- Imaging
- Deconvolution
- Calibration

Astronomical Techniques II : **Lecture 11**

Ruta Kale

Low Frequency Radio Astronomy (Chp. 12)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy
Synthesis imaging in radio astronomy II, Chp 7, 8
For correlators:

- Talk by Adam Deller
  https://nmt.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=42804ec9-3b6c-4b40-8978-a920010eb3fa

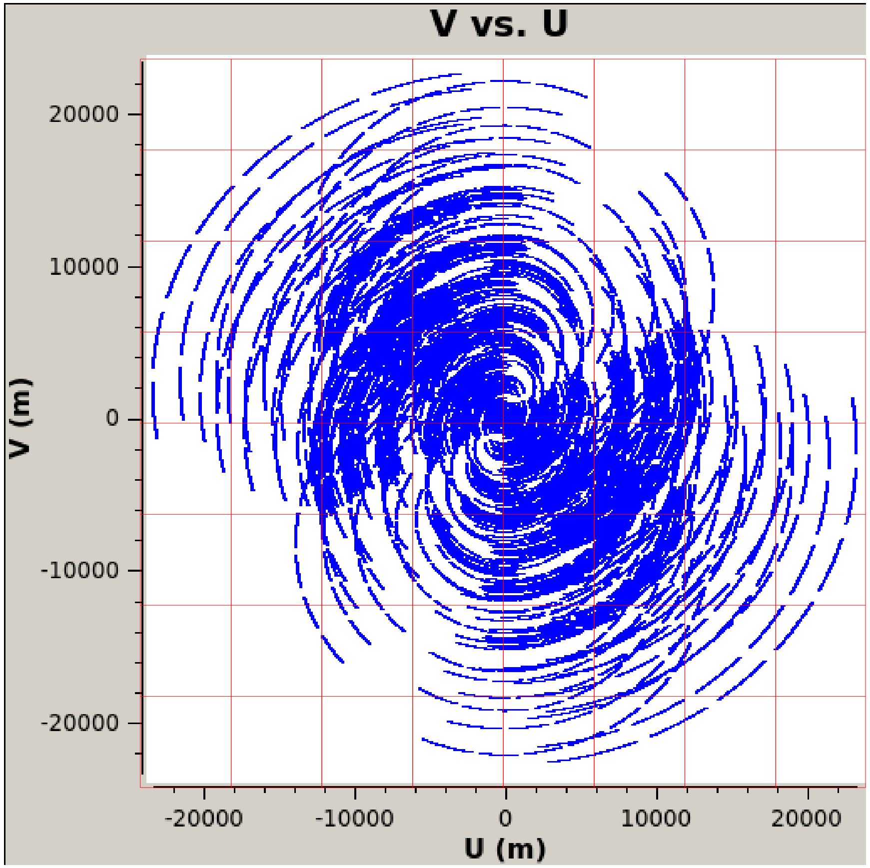
Gridding the visibilities

Gridding by convolution: *convolve the weighted sampled visibility with some suitable function and then sample this function on the desired grid.*
Value assigned at each grid point will be an average of the local values.

$$C * V^W$$

$$\sum_{k=1}^{M} C(u_c - u_k, v_c - v_k) V^W(u_k, v_k)$$

$$V^R = R\left(C * V^W\right) = R\left(C * (WV')\right)$$



The “dirty image” can be given by

$$\widetilde{I}^D = \mathfrak{F}R * \left[(\mathfrak{F}C)\left(\mathfrak{F}V^W\right)\right]$$

$$= \mathfrak{F}R * \left[(\mathfrak{F}C)\left(\mathfrak{F}W * \mathfrak{F}V'\right)\right]$$

$$R(u,v) = \text{III}(u/\Delta u, v/\Delta v)$$

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$$R(u,v) = \mathrm{III}(u/\Delta u, v/\Delta v)$$

$$(\mathfrak{F}R)(l,m) = \Delta u\,\Delta v \mathrm{III}(l\Delta u, m\Delta v) = \Delta u\,\Delta v \sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} \delta(j - l\Delta u, k - m\Delta v)$$

- Aliasing is introduced due to convolution with the scaled sha function.

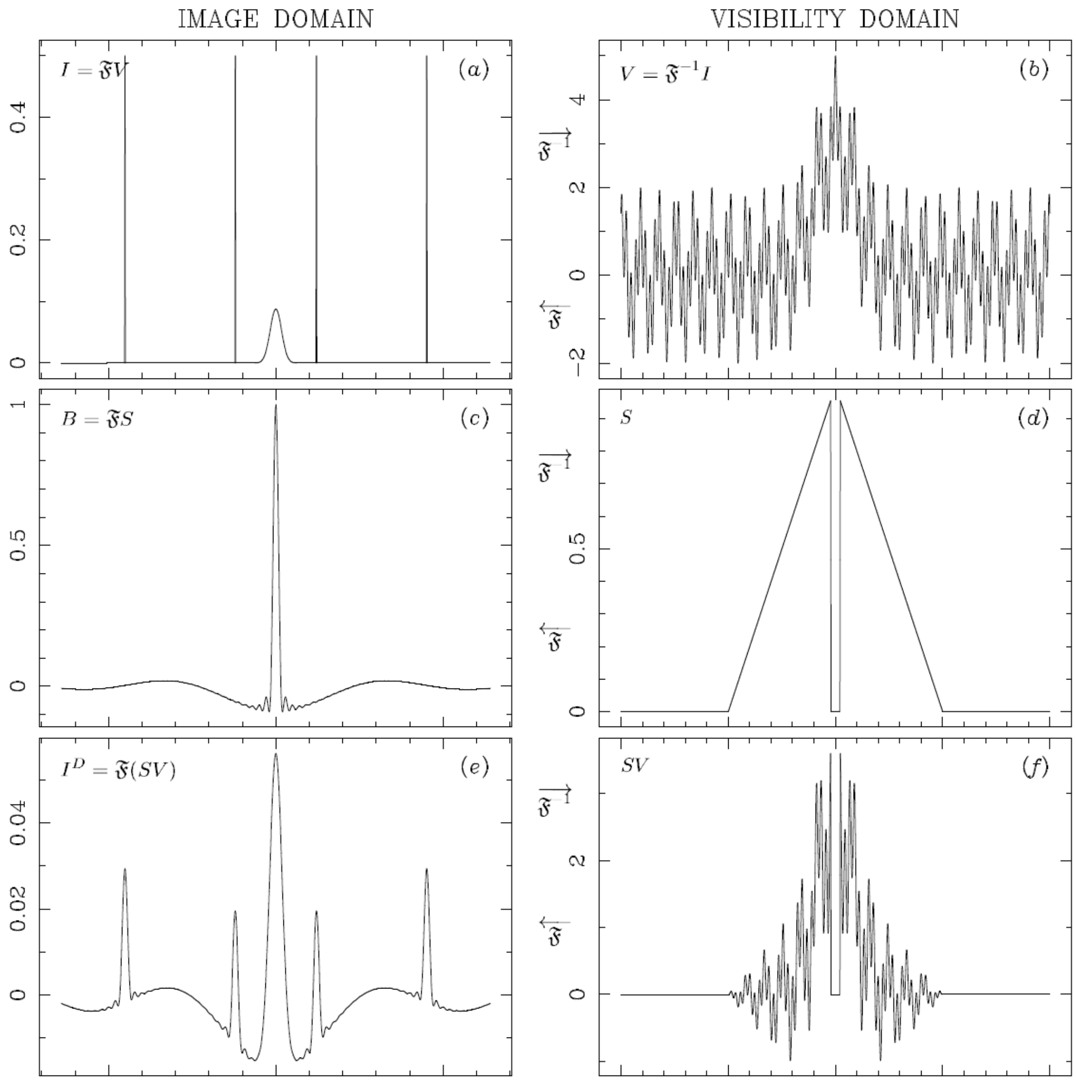
- Resampling makes dirty image a periodic function of l and m of period 1/∆u and 1/∆v.

