwidth



τ_g = b sin(θ)/c

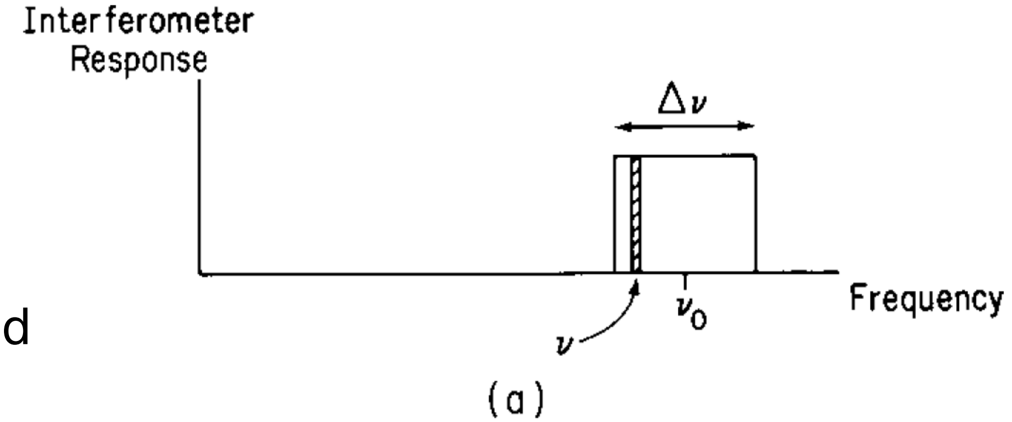
$$r = A_0|V| \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

$$= A_0|V|\Delta\nu \frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

- Bandwidth leads to a modulation of the fringe with a sinc function.
- Introduction of delay tracking to remove this effect: however it is only valid for the delay tracking centre.

Bandwidth smearing

The bandwidth over which the signal that is delay tracked only at the central frequency is averaged and this lead to blurring in the image.



u_0,v_0 for the central frequency and u and v for another frequency.

$$(u_0, v_0) = \left(\frac{\nu_0}{\nu}u, \frac{\nu_0}{\nu}v\right)$$

$$V(u,v) \rightleftharpoons I(l,m)$$

Similarity theorem of FT

$$V\left(\frac{\nu_0}{\nu}u, \frac{\nu_0}{\nu}v\right) \rightleftharpoons \left(\frac{\nu}{\nu_0}\right)^2 I\left(\frac{\nu}{\nu_0}l, \frac{\nu}{\nu_0}m\right)$$

Bandwidth smearing

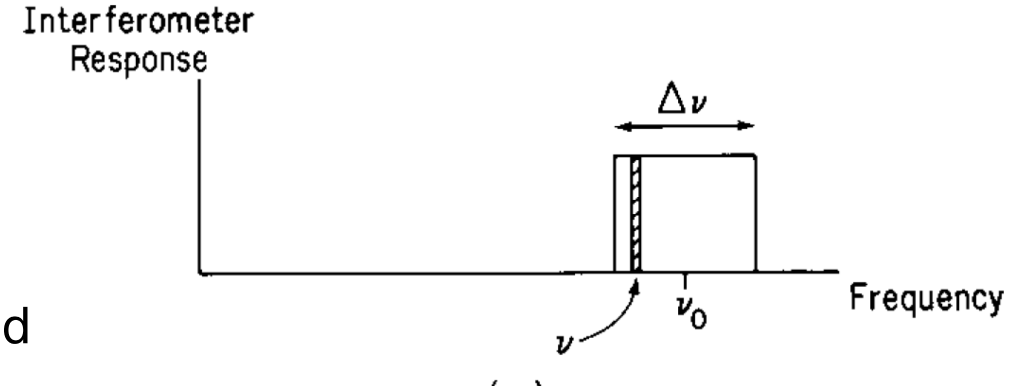
The bandwidth over which the signal that is delay tracked only at the central frequency is averaged and this lead to blurring in the image.

u_0,v_0 for the central frequency and u and v for another frequency.

