# Astronomical Techniques II
# Lecture 7 - *u*-*v* Coverage and Array Design

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# Response of an interferometer

- Geometric delay - $\tau_g = \frac{\vec{b}.\vec{s}}{c}$
- Correlator output - $r(\tau_g) =< V_1(t)\ V_2(t) >$
- $V_1 = v_1\ cos\ 2\pi\nu(t - \tau_g);\ V_2 = v_2\ cos\ 2\pi\nu t;$
- $r(\tau_g) = v_1 v_2\ cos\ 2\pi\nu\tau_g$

# Response to a Brightness distribution

- $dr = A(\vec{s})\ B(\vec{s})\ \Delta\nu\ \Delta\Omega\ cos\ 2\pi\nu\tau_g$
- $r(\tau_g) = \int_\Omega A(\vec{s})\ B(\vec{s})\ \Delta\nu\ cos\ 2\pi\nu\tau_g\ d\Omega$
- $r(\tau_g) = \Delta\nu\ \int_\Omega A(\vec{s})\ B(\vec{s})\ cos\ \frac{2\pi\nu\vec{b}\cdot\vec{s}}{c}\ d\Omega$

# Phase Tracking Center

- $\vec{s} = \vec{s_0} + \vec{\sigma}$

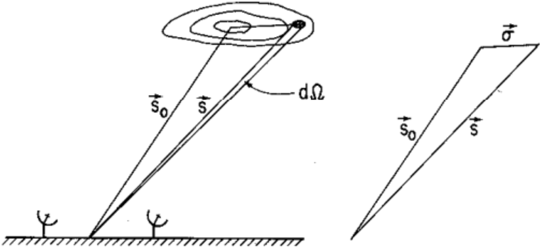


**Figure 2-2.** Position vectors used in deriving the interferometer response to a source. The source is represented by the contours of radio brightness $I(\mathbf{s})$ on the sky.

# Visibility

- $V = |V|\ e^{i\phi_V} = \int_\Omega A_N(\vec{\sigma})\ B(\vec{\sigma})\ e^{-2\pi i\ \nu\ \vec{b}.\vec{\sigma}/c}\ d\Omega$
- $A_N(\vec{\sigma}) = A(\vec{\sigma})/A_0$
- ...
- $r = A_0\ \Delta\nu\ |V|\ cos\ (2\pi\nu\frac{\vec{b}.\vec{s_0}}{c} - \phi_V)$

# Van Cittert-Zernike Theorem

- Relates FT of *Mutal Coherence Function* to the Brightness distribution of a distant source.

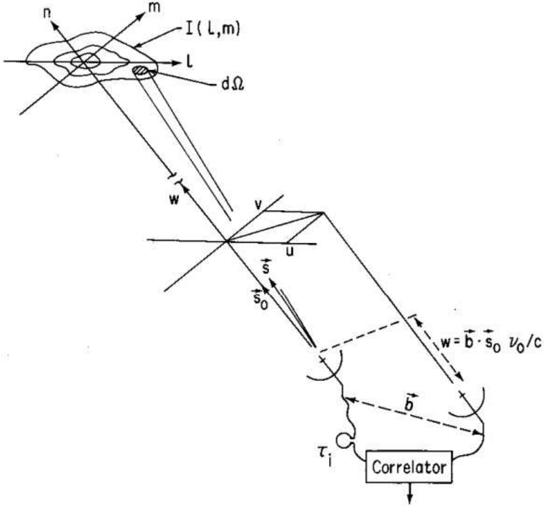
# Effect of bandwidth

- $dr = A_0 \, |V| \, cos(2\pi\nu\tau_g - \phi_V) \, d\nu$
- $r = A_0 \, |V| \, \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} cos(2\pi\nu\tau_g - \phi_V) \, d\nu$
- $r = A_0 \, |V| \, \frac{sin \, \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \, cos(2\pi\nu_0\tau_g - \phi_V)$

# Misc.

- Delay tracking - automated compensation for $\tau_g$
- Frequency Conversion (mixing) - bringing the signal to an easier to handle (lower) frequency
- Complex Correlator

# Geometric Relationship



# Geometric Relationship

- $\vec{D_\lambda} \cdot \vec{s_0} = w$
- $\vec{D_\lambda} \cdot \vec{s} = (ul + vm + wn);\ n = \sqrt{1 - l^2 - m^2}$
- $d\Omega = \frac{dl\ dm}{\sqrt{1-l^2-m^2}}$

