- Imaging

Astronomical Techniques II : Lecture 10

Ruta Kale

Low Frequency Radio Astronomy (Chp. 12)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy
Synthesis imaging in radio astronomy II, Chp 7
For correlators:

- Talk by Adam Deller https://nmt.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=42804ec9-3b6c-4b40-8978-a920010eb3fa

Correlator examples

GMRT, ALMA have an FX correlator.

VLA, IRAM have an XF correlator

IRAM: Institut de Radio Astronomie Millimetrique

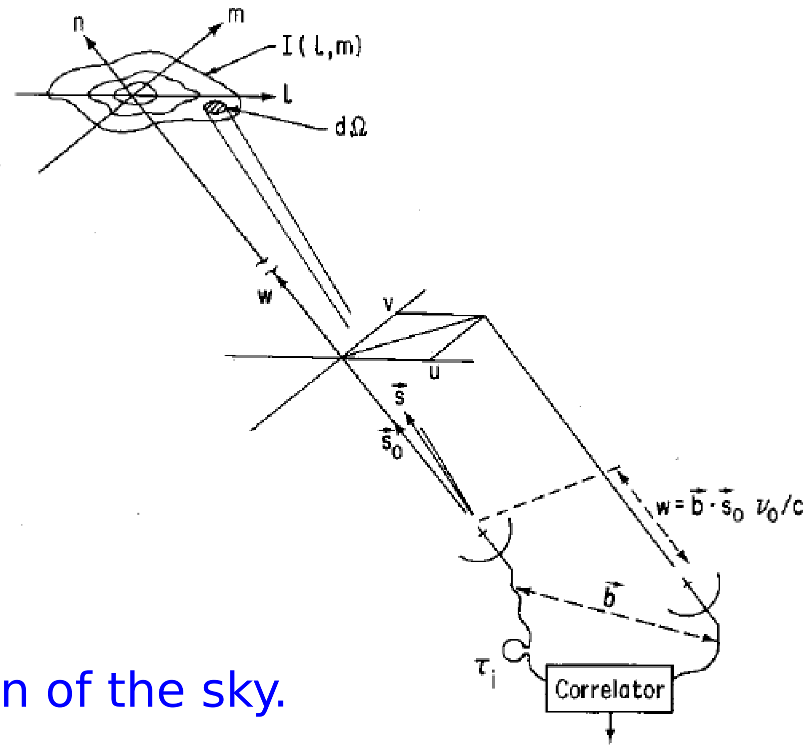
Imaging

$$\mathcal{A}(l,m)I(l,m) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(u,v)e^{2\pi i(ul+vm)}\, du\, dv$$

2-D relationship holds while:

$$\left|\frac{\Delta\nu}{c}\,\mathbf{b}\cdot(\mathbf{s}-\mathbf{s_0})\right| \ll 1$$

$$\left|w(l^2+m^2)\right| \ll 1$$



Observations are confined to a small region of the sky.

Imaging

$$\mathcal{A}(l,m)I(l,m) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$

Primary beam

$$\left|\frac{\Delta\nu}{c}\mathbf{b}\cdot(\mathbf{s}-\mathbf{s_0})\right| \ll 1$$

$$|w(l^2+m^2)| \ll 1$$

Imaging

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Discrete measurements:

$$(u_k, v_k),\ k = 1, \ldots, M$$

M depends on the number of antennas in an array
For an array of 30 antennas like the GMRT, M ?

Imaging

$$I(l,m) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$

Image

Visibilities

$$I^D(l,m) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(u,v)V'(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$

"Dirty" image

Sampling function

Observed visibilities

Imaging

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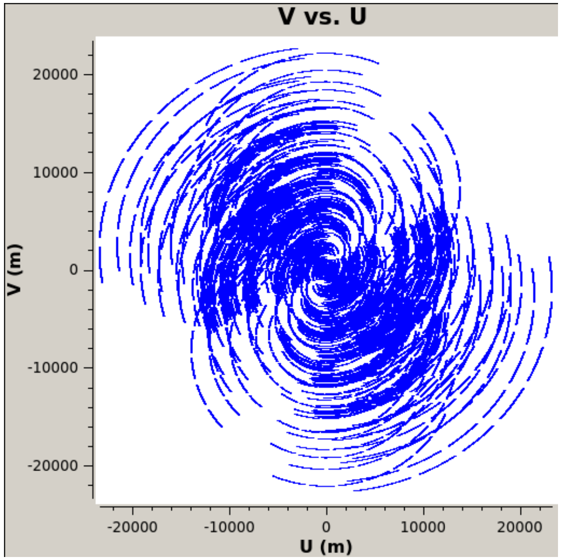
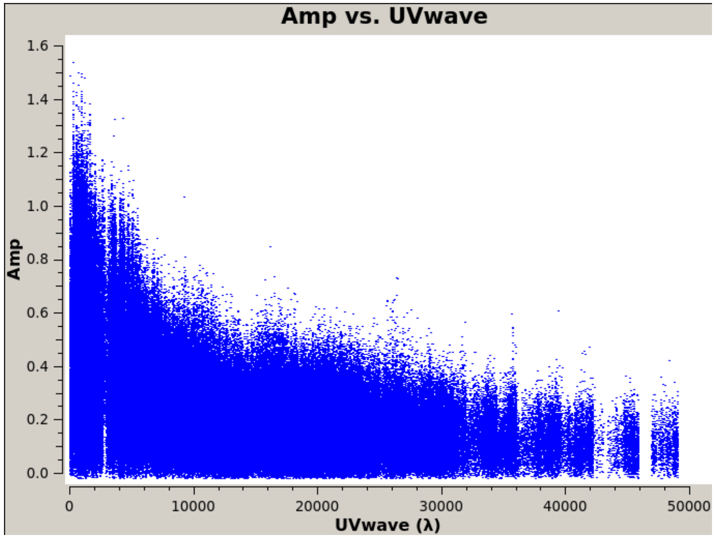


Sampling



Visibilities (complex numbers)
Only amp. shown here

Imaging

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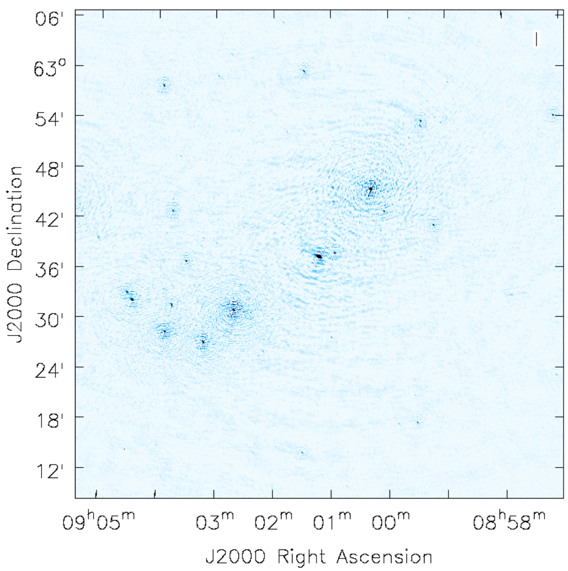
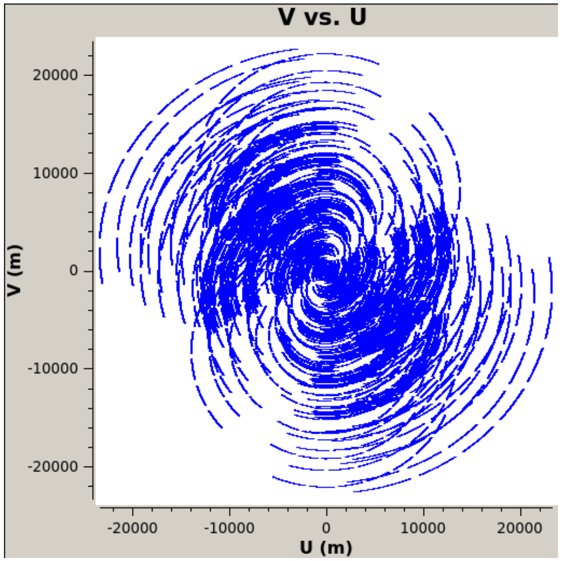
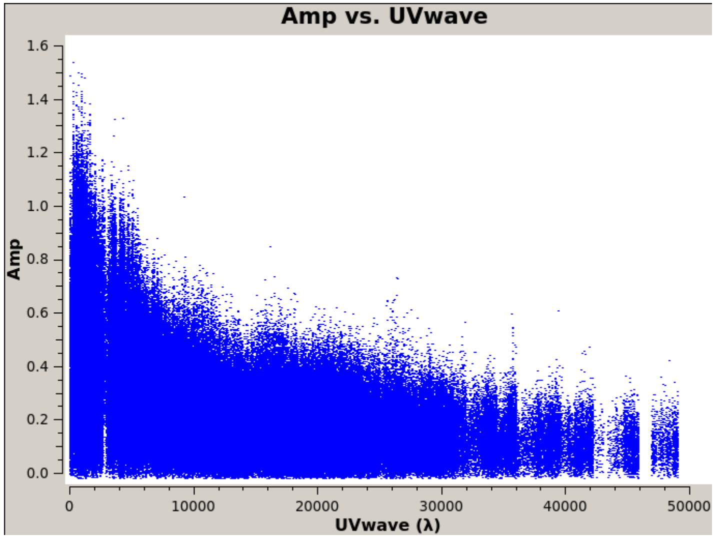


Image



Sampling



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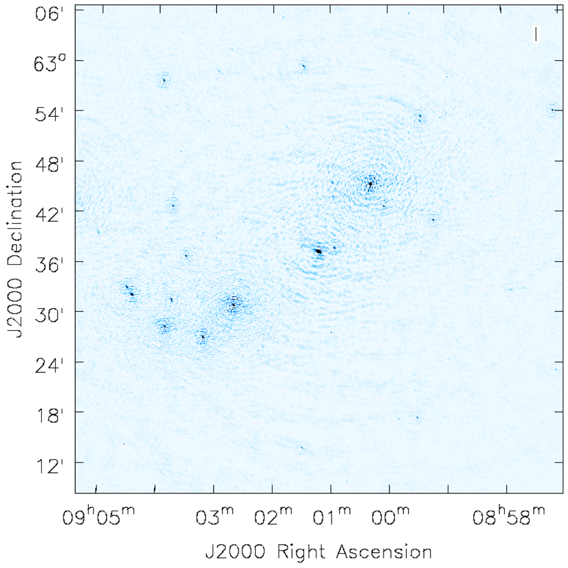

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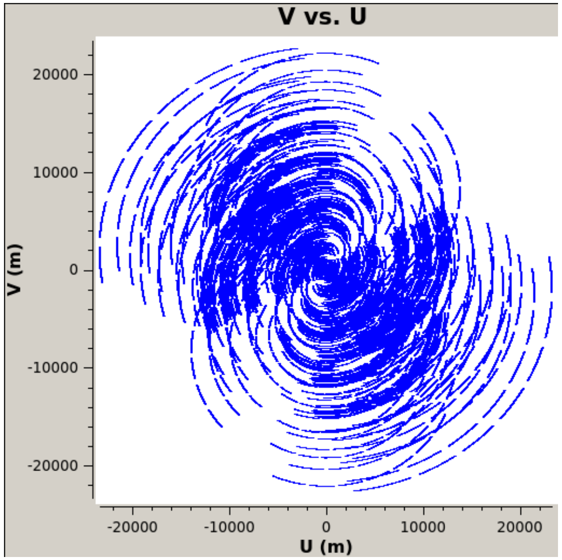

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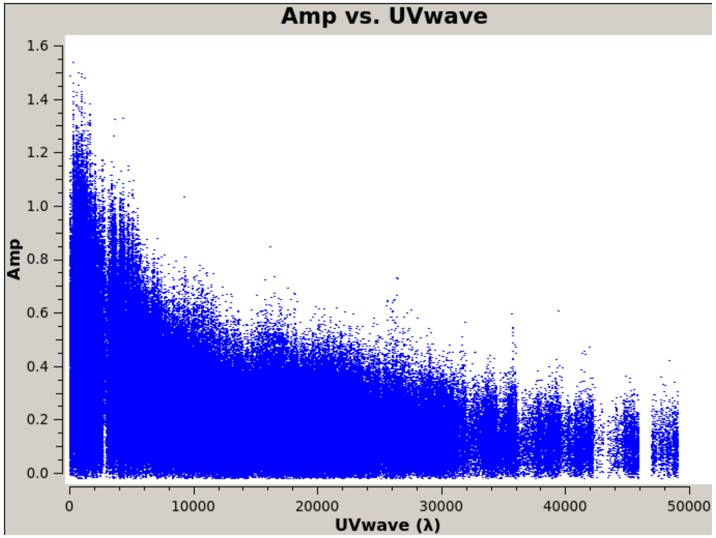

Observed visibilities (complex numbers) Only amp. Shown.