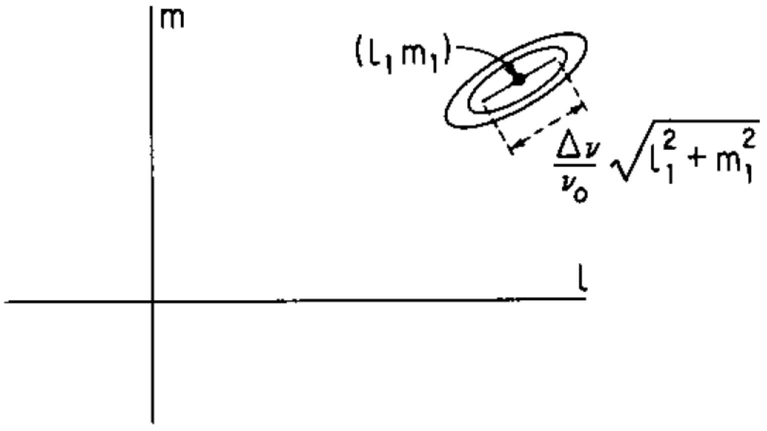
$$V\left(\frac{\nu_0}{\nu}u, \frac{\nu_0}{\nu}v\right) \rightleftharpoons \left(\frac{\nu}{\nu_0}\right)^2 I\left(\frac{\nu}{\nu_0}l, \frac{\nu}{\nu_0}m\right)$$

Range of variation in the coordinates decided by ν/ν_0

Introduces a *radial smearing* proportional to their distance from the tracking centre.



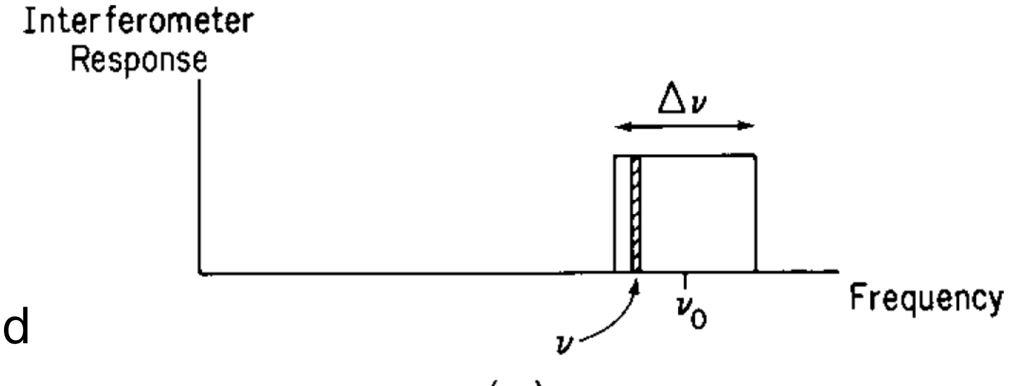
(a)



(b)

Bandwidth smearing

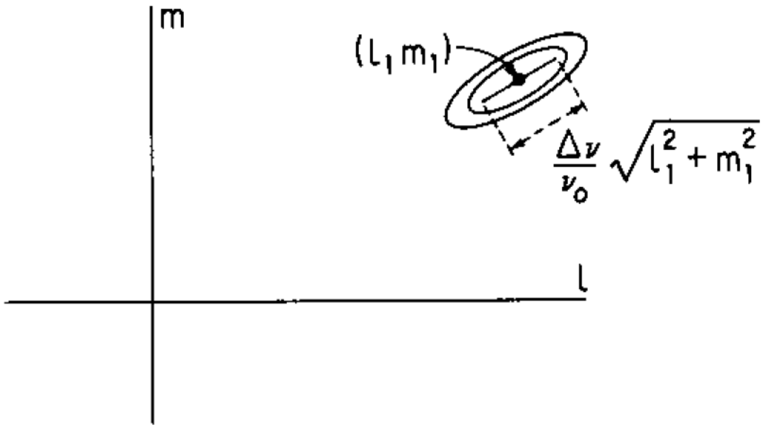
The bandwidth over which the signal that is delay tracked only at the central frequency is averaged and this lead to blurring in the image.



(a)

u_0,v_0 for the central frequency and u and v for another frequency.

$$V\left(\frac{\nu_0}{\nu}u, \frac{\nu_0}{\nu}v\right) \rightleftharpoons \left(\frac{\nu}{\nu_0}\right)^2 I\left(\frac{\nu}{\nu_0}l, \frac{\nu}{\nu_0}m\right)$$



(b)

Range of variation in the coordinates decided by ν/ν_0

Introduces a *radial smearing* proportional to their distance from the tracking centre.
Will become significant when it becomes of the order of the synthesized beam.