- $\vec{s} = \vec{s_0} + \vec{\sigma}$
  $\implies \vec{D_\lambda} \cdot \vec{\sigma} = \vec{D_\lambda} \cdot \vec{s} - \vec{D_\lambda} \cdot \vec{s_0}$
- $\vec{D_\lambda} \cdot \vec{s} = ul + vm + w(\sqrt{1 - l^2 - m^2})$
- $\mathcal{V}(u, v, w) =$
  $$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_N(l, m)\ B(l, m)\ e^{-i2\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]} \frac{dl\ dm}{\sqrt{1-l^2-m^2}}$$

- Thompson, Moran, Swenson - Chap. 3

# Small FoV approximation

- $w(\sqrt{1 - l^2 - m^2} - 1) \sim -\frac{1}{2}(l^2 + m^2)w << ul + vm$
- $\mathcal{V}(u, v, w) \sim \mathcal{V}(u, v, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_N(l, m)\ B(l, m)\ e^{-i2\pi[ul+vm]} \frac{dl\ dm}{\sqrt{1-l^2-m^2}}$

# Impact of the $w$ term

- Phase error - $\Delta\phi = \pi\ w\ (l^2 + m^2)$

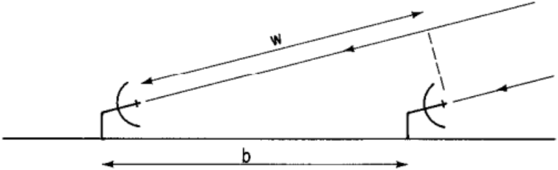


**Figure 2-10.** Comparison of the $w$-component and the antenna spacing when the direction of the source is close to that of the baseline. This condition can occur when the source is rising or setting.

- $\dfrac{1}{\theta_{HPBW}} \sim \dfrac{b_{max}}{\lambda} \sim w_{max}$; $\theta_{HPBW}$- Synthesised Beam
- $\Delta\phi_{max} \sim \pi \left(\dfrac{\theta_F}{2}\right)^2 \dfrac{1}{\theta_{HPBW}}$; $\theta_F$ - size of the Map

# Earth Rotation Synthesis Geometry

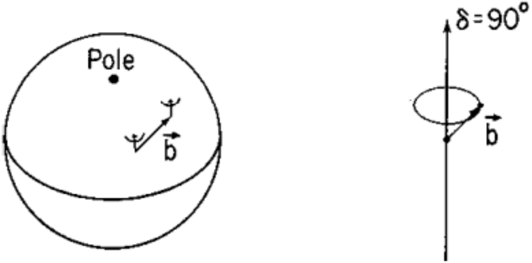


**Figure 2-8.** As the Earth rotates, the baseline vector $\mathbf{b}$, which represents the spacing of the two antennas, traces out a circular locus in a plane normal to the direction of declination ($\delta$) equal to 90°. If the antennas are in an East–West line on the Earth, then the vector $\mathbf{b}$ is normal to the rotation axis.

# Coordinate Frame

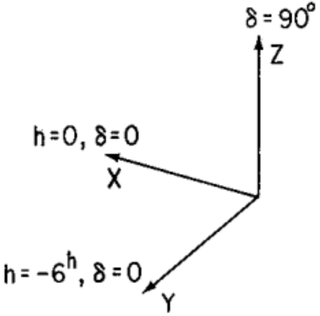


**Figure 2-11.** Coordinate system for specification of baseline parameters. $X$ is the direction of the meridian at the celestial equator, $Y$ is toward the East, and $Z$ toward the North celestial pole.

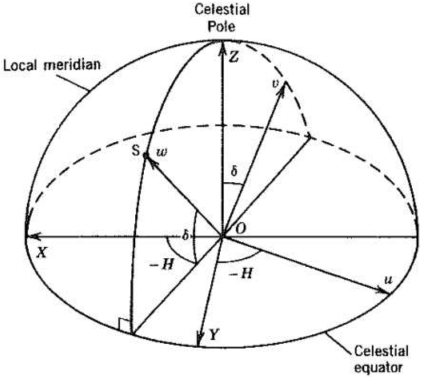
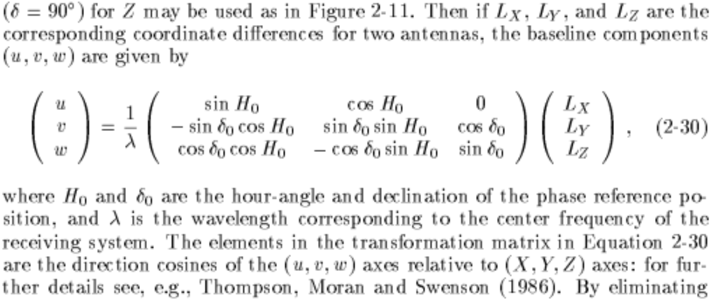