Imaging

$$I^D(l,m) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(u,v)V'(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$

Direct Fourier Transform and Fast Fourier Transform : two methods of imaging

Direct Vs Discrete Fourier Transform

Due to computational advantages fast algorithms to find the Discrete Fourier Transform (DFT) are most commonly used in radio astronomy (algorith for DFT: Fast Fourier Transform).

Application of FFTs requires bringing data to regular grid and then performs the transform.

Only in special cases where number of antenna elements are few, the "direct Fourier Transform" is used.

Imaging

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Direct Fourier Transform and Fast Fourier Transform : two methods of imaging

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Direct Fourier Transform and FFT : two methods of imaging

Direct Fourier Transform: $\frac{1}{M}\sum_{k=1}^{M} V'(u_k, v_k)e^{2\pi i(u_k l + v_k m)}$

To be evaluated at every point of a NxN grid.

Number of multiplications needed to evaluate are ~$2MN^2$

M and N are of the same order and thus the number of multiplications needed are ~N^4

Imaging

$$I^D(l,m) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(u,v)V'(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$

Direct Fourier Transform and FFT : two methods of imaging

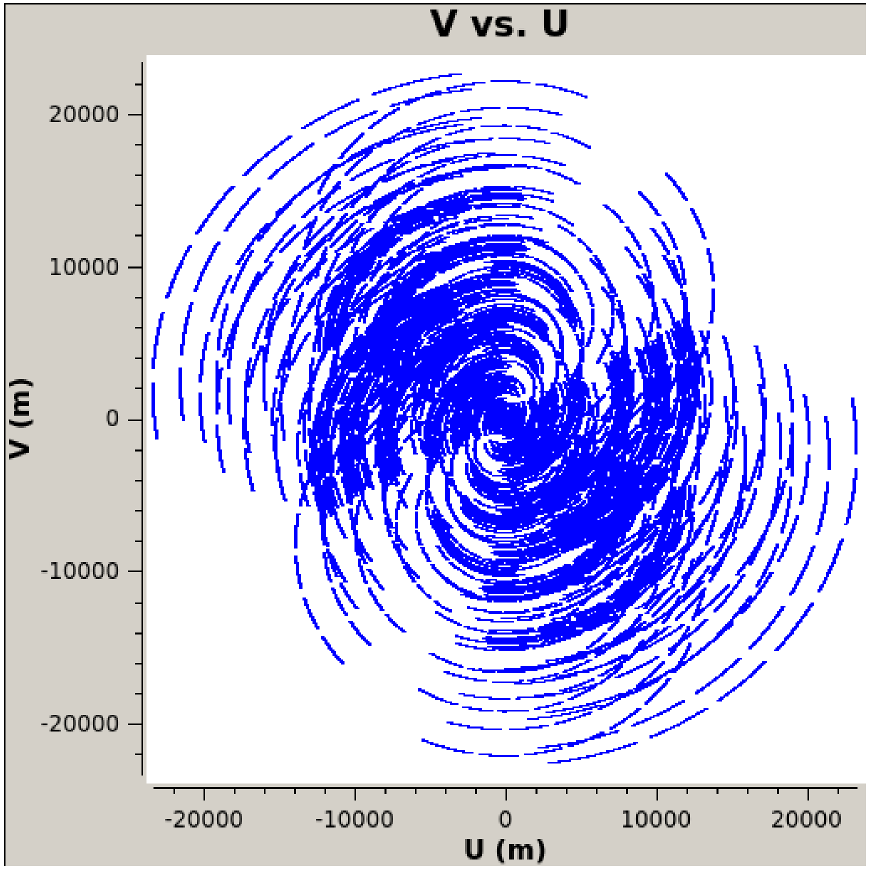
Direct Fourier Transform: $\frac{1}{M}\sum_{k=1}^{M} V'(u_k,v_k)e^{2\pi i(u_k l+v_k m)}$

Fast Fourier Transform: interpolation of the data onto a regular grid and then apply FFT algorithm.

The interpolation of data onto a grid is referred to as “gridding”.

Fast Fourier Transform

Sampling



Fast Fourier Transform

Requires the data to be on a regular grid.

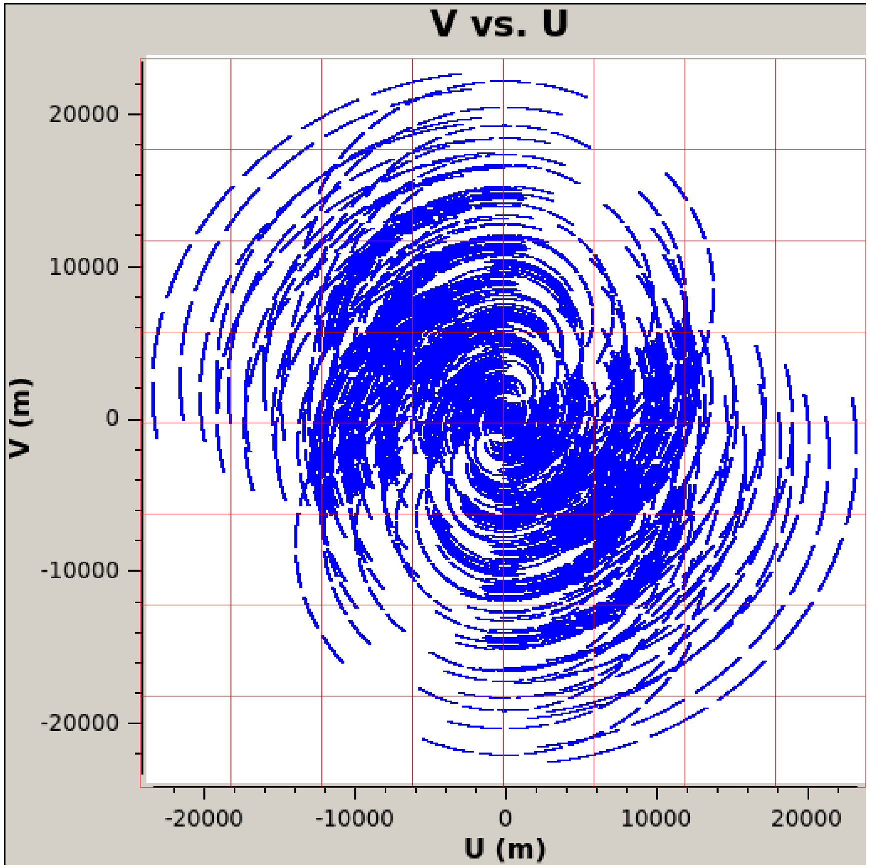
Gridding

To bring the data to a regular grid required ~ N operations.

Further the FFT algorithms only require ~ $N^2 \log_2 N$ operations. (E. g. Cooley-Tukey algorithm)

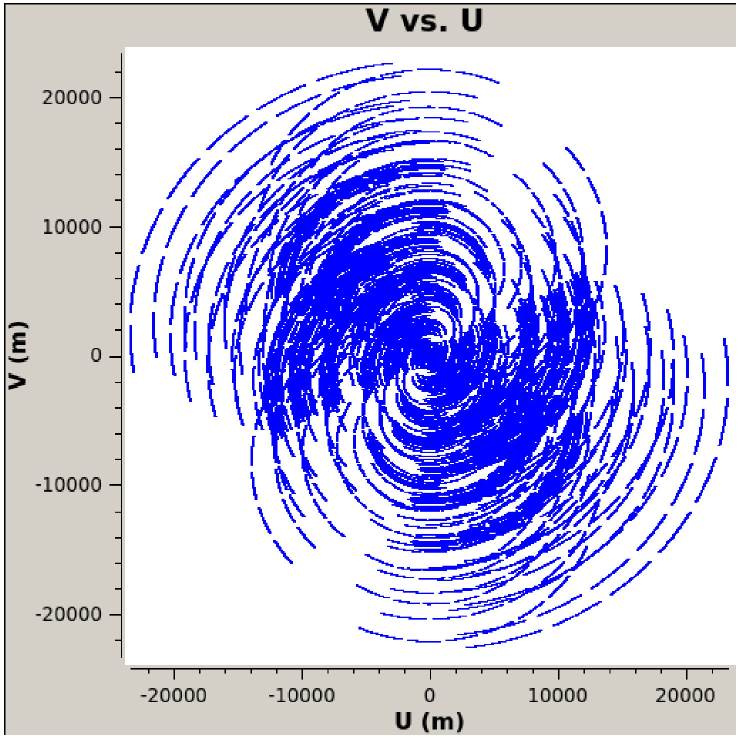
Compare this with N^4 for the DFT case

In most common situations, FFTs are used.



Sampling and the point source response or the beam

$$I^D(l,m) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(u,v)V'(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$



Sampling function:

$$S(u,v) = \sum_{k=1}^{M} \delta(u-u_k, v-v_k)$$

Sampled visibilities:

$$V^S(u,v) \equiv \sum_{k=1}^{M} \delta(u-u_k, v-v_k)V'(u_k,v_k)$$

$$V^S = SV'$$

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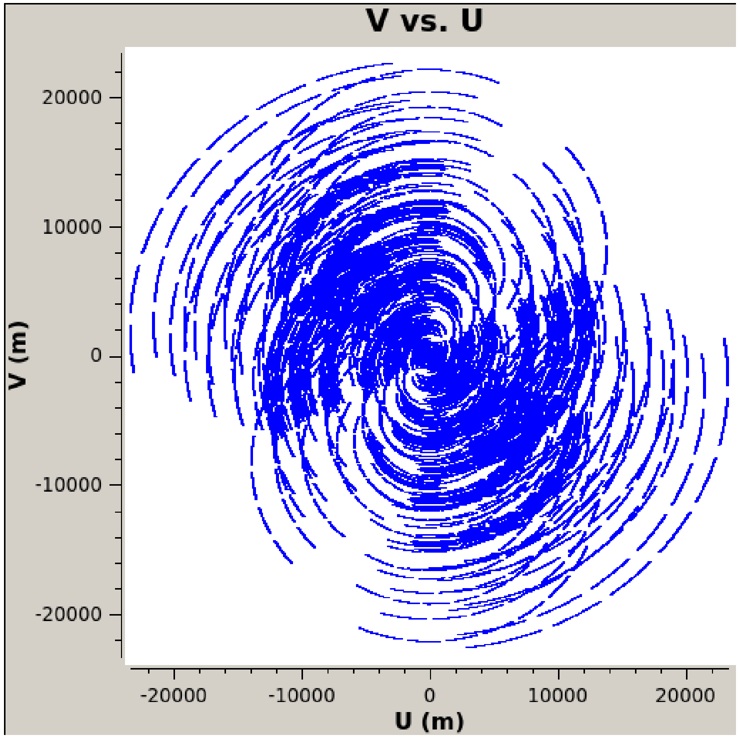

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$V^S = SV'$ → $I^D = \mathfrak{F}V^S = \mathfrak{F}(SV')$