Graphical representation

Model source: symmetric

Synthesized beam

Dirty image if a direct FT is computed



Model Visibilities: Real and even due to symmetry

Sampling: central hole, falling density towards the outskirts

Sampled visibilities

SIRA, Fig. 7-5

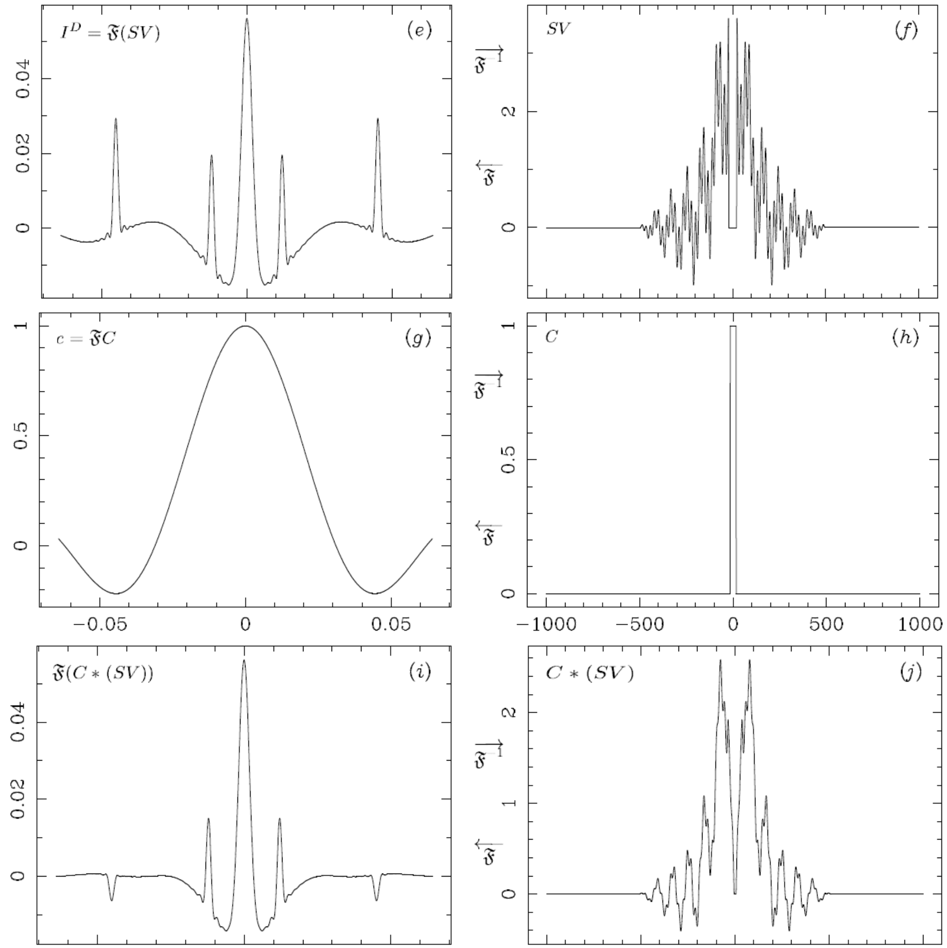
Sampled visibilities

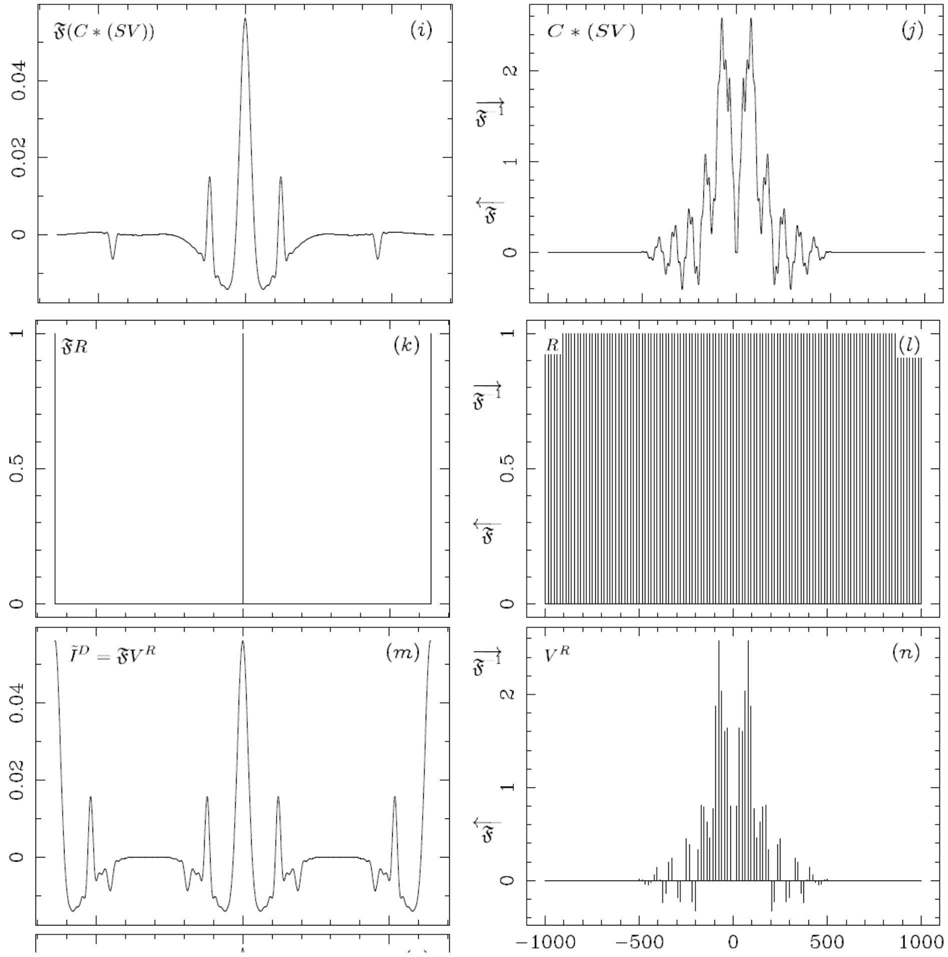
FT of the convolution function

Convolution function

Effect in the image domain

Convolved sampled visibilities



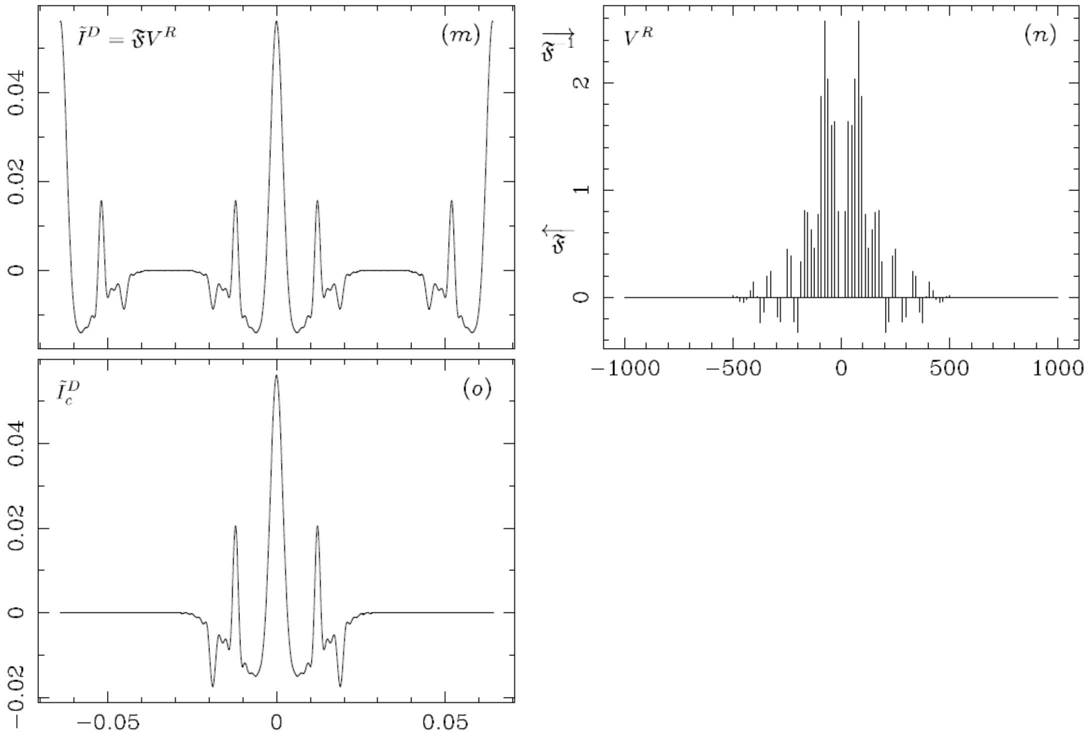


Resampling

Dirty image: aliasing

Resampled visibility

Divide by the FT of the convolution function



This image is far from satisfactory representation of the actual distribution: can do better than this by deconvolution.