**Figure 4.2** Relationships between the $(X, Y, Z)$ and $(u, v, w)$ coordinate systems. The $(u, v, w)$ system is defined for observation in the direction of the point $S$, which has hour angle and declination $H$ and $\delta$. As shown, $S$ is in the eastern half of the hemisphere and $H$ is therefore negative. The direction cosines in the transformation matrix in Eq. (4.1) follow from the relationships in this diagram. The relationship in Eq. (4.2) can also be derived if we let $S$ represent the direction of the baseline and put the baseline coordinates $(h, d)$ for $(H, \delta)$.

# Antenna Spacing Coordinates and *u, v, w*



($\delta = 90°$) for $Z$ may be used as in Figure 2-11. Then if $L_X$, $L_Y$, and $L_Z$ are the corresponding coordinate differences for two antennas, the baseline components $(u, v, w)$ are given by

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin \delta_0 \cos H_0 & \sin \delta_0 \sin H_0 & \cos \delta_0 \\ \cos \delta_0 \cos H_0 & -\cos \delta_0 \sin H_0 & \sin \delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}, \qquad (2\text{-}30)$$

where $H_0$ and $\delta_0$ are the hour-angle and declination of the phase reference position, and $\lambda$ is the wavelength corresponding to the center frequency of the receiving system. The elements in the transformation matrix in Equation 2-30 are the direction cosines of the $(u, v, w)$ axes relative to $(X, Y, Z)$ axes: for further details see, e.g., Thompson, Moran and Swenson (1986). By eliminating

# Misc. comments

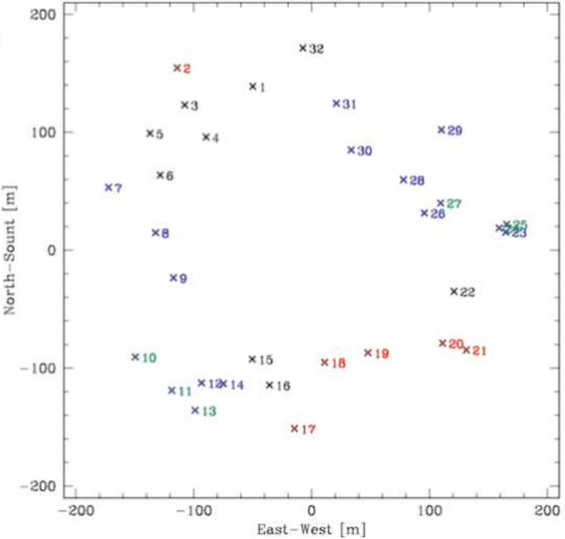
- Fringe Frequency
- Locus of a u,v track
- An East-West baseline
- Source at $\delta_0 = 0^\circ$
- Source at $\delta_0 = 90^\circ$