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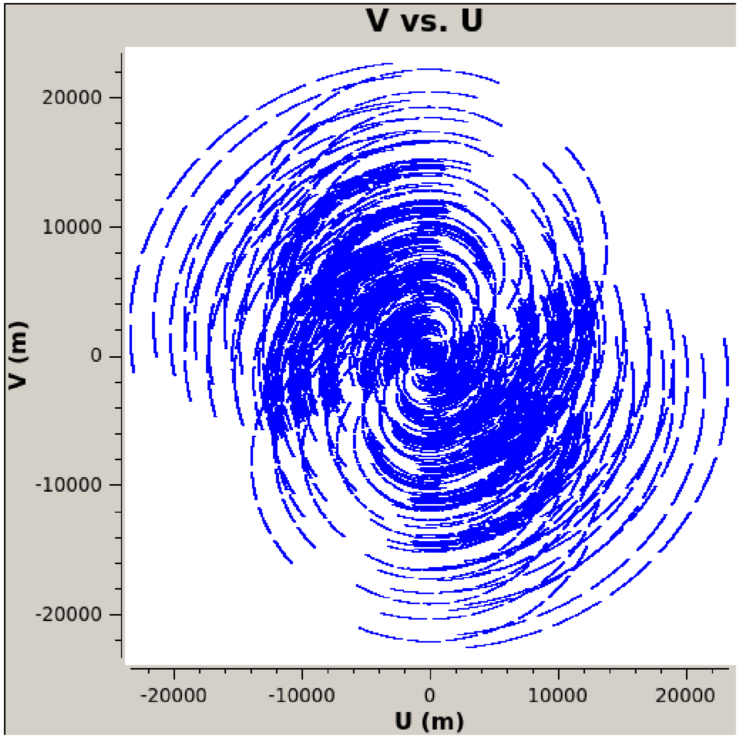
FT of a product of functions, is the convolution of their FTs,

Ref. FT text book
e. g. Bracewell



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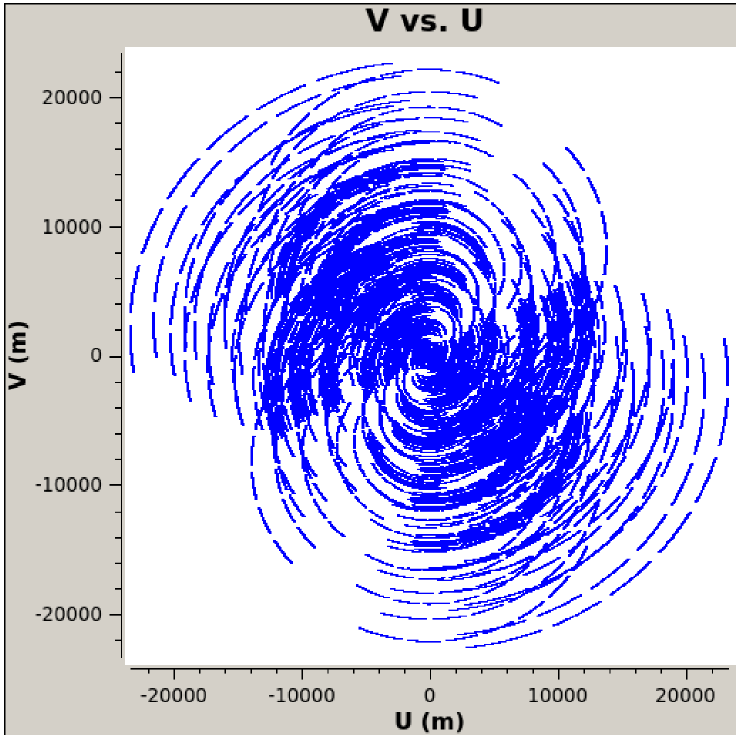


* symbol denotes convolution

Sampling and the point source response or the beam

$$I^D = \mathfrak{F} V^S = \mathfrak{F}(SV')$$

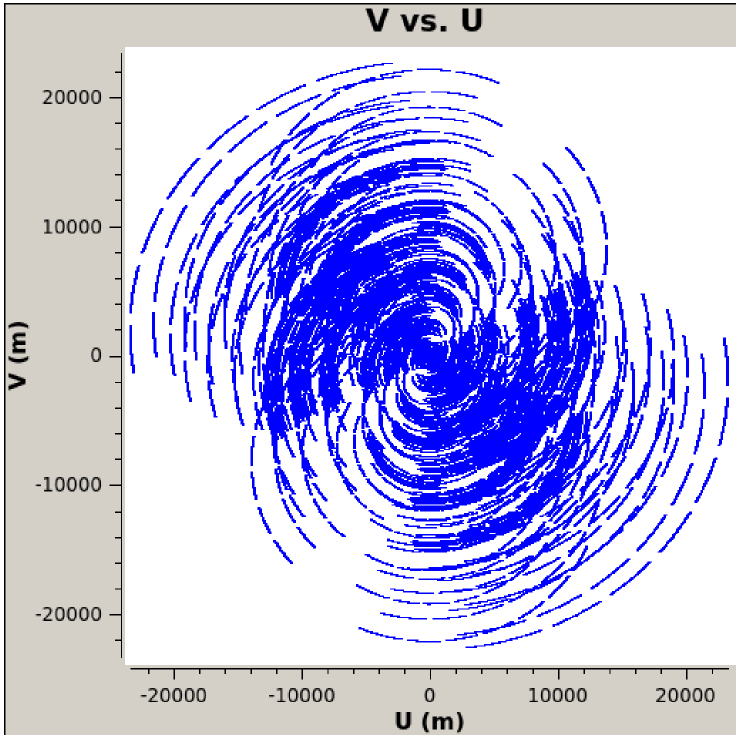
$$I^D = \mathfrak{F} S * \mathfrak{F} V'$$

For a point source of unit flux density, located at l_0, m_0

$$|V'(u,v)| \equiv 1$$

Assuming there is no other noise

FT of this ?



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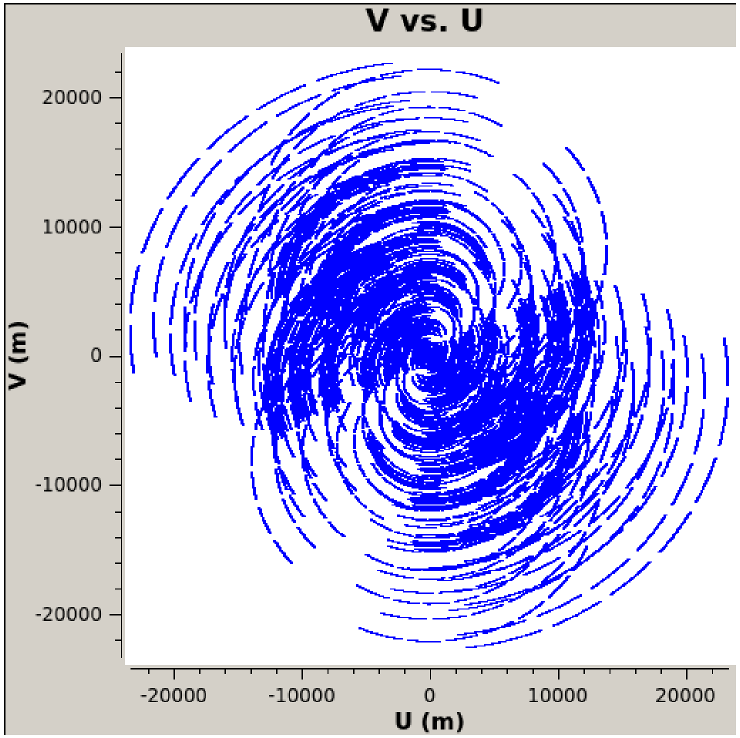
FT of this will be a delta function.



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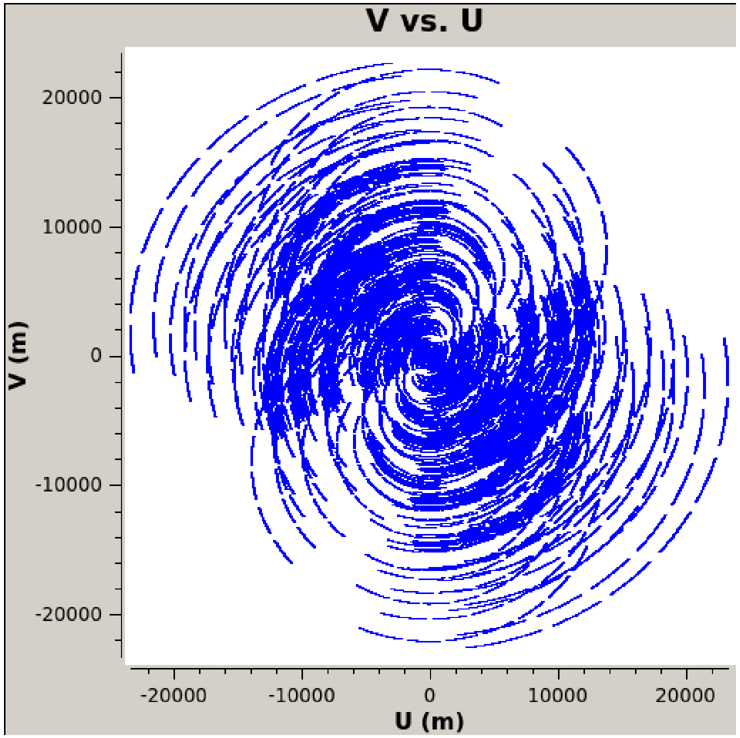
$$\mathfrak{F} V'(l,m) = \delta(l - l_0, m - m_0)$$



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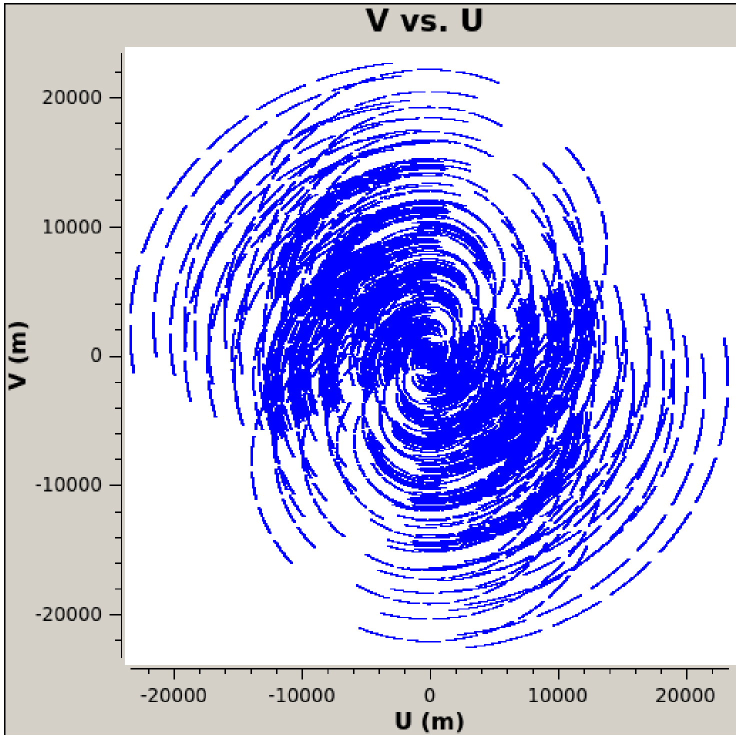
$$I^D = \mathfrak{F}S * \delta(l - l_0, m - m_0) = \mathfrak{F}S$$



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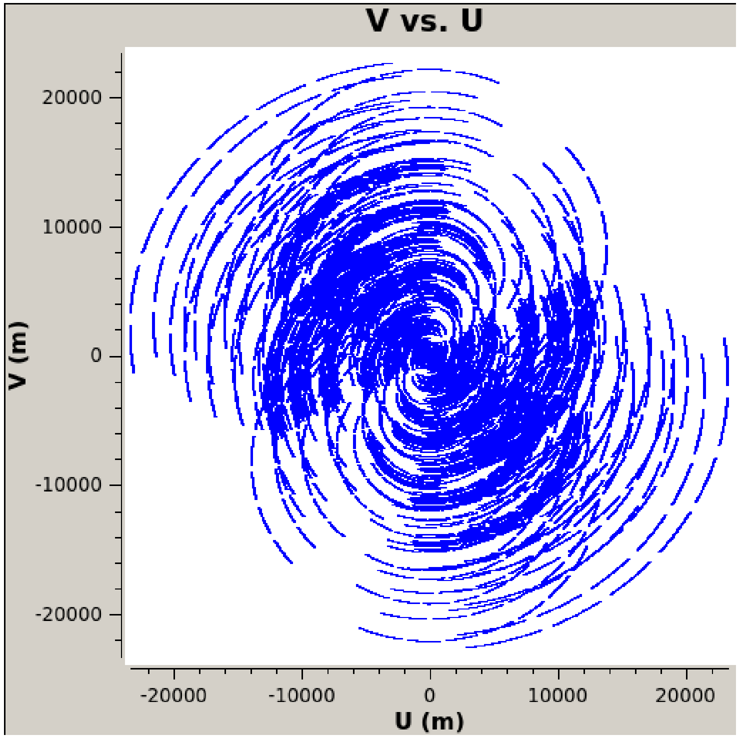