Choice of the gridding convolution function

Desired choices to avoid aliasing:

a) image is large enough to include any sources at the edges.

b) avoid under sampling

c) use a gridding convolution function whose Fourier transform drops off rapidly outside the image.

C is chosen to be real and even. C is separable C(u)C(v).

1. a pillbox function
2. truncated exponential
3. a truncated sinc function
4. an exponential multiplied by a truncated sinc
5. a truncated spheroidal

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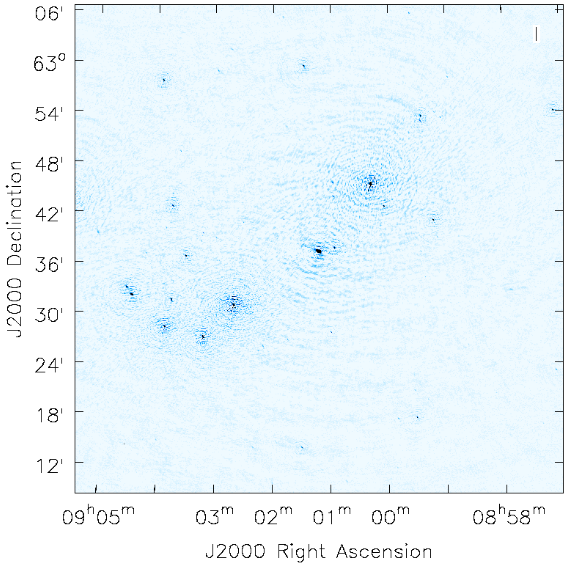
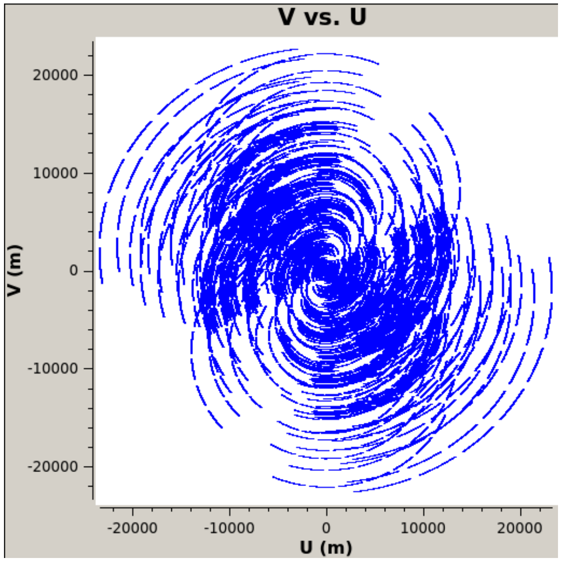
See Figs. 7-6, 7-7 and 7-8 in SIRA II for more discussion on the choice of gridding function.