# Design of Arrays

- Redundancy
- Sampling in the $u$-$v$ plane
- Weighting

# MWA Prototype: array configuration

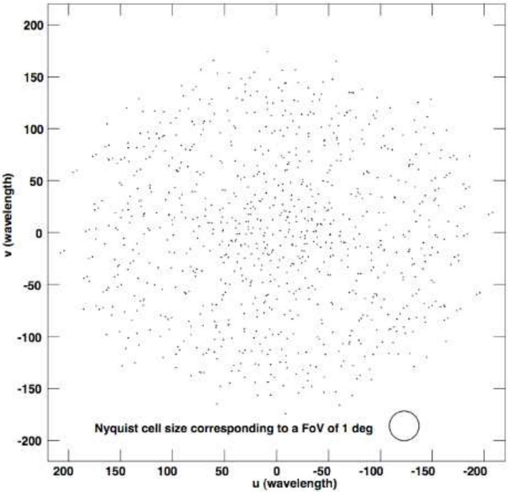
Locations of
Antenna Tiles in
32T Array



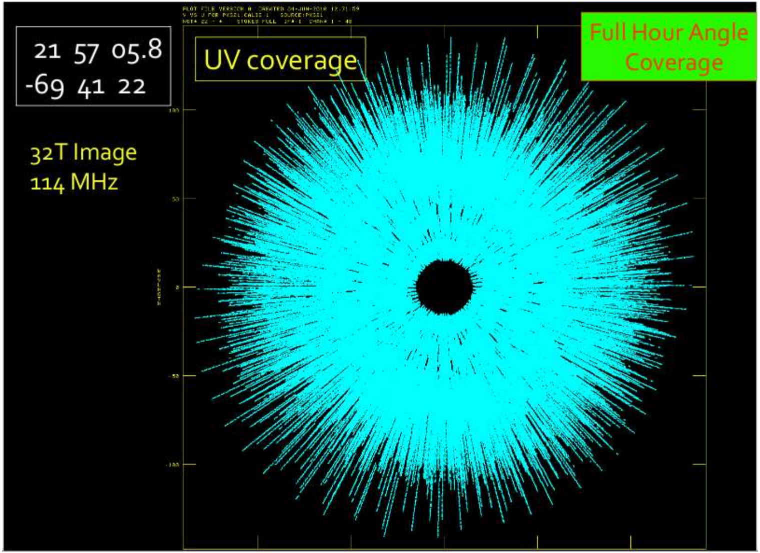
# MWA Prototype: instantaneous uv coverage

Instantaneous 32T uv-coverage

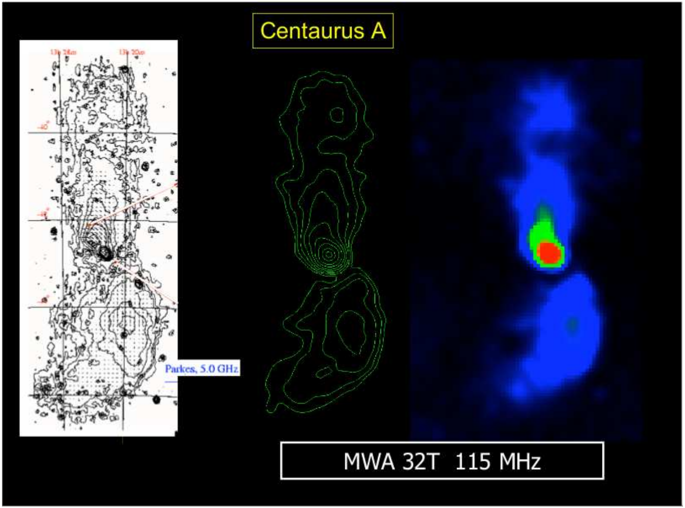
Oberoi, Matthews, et al



# MWA Prototype: Rotation+Frequency synthesis



# MWA Prototype: Centaurus A



Ben McKinley and Frank Briggs et al. (2013)

# ASKAP array configuration

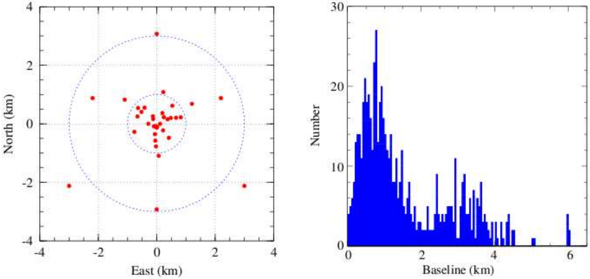
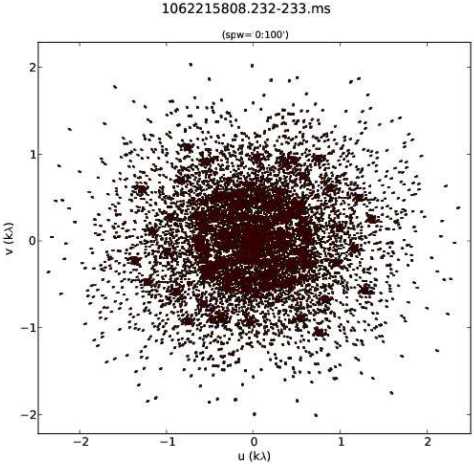
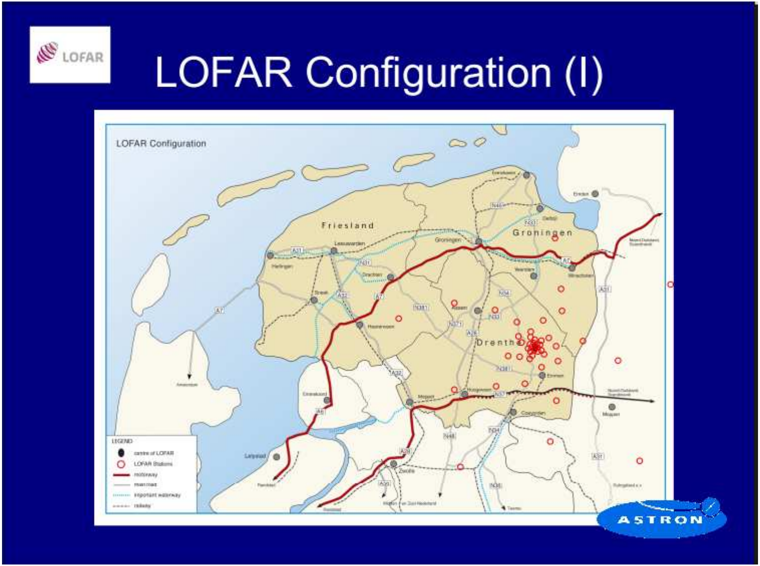