Synthesized beam:

$$B = \mathfrak{F}S$$

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Sampling and the point source response or the beam

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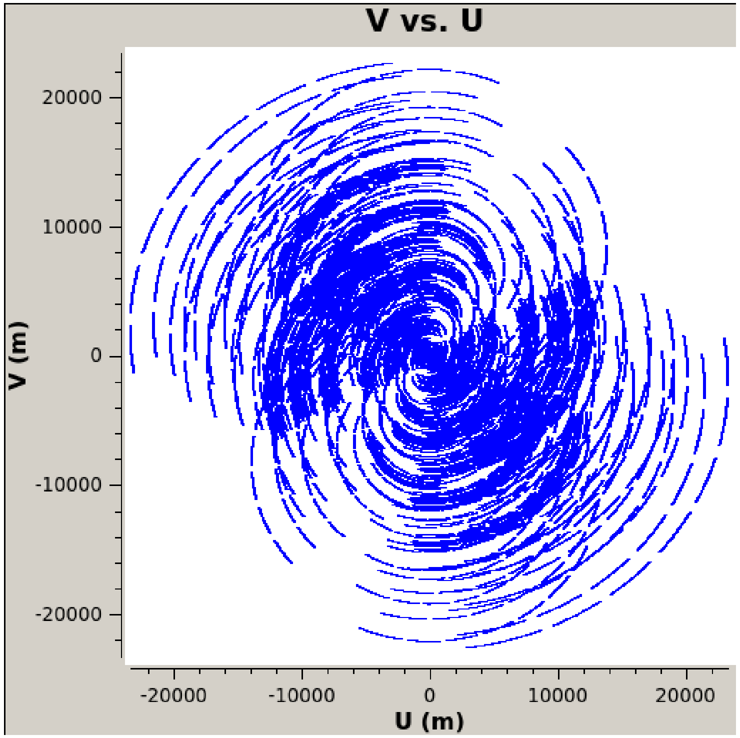
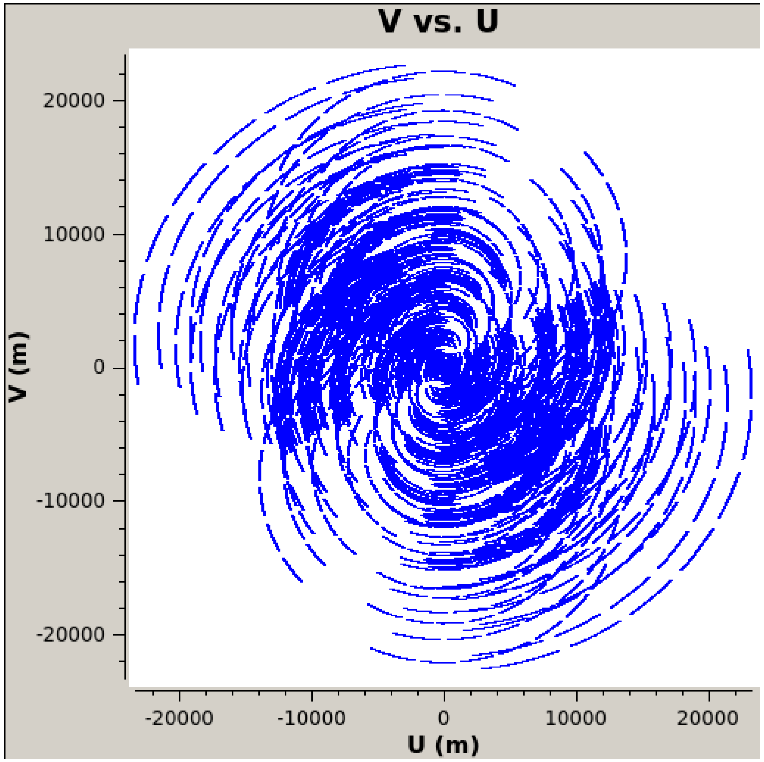


Point source response of the array:

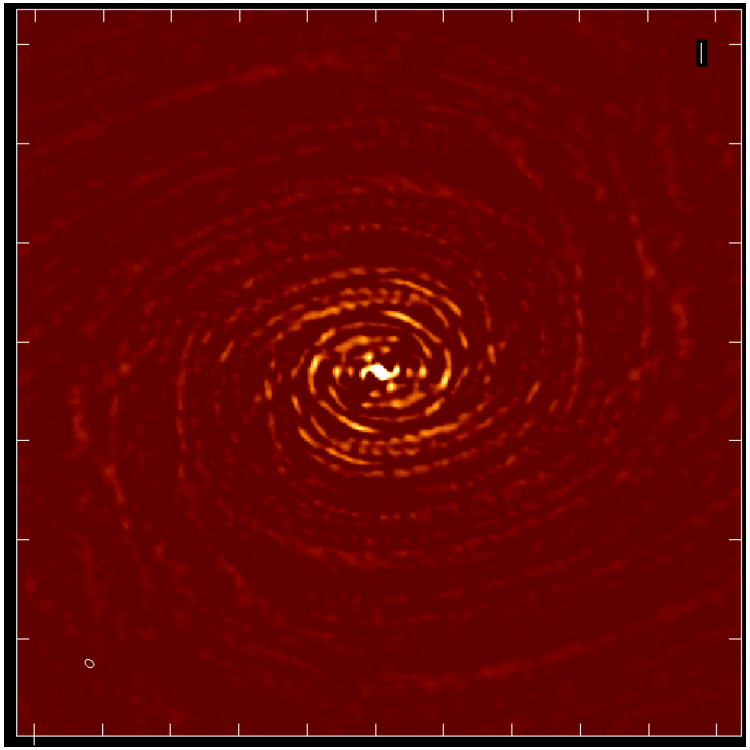
Synthesized beam: $B = \mathfrak{F}S$

* symbol denotes convolution

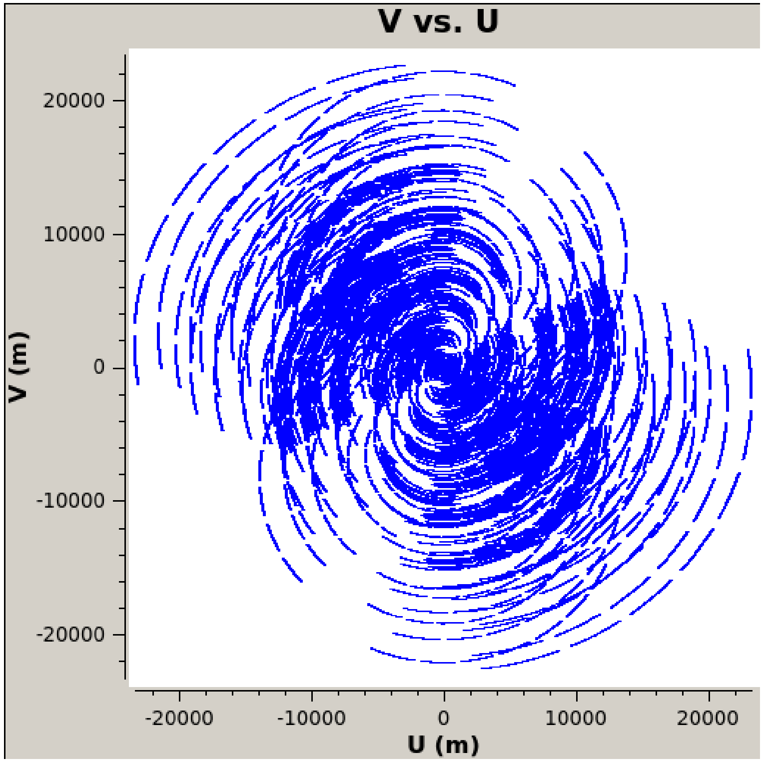
Synthesized beam



FT →



Synthesized beam



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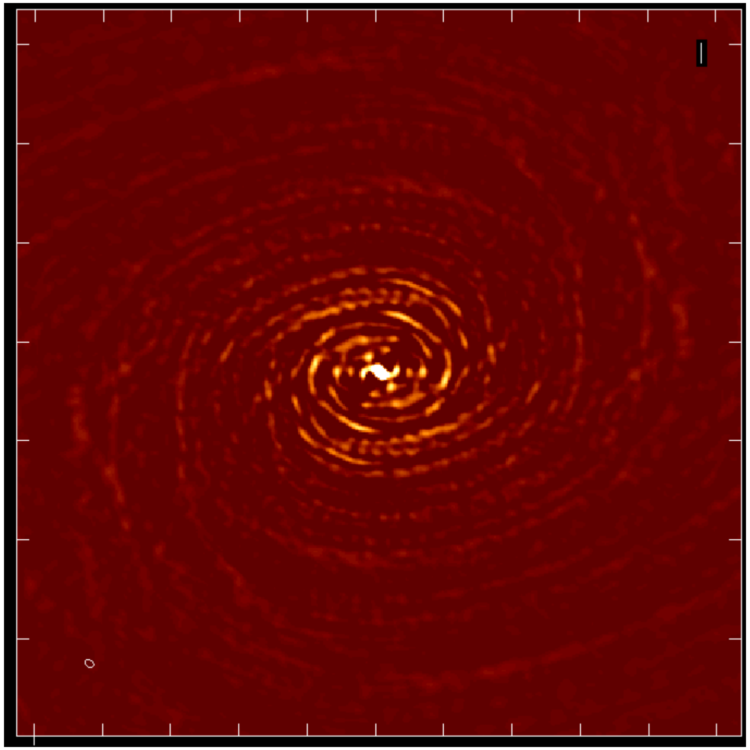

Desirable characteristics: Low and uniform sidelobes; high resolution

No unique approach to get all of this. Choice according to the science requirement.

Weighting: control the shape of the beam

$$S(u,v) = \sum_{k=1}^{M} \delta(u - u_k, v - v_k)$$

$$B = \mathfrak{F} S$$

Introduce a weighted sampling distribution:

Weighting: control the shape of the beam

$$S(u,v) = \sum_{k=1}^{M} \delta(u - u_k, v - v_k)$$

$$B = \mathfrak{F} S$$

Introduce a weighted sampling distribution:

$$W(u,v) = \sum_{k=1}^{M} R_k T_k D_k \delta(u - u_k, v - v_k)$$

T_k = tapering function
D_k = density weighting
R_k = reliability weight

Weighting: control the shape of the beam

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$$V^S(u,v) \equiv \sum_{k=1}^{M} \delta(u-u_k, v-v_k) V'(u_k, v_k)$$

Weighted visibilities

$$V^W = WV'$$

$$V^W(u,v) = \sum_{k=1}^{M} R_k T_k D_k \delta(u-u_k, v-v_k) V'(u_k, v_k)$$

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If the sampling were a smooth function like a Gaussian we would have no sidelobes.
However it is like a bunch of delta functions – often with large gaps in between.

In an array: typically data points are in the inner region of the uv-plane and are sparse outside – gives rise to more weight to shorter spacings.

Briggs 1995 (PhD thesis: detailed treatment of weighting of visibilities)

Weighting: control the shape of the beam

$$W(u,v) = \sum_{k=1}^{M} R_k T_k D_k \delta(u - u_k, v - v_k)$$

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$$V^W(u,v) = \sum_{k=1}^{M} R_k T_k D_k \delta(u - u_k, v - v_k) V'(u_k, v_k)$$

Tapering weights are used to downweight the data at the outer edge.
Density weights are used to lessen the effect of non-uniform density of sampling in the uv-plane.

The weights are factored into components arbitrarily - only for convenience.

Briggs 1995 (PhD thesis: detailed treatment of weighting of visibilities)

Weighting: control the shape of the beam

$$W(u,v) = \sum_{k=1}^{M} R_k T_k D_k \delta(u - u_k, v - v_k)$$

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T_k = tapering function, separable into u and v dependent parts.