Imaging

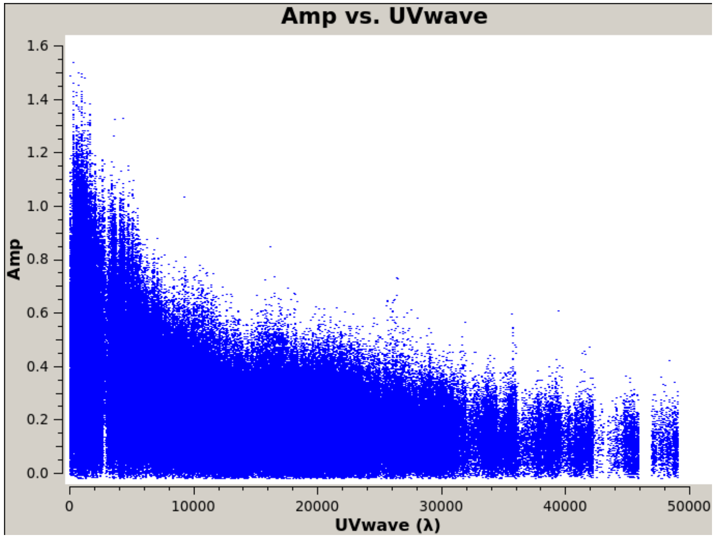
$$I^D(l,m) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(u,v)V'(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$



“Dirty” image



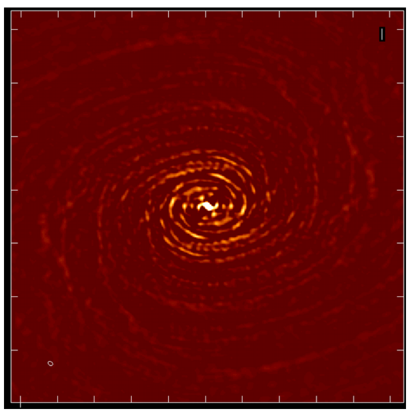
Sampling



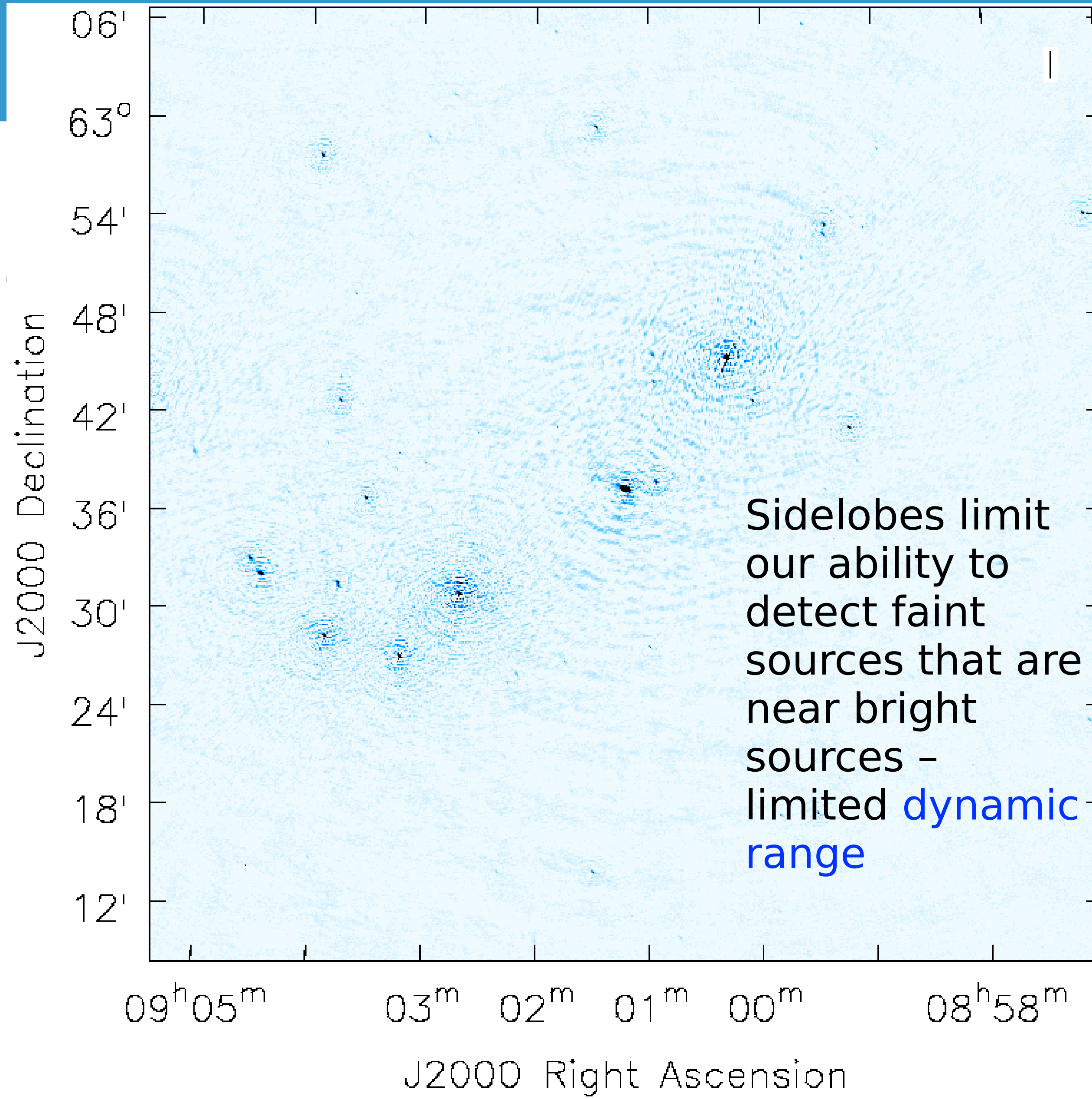
Observed visibilities (complex numbers) Only amp. Shown.

Imaging

$$I^D(l,m) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}$$



Point source response or synthesized beam



Sidelobes limit our ability to detect faint sources that are near bright sources – limited dynamic range

"Dirty" image

Imaging: deconvolution

$$V(u,v) = \int\int_S I(l,m) e^{-2\pi i(ul+vm)}\, dl\, dm$$

Only a finite number of measurements (noisy too) of the visibilities are available; thus recovering *I(l,m)* has limitations.