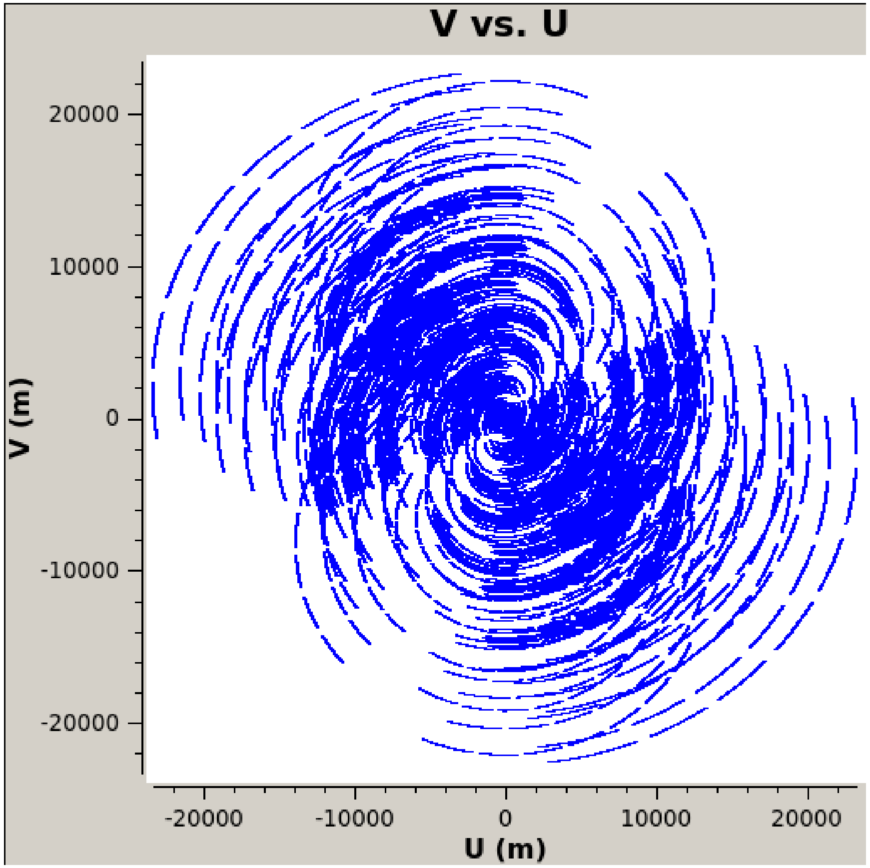
$$T(u,v) = T_1(u) T_2(v)$$

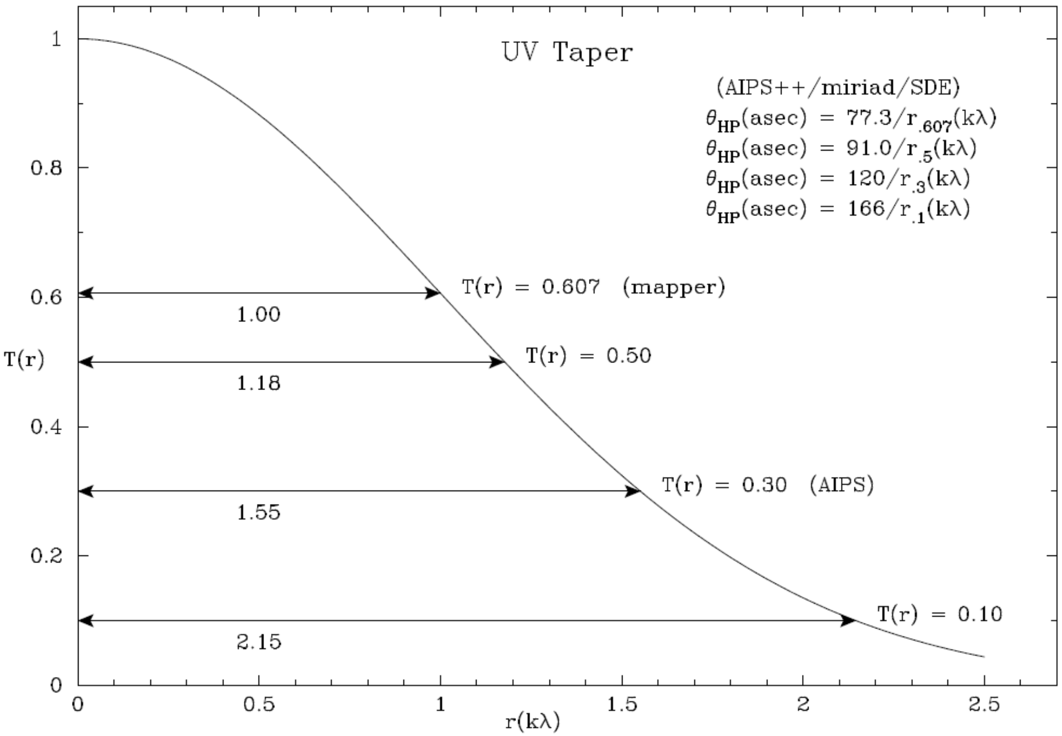
$$T_k = T(r_k) \qquad r_k \equiv \sqrt{u_k^2 + v_k^2}.$$

A Gaussian taper, for example:

$$T(r) = \exp(-r^2/2\sigma^2)$$

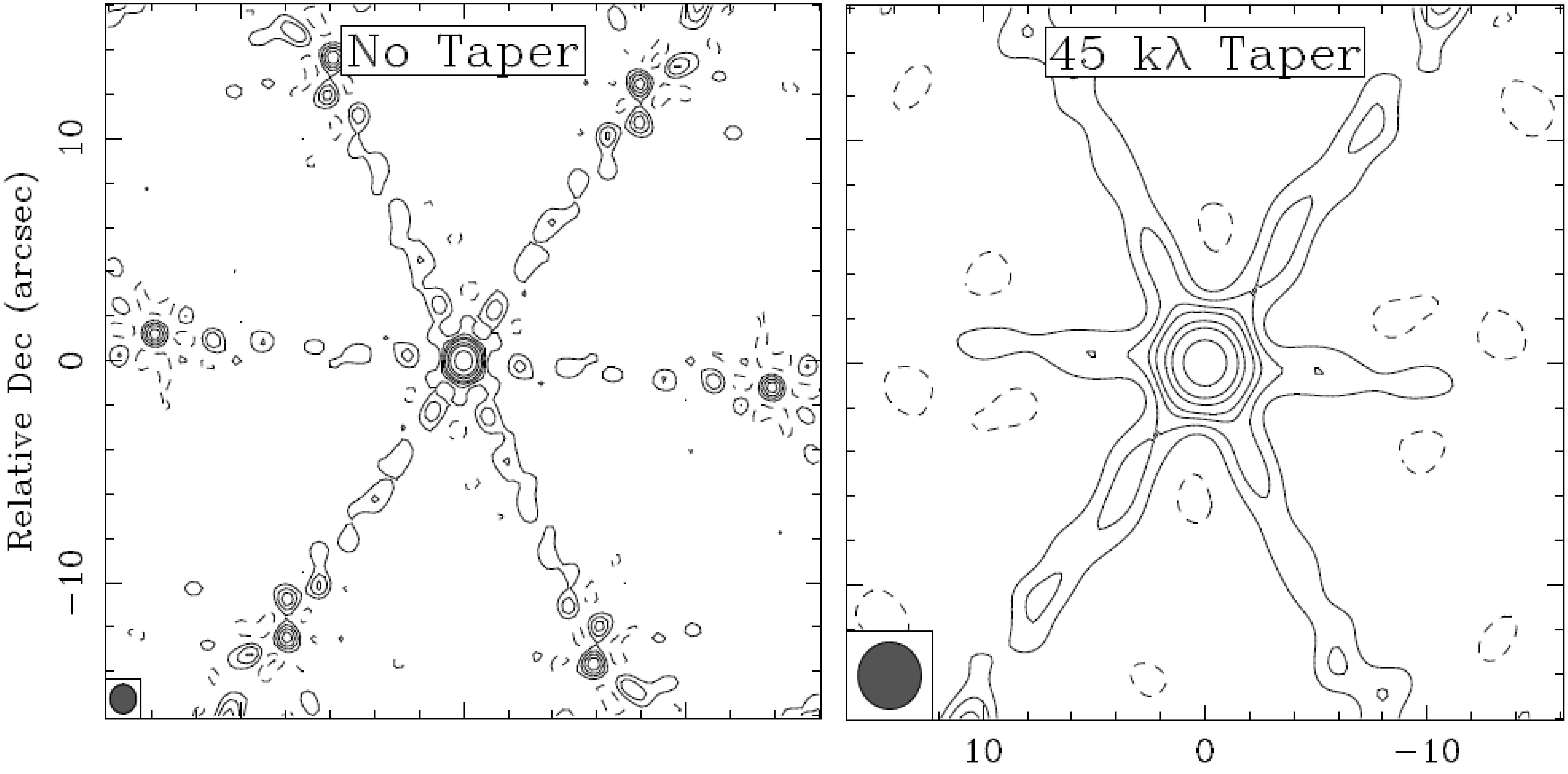
Tapering





The synthesized beam width will change depending on the choice of the taper.

Tapering example



Density weighting

Natural weights

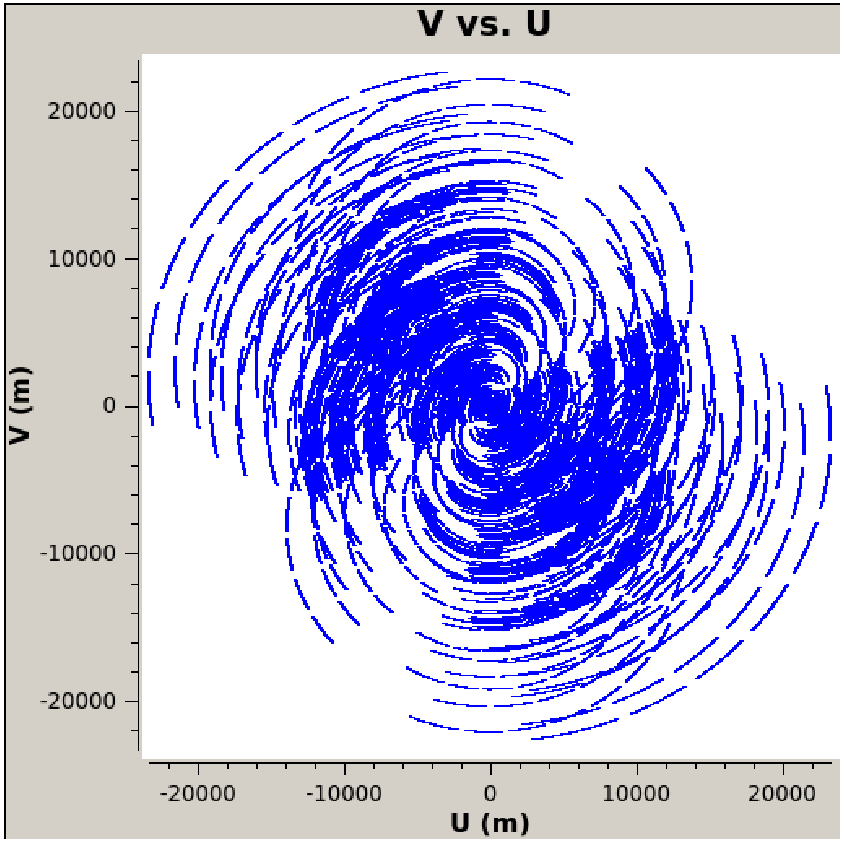
$$D_k = 1$$

Uniform weights

$$D_k = \frac{1}{N_s(k)}$$

$N_s(k)$ is the number of points within a symmetric region in (u,v) of width s centered on k^{th} point.

N_s is the number of points within a grid cell.