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A model with a finite number of parameters is needed.

Imaging problem is “ill-posed” or “underdetermined” or “ill conditioned”: fewer equations than unknowns.

Thus “additional” information - “a priori information” must be used.

Imaging: deconvolution

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Deconvolution methods originate in a “deceptively” simple idea proposed by J. Hogbom (A&AS, 15, 417, 1974) – turned out to be a breakthrough for radio astronomy!

Imaging: deconvolution

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Only a finite number of measurements (noisy too) of the visibilities are available; thus recovering *I(l,m)* has limitations.

A model with a finite number of parameters is needed.
A general purpose model of the sky is that of a 2-D grid of delta functions with strengths $\widehat{I}(p\Delta l, q\Delta m)$ can be considered.

Idea of deconvolution

Consider that sky is composed of a number of isolated point sources.

In the dirty image – each source is like the synthesized beam scaled to the strength of the source.

The effect of the synthesized beam is modifying the position and strength of the source.

We consider the brightest source and assume that its position is correct. We subtract the convolved source from the image.

Go to the next brightest source – repeat and go on until you find no peak remaining in the image.

Put back the list of “components” that you found after convolution with a Gaussian beam of width same as your synthesized beam.

Imaging: deconvolution

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Only finite measurements of the visibilities are available; thus recovering *I(l,m)* has limitations.
A model with a finite number of parameters is needed.
A general purpose model of the sky is that of a 2-D grid of delta functions with strengths $\widehat{I}(p\Delta l, q\Delta m)$ can be considered.

The visibility predicted by this model is given by:

$$\widehat{V}(u,v) = \sum_{p=1}^{N_l} \sum_{q=1}^{N_m} \widehat{I}(p\Delta l, q\Delta m) e^{-2\pi i(pu\Delta l + qu\Delta m)}$$

Imaging: deconvolution

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N_l and N_m are pixels on each side. And the range of the uv points sampled are required to be:

$$\Delta l \leq \frac{1}{2u_{\max}},\ \Delta m \leq \frac{1}{2v_{\max}}$$

~Pixel size in the image

$$N_l\Delta l \geq \frac{1}{u_{\min}},\ \text{and}\ N_m\Delta m \geq \frac{1}{v_{\min}}$$

~Size of the image

One can estimate source features with widths in the range:

Minimum= $\mathcal{O}(1/\max(u,v))$ Maximum= $\mathcal{O}(1/\min(u,v))$

$N_l N_m$ free parameters that are the cell flux densities.

Imaging: deconvolution

$$\widehat{V}(u,v) = \sum_{p=1}^{N_l} \sum_{q=1}^{N_m} \widehat{I}(p\Delta l, q\Delta m) e^{-2\pi i (pu\Delta l + qu\Delta m)}$$

The measurements constrain the model such that at the sampled u,v points

$$V(u_k, v_k) = V\widehat{(u_k, v_k)} + \epsilon(u_k, v_k)$$

$\epsilon(u_k, v_k)$ is a complex, normally distributed random error due to receiver noise.

At the points in the plane where no sample was taken the model is free to take on any value.

Imaging: deconvolution

$V(u_k, v_k) = \widehat{V(u_k, v_k)} + \epsilon(u_k, v_k)$ can be expressed as a multiplicative relation

$$V(u, v) = W(u, v)(\widehat{V}(u, v) + \epsilon(u, v))$$

W is the weighted sampling function;

$$W(u, v) = \sum_k W_k \delta(u - u_k, v - v_k)$$

Non-zero only for the sampled points.

Imaging: deconvolution

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Non-zero only for the sampled points.

In the image plane this translates to a convolution relation:

$$I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \widehat{I}_{p',q'} + E_{p,q}$$

Noise image

Dirty image

$$I^D_{p,q} = \sum_k W(u_k, v_k)\ \mathrm{Re}\left(V(u_k, v_k) e^{2\pi i (p u_k \Delta l + q v_k \Delta m)}\right)$$

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Dirty image

Dirty beam

Imaging : deconvolution

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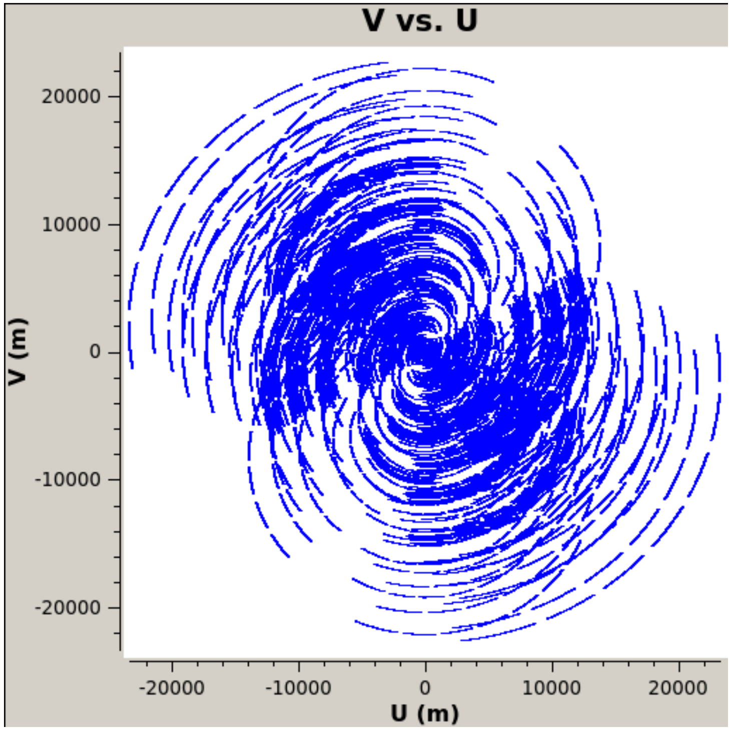
$$I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \widehat{I}_{p',q'} + E_{p,q}$$

The model when convolved with the point spread function (dirty beam) corresponding to the sampled weighted (u,v) coverage should yield the dirty image.



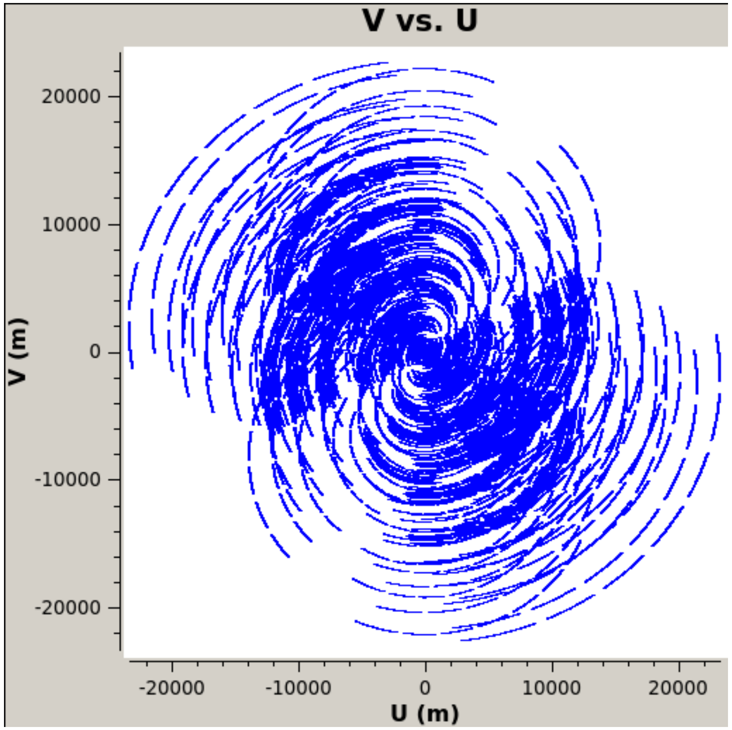
Principal solution and invisible distributions

$$I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \widehat{I}_{p',q'} + E_{p,q}$$

Is the solution unique ?
Solution not unique in the absence of boundary conditions.
Existence of "homogenous" solutions: called invisible distributions in radio astronomy.

Invisible distribution is that which has non-zero amplitude in only the unsampled spatial frequencies.

Also called as "ghosts"!



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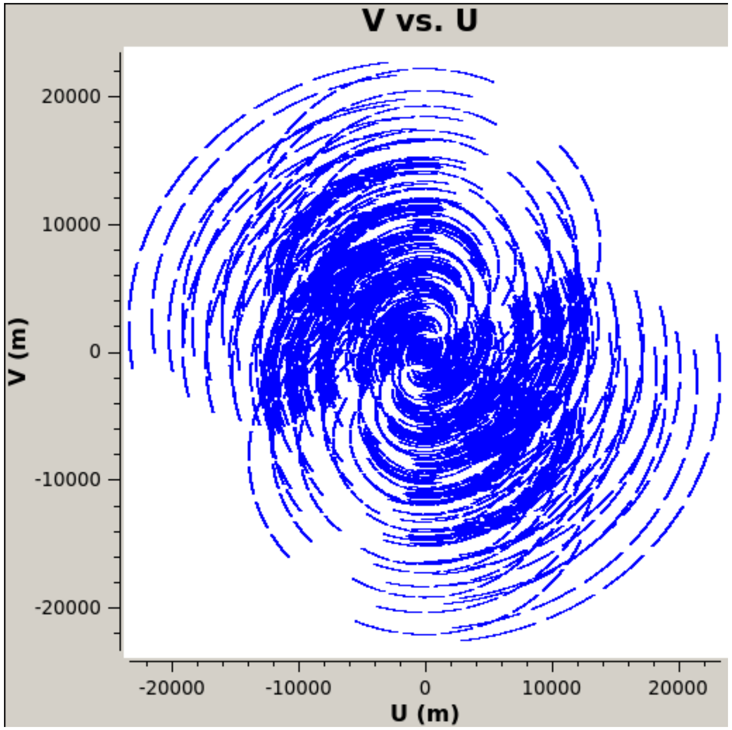
Existence of “homogenous” solutions: called invisible distributions in radio astronomy.

Invisible distributions arise due to :
- Limit on the extent of u,v coverage.
- Holes in the u,v coverage



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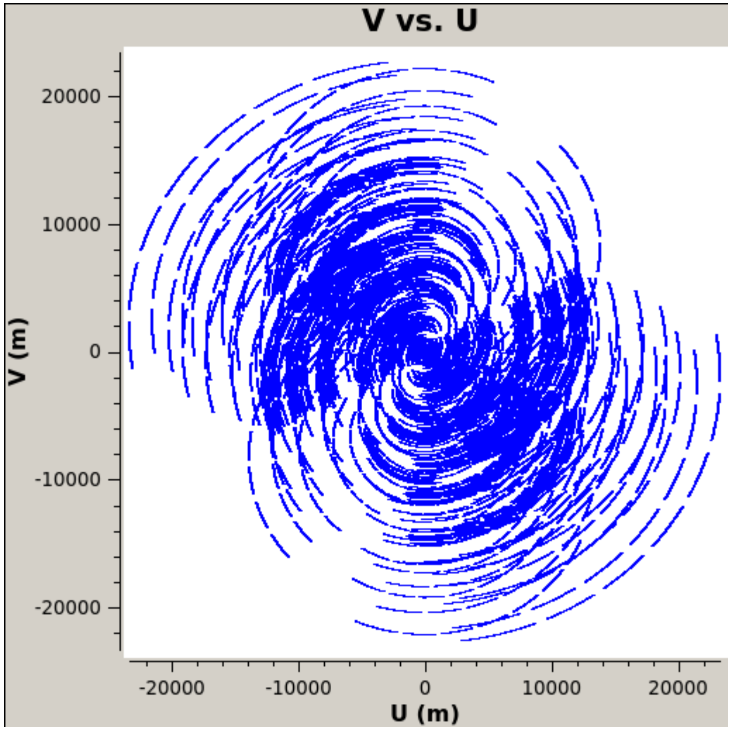
Existence of “homogenous” solutions: called invisible distributions in radio astronomy.

Invisible distribution is that which has non-zero amplitude in only the unsampled spatial frequencies.

- The solution having zero amplitude in all the unsampled spatial frequencies is called the *principal solution*.



Problems with the principal solution

- The solution having zero amplitude in all the unsampled spatial frequencies is called the *principal solution*.

Principal solution is not enough as we cannot make out the if the source is a point source or is shaped like the beam !

Also it will change as we change the visibilities. *We need a method to estimate the visibilities in the unsampled range.*

We can use the information at the total intensity of the source must be positive.

Use of a priori information is the key to making an image of the sky.

Deconvolution algorithms use this to obtain better estimates of the sky than given by the principal solution.

Deconvolution: non-linear, iterative image re-construction

CLEAN algorithm : Hogbom 1974

The CLEAN algorithm (Högbom 1974)

- Provides a solution to the convolution equation by representing any source as a collection of point sources. An iterative approach is used to find the positions and strengths of the point sources.