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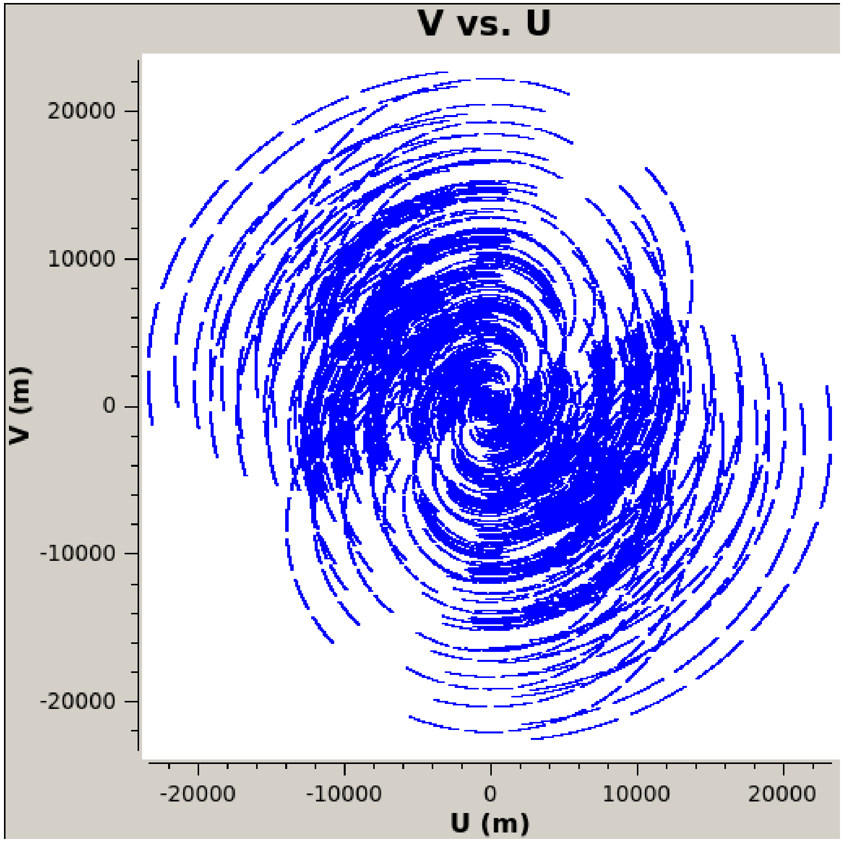
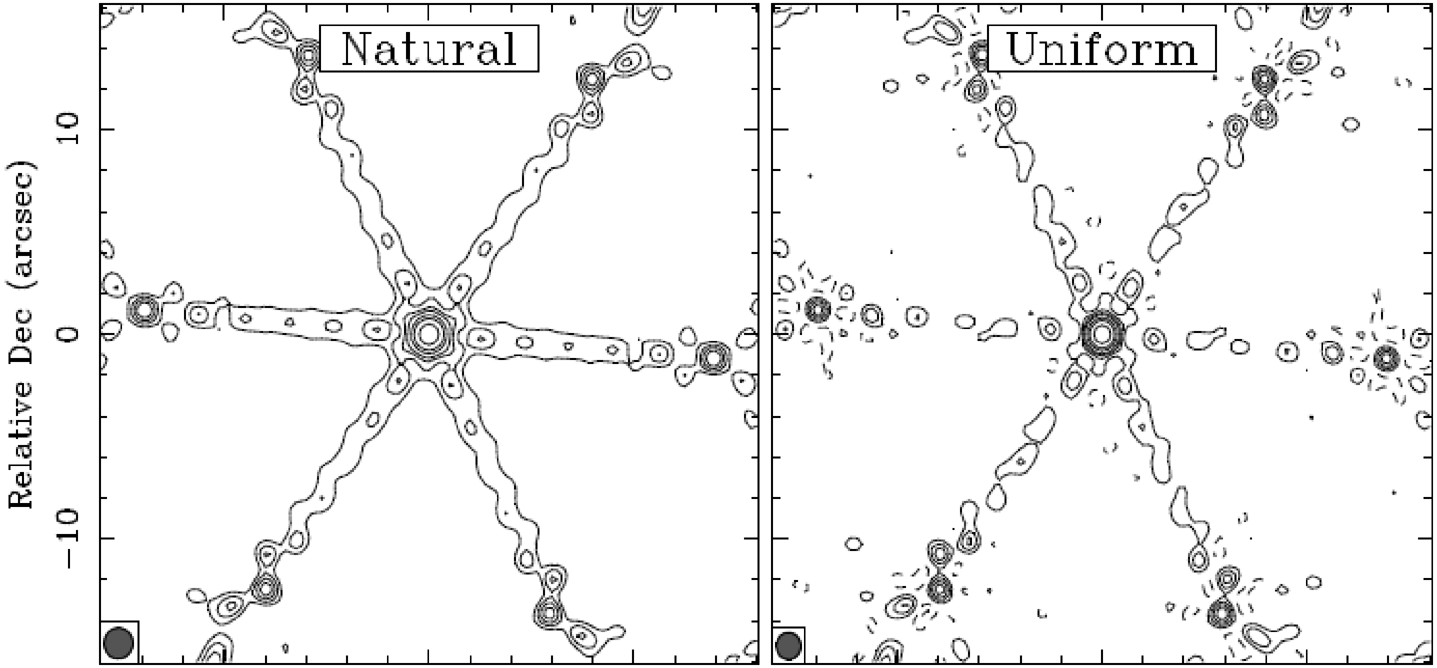


Robust weighting: hybrid form of weighting: uses minimisation of summed sidelobe power and thermal noise.

Density weights example

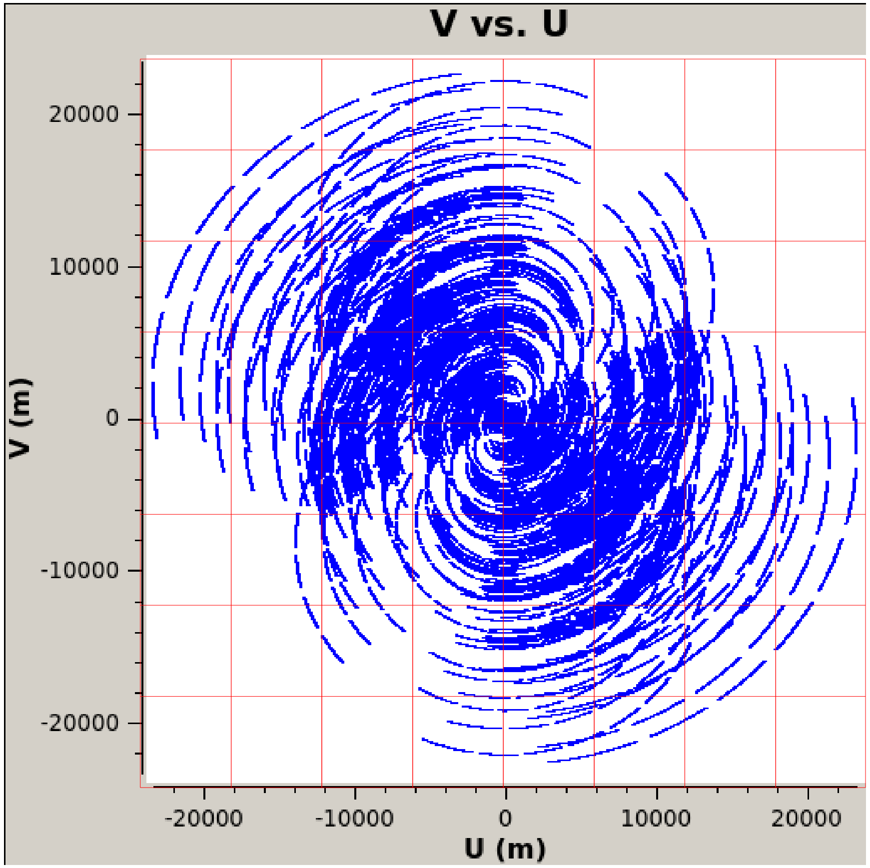


Gridding the visibilities

Motivated by the fact that we want to take full advantage of the FFT algorithms.

We want the data on a “grid” that is uniformly spaced with a power of two points on each side.

Interpolation procedure needed to bring the data onto a grid.

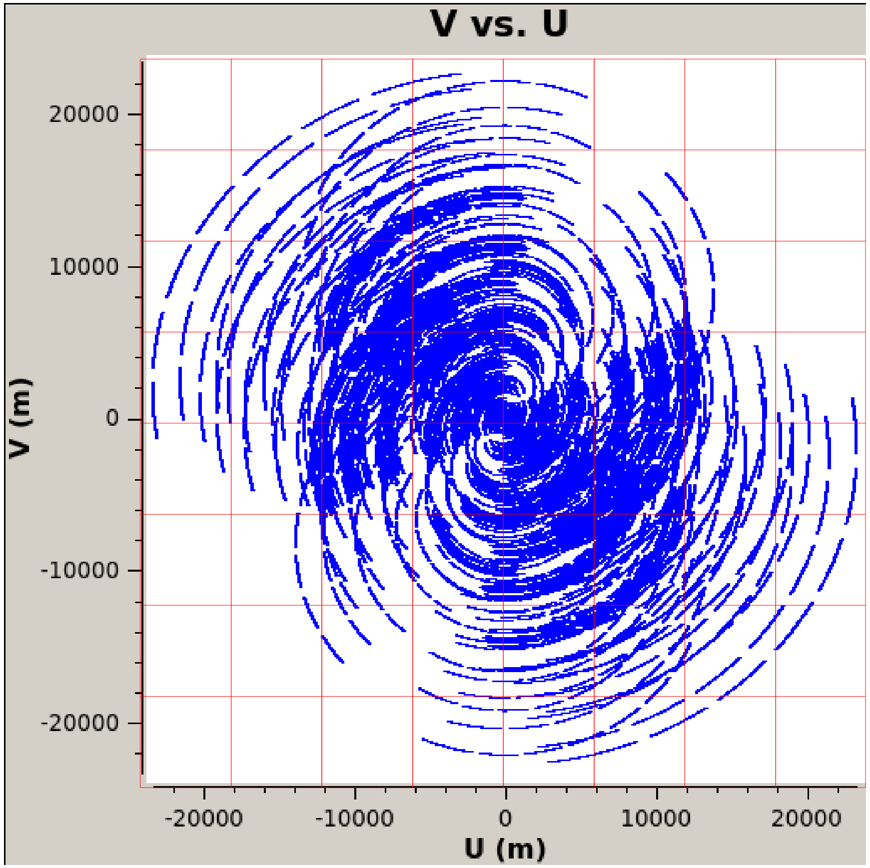


Gridding the visibilities

Gridding by convolution: *convolve the weighted sampled visibility with some suitable function and then sample this function on the desired grid.*
Value assigned at each grid point will be an average of the local values.

$$C * V^W$$

$$\sum_{k=1}^{M} C(u_c - u_k, v_c - v_k) V^W(u_k, v_k)$$



Resampling

$$V^R = R\left(C * V^W\right) = R\left(C * (WV')\right)$$

$$R(u,v) = \mathrm{III}(u/\Delta u, v/\Delta v) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - u/\Delta u, k - v/\Delta v)$$

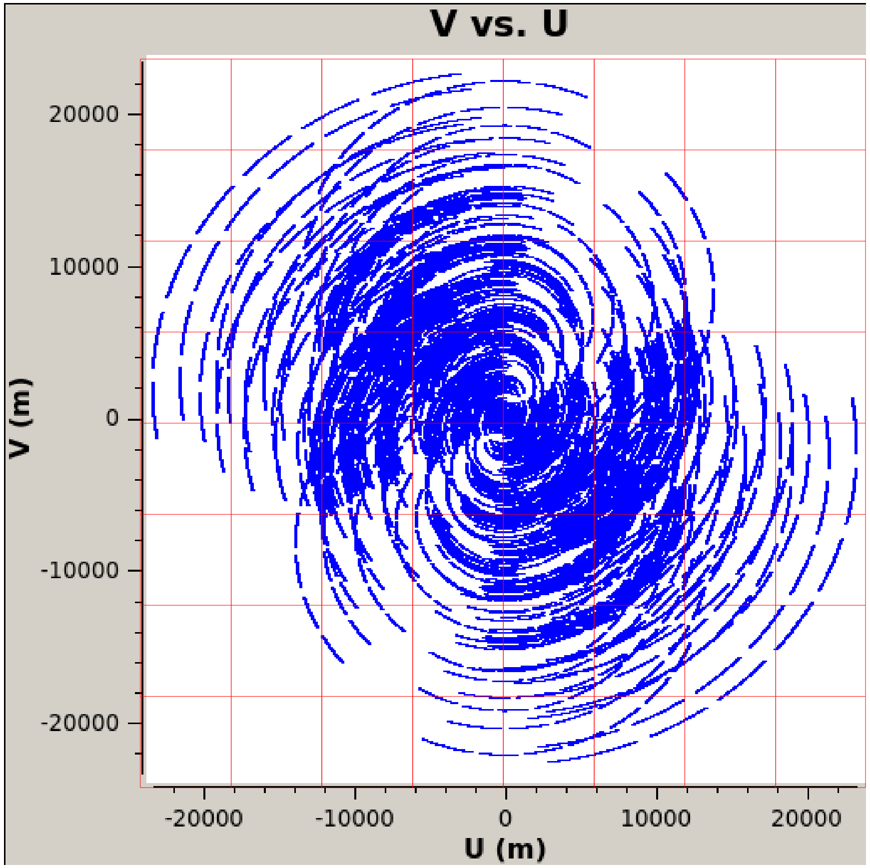
Gridding the visibilities

Gridding by convolution: *convolve the weighted sampled visibility with some suitable function and then sample this function on the desired grid.*
Value assigned at each grid point will be an average of the local values.