- It makes use of the fact that the dirty beam is known and thus we can remove features of it and tell them apart from a real source.

- The final “deconvolved” image is called CLEAN image – it is the sum of the point source components convolved with the CLEAN beam – chosen usually to be a Gaussian.

Högbom's CLEAN algorithm

1. Find the position and strength of the brightest point in the dirty image, $I^{D}_{p,q}$.

2. Multiply the peak with the dirty beam B and a "damping factor" (loop gain) and subtract from the dirty image.

3. Save the position and strength of the peak in a "model image".

4. Go to (1) and repeat for the next peak until there is no peak above a user specified level.

Finally one will have "residual" image.

5. Convolve the model image with an idealized CLEAN beam (Gaussian fitted to the central peak of the dirty beam) to form a CLEAN image.

6. Add the residuals and the CLEAN image.

CLEAN algorithm

Clark (1980) CLEAN: use of psf patches

Minor cycle:beam patch to select for components; proceeds like Hogbom CLEAN
Major cycle: Point source model is transformed via FFT, transformed back and subtracted from dirty image.

Cotton-Schwab CLEAN: Periodically predict model-visibilities, calculate residual visibilities and re-grid – major and minor cycles: works on ungridded visibilities
Minor cycle: each field cleaned independently but in major cycle components from all the fields are removed – relevant for the non-coplanar baselines case.

Högbom's CLEAN algorithm

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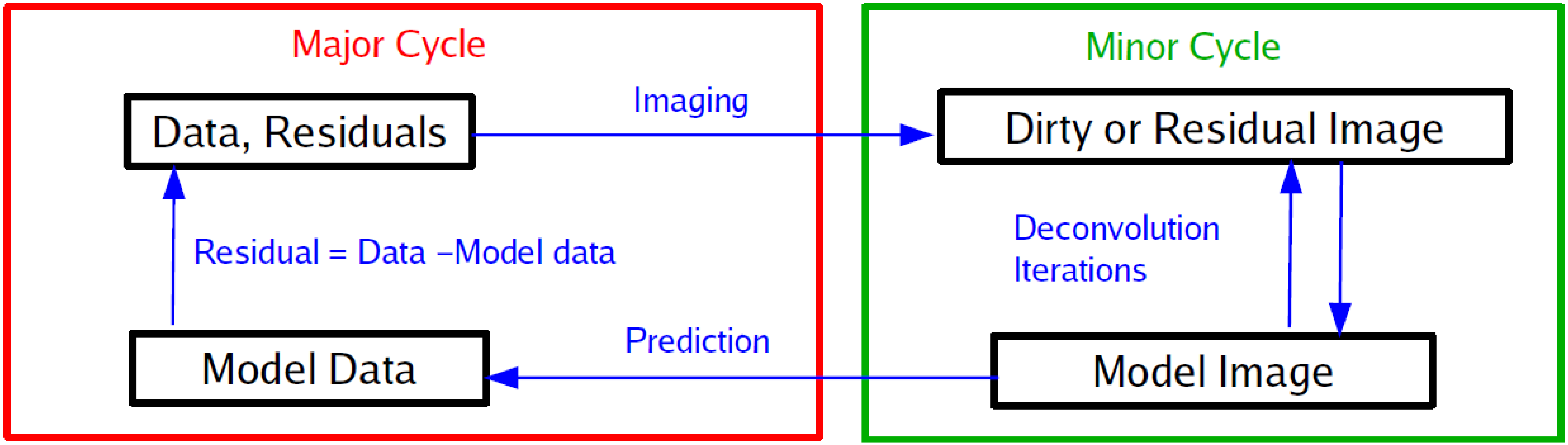
Major and minor cycles



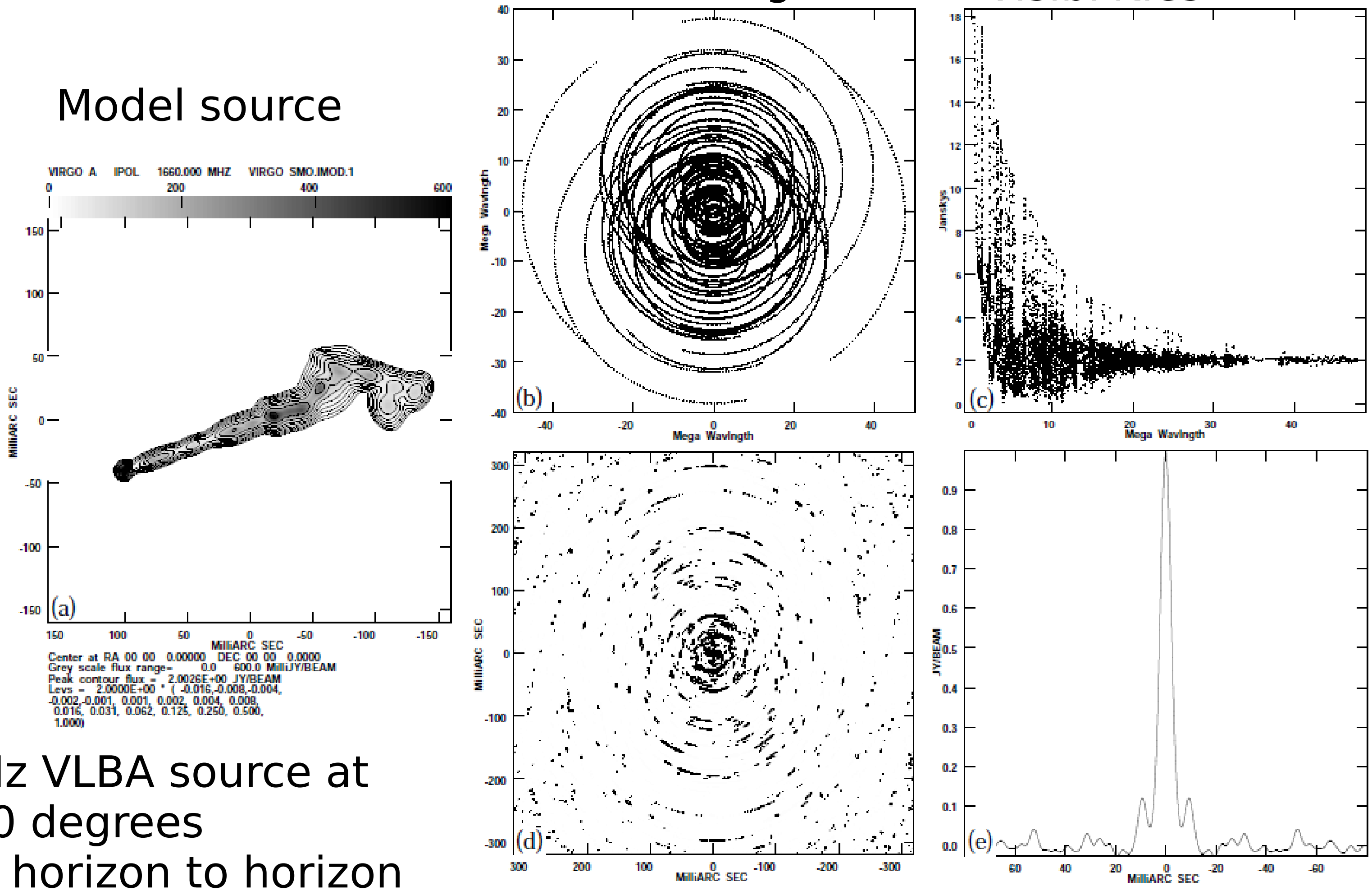
Schematic from U. Rau

Example

uv-coverage

Visibilities

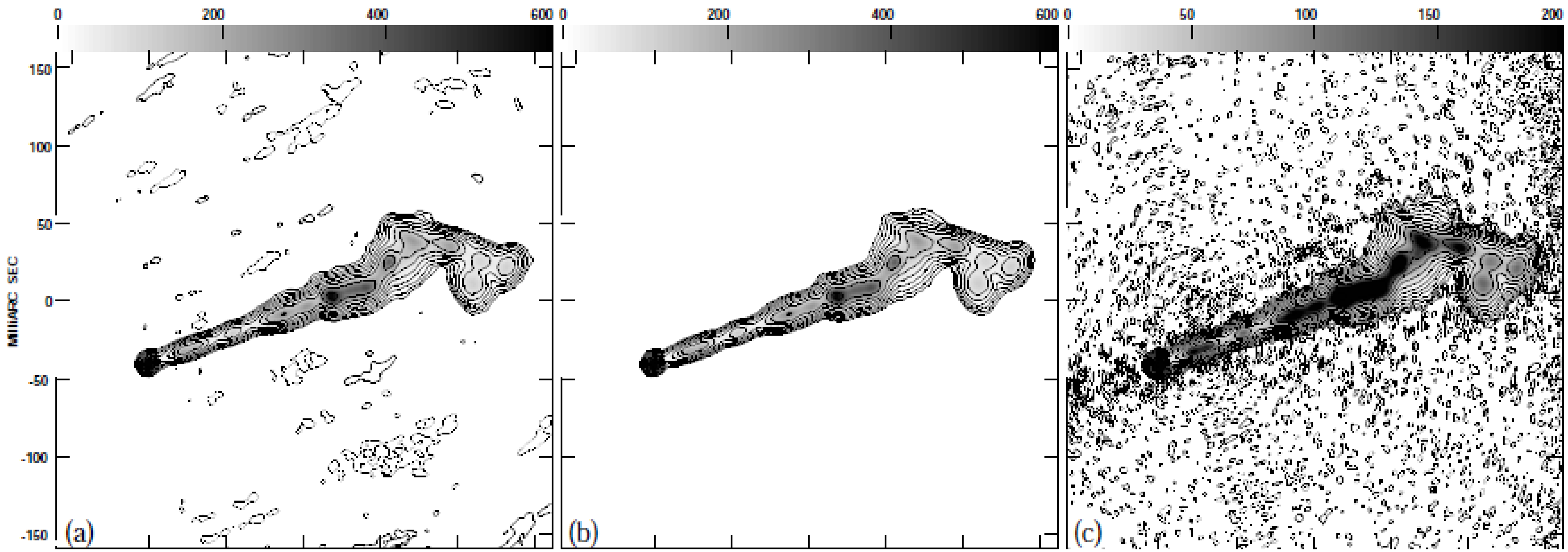
Model source



1.6 GHz VLBA source at dec. 50 degrees
Nearly horizon to horizon coverage.

Dirty beam

Dirty beam



Restored CLEAN image;
CLEANing without constraint.

Restored CLEAN image;
CLEANing with a constraint to be within the region of the source.

Same as panel b but with contours drawn starting at 10 times lower level to show the pattern in the rest of the image.

Softwares implementing CLEAN

NRAO CASA: Common Astronomy Software Applications

NRAO AIPS: Astronomical Image Processing System

ATNF MIRIAD

Alternatives to CLEAN

Maximum Entropy Method (MEM)

In the problem of deconvolution we are trying to select one answer from many possible answers – basically one image from the many possible that can fit the visibilities.

MEM uses a statistical approach to find the most likely image.

We will discuss MEM and other more recent approaches in the last lecture in this course.

Purpose of calibration

To remove the effects of
a) *instrumental* factors and b) *atmospheric* factors in the measurements.

- Such factors largely depend on individual antennas or pairs of antennas. Thus these should be removed before performing the imaging.

Calibration

- Role of calibrators:

In order to measure the effect of instrumental factors, one needs to observe something for which one can predict the visibility.

Data taken towards calibrator sources are useful here. Calibrators are usually *bright, unresolved* sources that dominate the field of view – and chosen to be located close to the position of the target. The point source response of interferometer is expected to be constant across baselines and thus is predictable.

Instrumental factors I (long term)

Among the instrumental factors there are some that vary only on timescales of several weeks or months. These include:

1. *Antenna position coordinates*
2. *Antenna pointing corrections* resulting from axis misalignments or other mechanical tolerances.
3. *Zero-point settings for the delays* – settings for which the delays from the antennas to the correlator inputs are equal.

Such parameters only vary with major changes such as antenna location change (e. g. movable antennas such as of VLA). Usually observatories make these corrections – individual observations do not need measurements.

Instrumental factors II (short term)

There are also instrument parameters that *vary over the course of a single observation* – on timescales of a few to several minutes or hours. Either these are predictable changes or need continuous monitoring.

Predictable changes:

- *atmospheric attenuation* as a function of zenith angle (~)
- *variation of antenna gain as a function of elevation*
- *shadowing* of one antenna by a neighboring antenna at low elevation angles. Generally data where shadowing affects the observation are discarded as the correction can be difficult due to effects of diffraction.

Instrumental factors III (short term)

There are also instrument parameters that vary over the course of a single observation – on timescales of a few to several minutes or hours.

Changes that need to be monitored during an observation:

- *variable atmospheric attenuation* as a function of zenith angle
- phase *variation in the local oscillator system*
- *variation of system noise temperature* due to changing ground pickup in the sidelobes.