$$C * V^W$$

Visibilities are a linear combination of M delta functions:

$$\sum_{k=1}^{M} C(u_c - u_k, v_c - v_k) V^W(u_k, v_k)$$

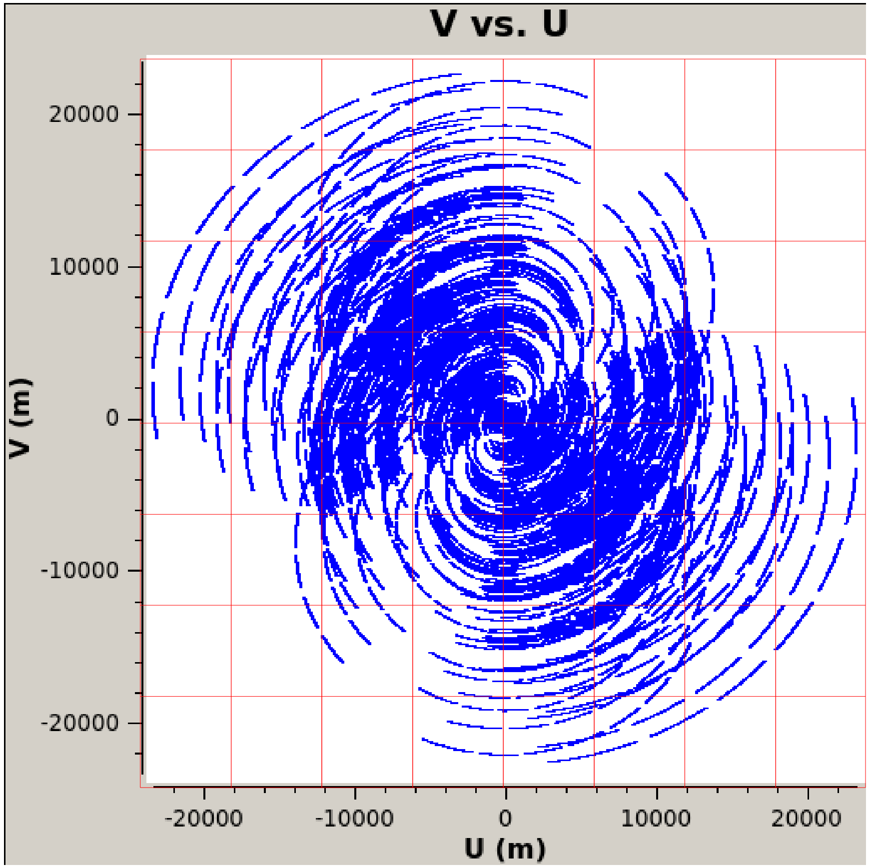


u_c, v_c is a grid point

Gridding the visibilities

Gridding by convolution: *convolve the weighted sampled visibility with some suitable function and then sample this function on the desired grid.*
Value assigned at each grid point will be an average of the local values.

$$\sum_{k=1}^{M} C(u_c - u_k, v_c - v_k) V^W(u_k, v_k)$$



Resampled visibility:

$$V^R = R\left(C * V^W\right) = R\left(C * (WV')\right)$$

Normalization of C in connected to the weighting scheme.

Gridding the visibilities

Gridding by convolution: *convolve the weighted sampled visibility with some suitable function and then sample this function on the desired grid.*
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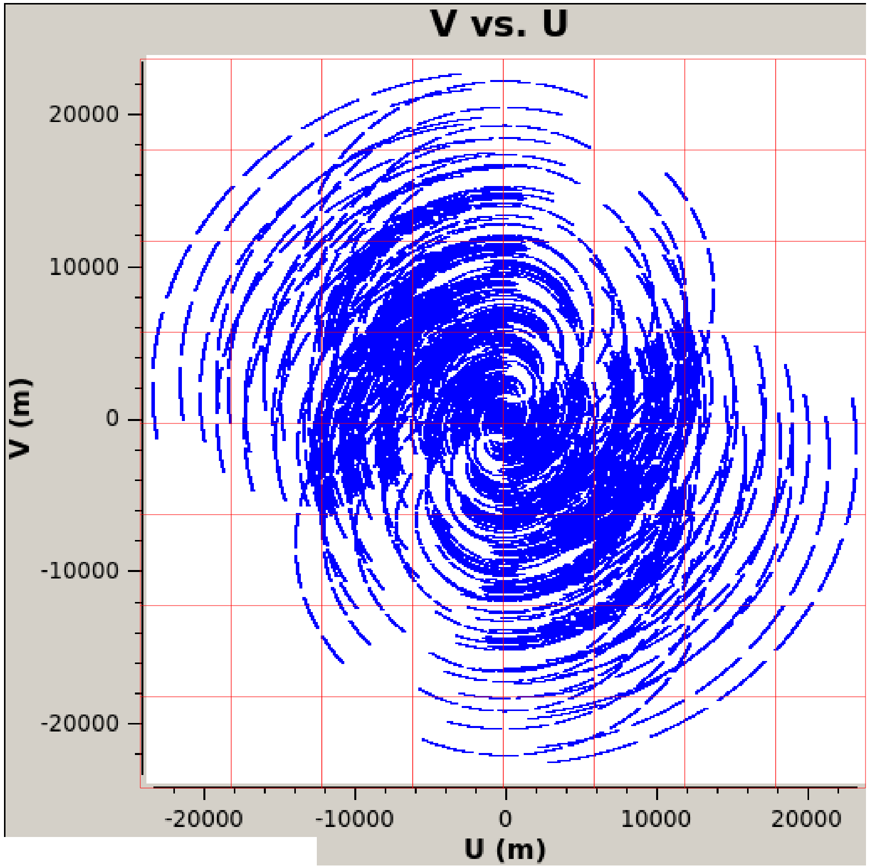
$$V^R = R\left(C * V^W\right) = R\left(C * (WV')\right)$$

Normalization of C in connected to the weighting scheme.

R is the “bed-of-nails” function or the sha Function: a train of delta functions



$$R(u,v) = \mathrm{III}(u/\Delta u, v/\Delta v) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - u/\Delta u, k - v/\Delta v)$$

$\mathfrak{F} V^R$ Can be evaluated using FFT

Gridding the visibilities

Gridding by convolution: convolve the weighted sampled visibility with some suitable function and then sample this function on the desired grid.
Value assigned at each grid point will be an average of the local values.

$$C * V^W$$

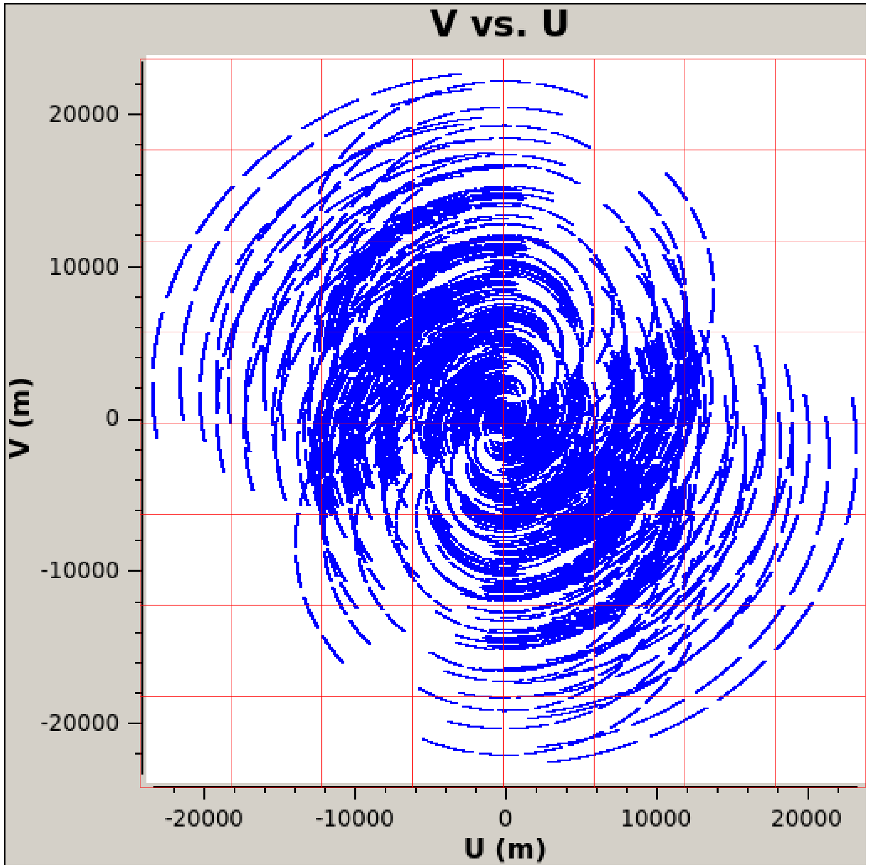
$$\sum_{k=1}^{M} C(u_c - u_k, v_c - v_k) V^W(u_k, v_k)$$

$$V^R = R\left(C * V^W\right) = R\left(C * (WV')\right)$$

The "dirty image" can be given by

$$\widetilde{I}^D = \mathfrak{F}R * \left[(\mathfrak{F}C)\left(\mathfrak{F}V^W\right)\right]$$

$$= \mathfrak{F}R * \left[(\mathfrak{F}C)\left(\mathfrak{F}W * \mathfrak{F}V'\right)\right]$$



$$R(u,v) = \text{III}(u/\Delta u, v/\Delta v)$$

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$$= \mathfrak{F}R * \left[(\mathfrak{F}C)\left(\mathfrak{F}W * \mathfrak{F}V'\right)\right]$$

$$R(u,v) = \text{III}(u/\Delta u, v/\Delta v)$$

$$(\mathfrak{F}R)(l,m) = \Delta u\,\Delta v \text{III}(l\Delta u, m\Delta v) = \Delta u\,\Delta v \sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} \delta(j - l\Delta u, k - m\Delta v)$$

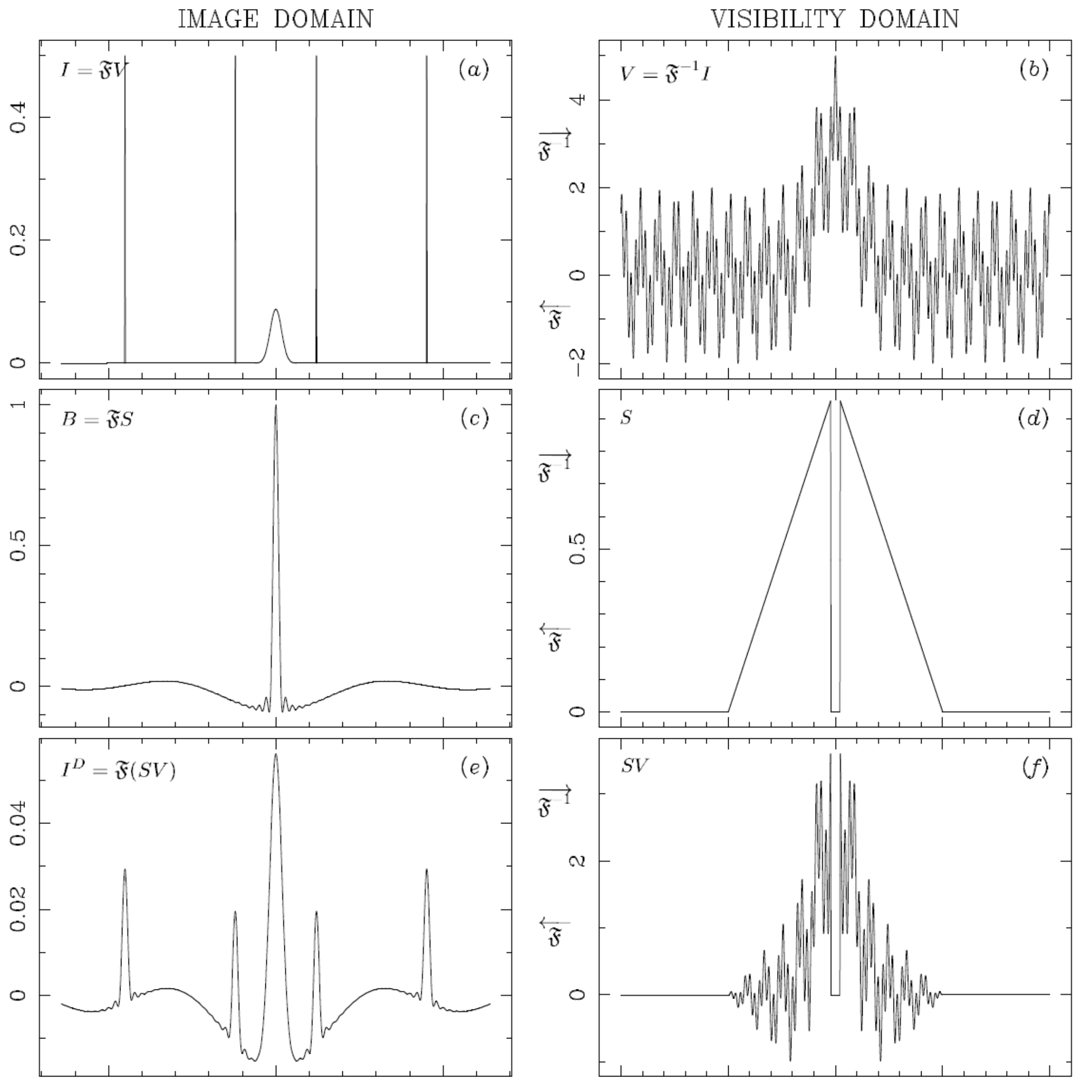
- Aliasing is introduced due to convolution with the scaled sha function.
- Resampling makes dirty image a periodic function of l and m of period $1/\Delta u$ and $1/\Delta v$.

Graphical representation

Model source: symmetric

Synthesized beam

Dirty image if a direct FT is computed

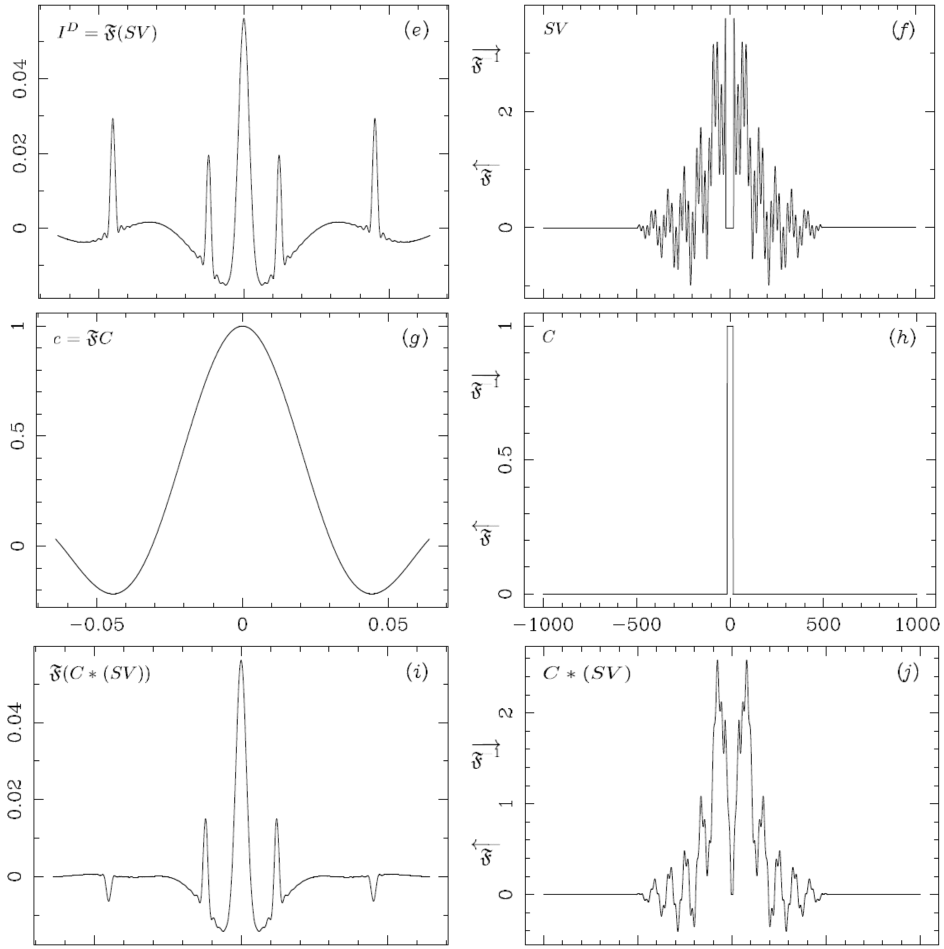


Model Visibilities: Real and even due to symmetry

Sampling: central hole, falling density towards the outskirts

Sampled visibilities

SIRA, Fig. 7-5



Sampled visibilities

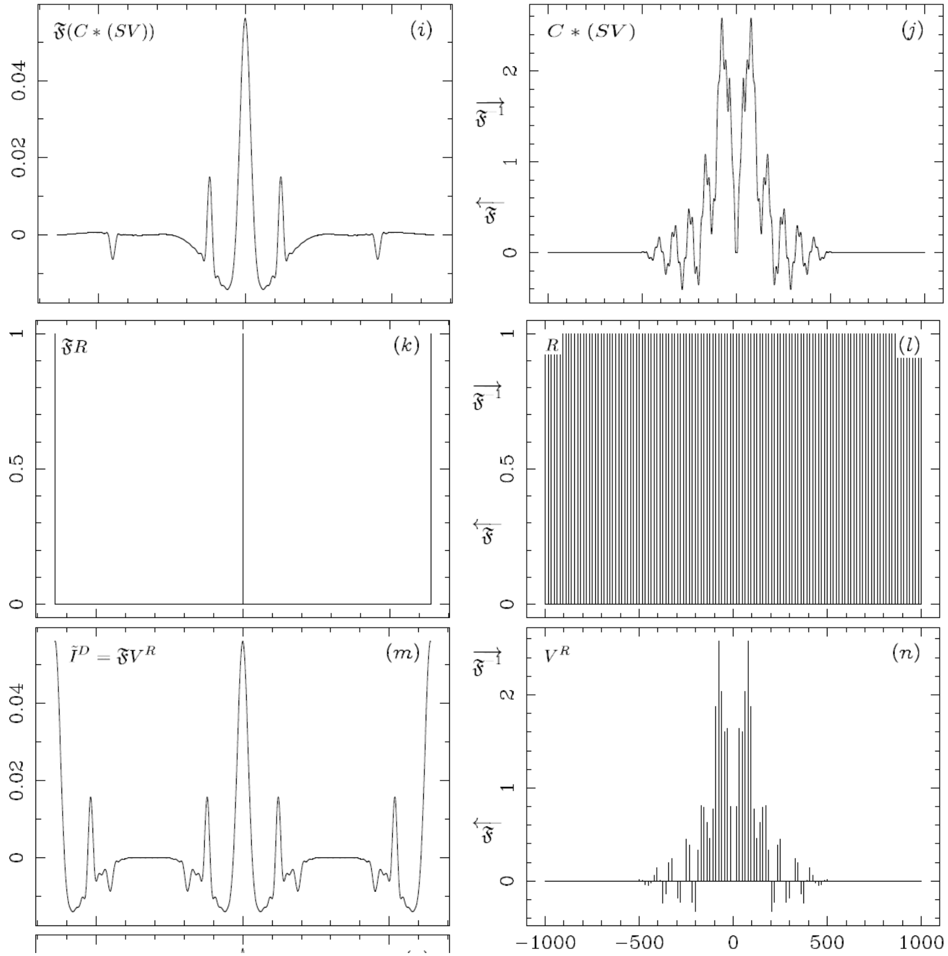
FT of the convolution function

Convolution function

Effect in the image domain

Convolved sampled visibilities

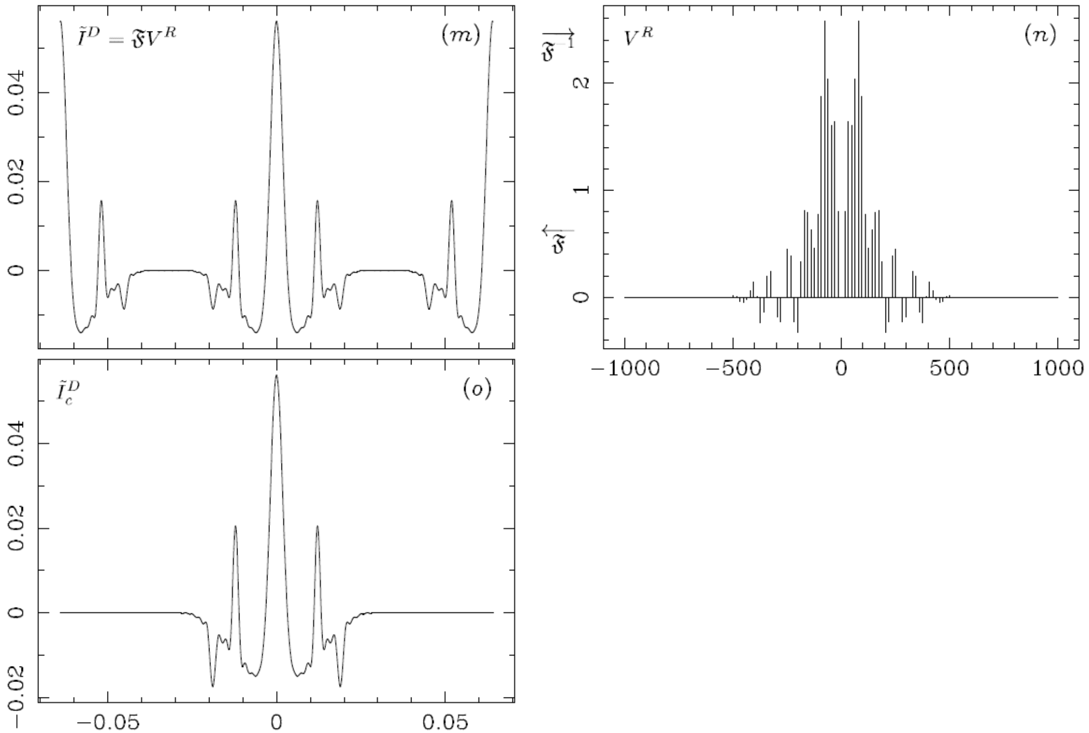
Dirty image: aliasing



Resampling

Resampled visibility

Divide by the FT of the convolution function



This image is far from satisfactory representation of the actual distribution: can do better than this by deconvolution.

Choice of the gridding convolution function

Desired choices to avoid aliasing:

a) image is large enough to include any sources at the edges.

b) avoid under sampling

c) use a gridding convolution function whose Fourier transform drops off rapidly outside the image.

C is chosen to be real and even. C is separable C(u)C(v).

1. a pillbox function
2. truncated exponential
3. a truncated sinc function
4. an exponential multiplied by a truncated sinc
5. a truncated spheroidal