**Figure 2:** Left: Layout of the 36 antennas of the initial ASKAP configuration (red dots). The blue circles have diameters of 2 and 6 km, respectively. Right: Histogram of telescope baseline lengths for the initial ASKAP configuration.

# MWA uvcoverage

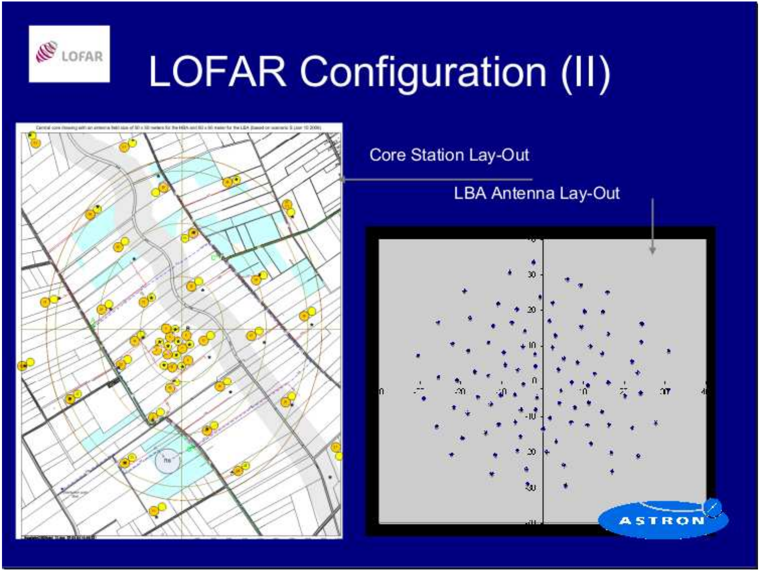


# LOFAR



LOFAR Configuration (I)

# LOFAR

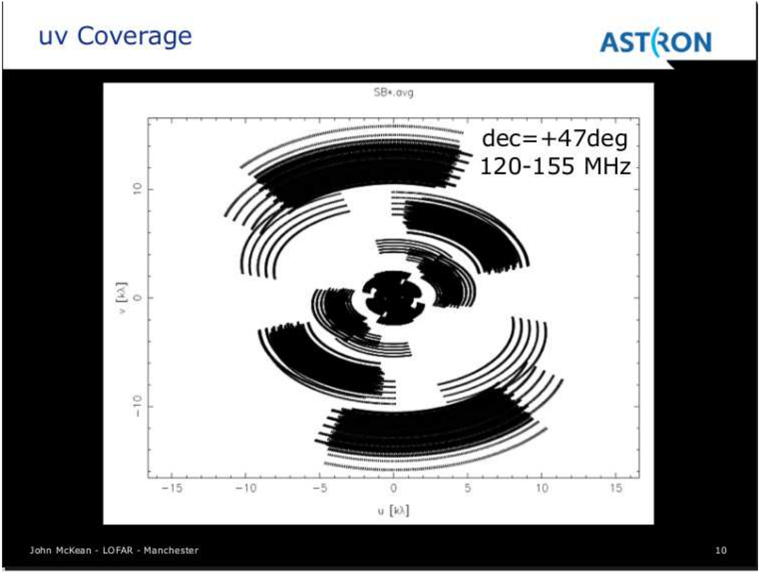


## LOFAR Configuration (II)

Core Station Lay-Out

LBA Antenna Lay-Out

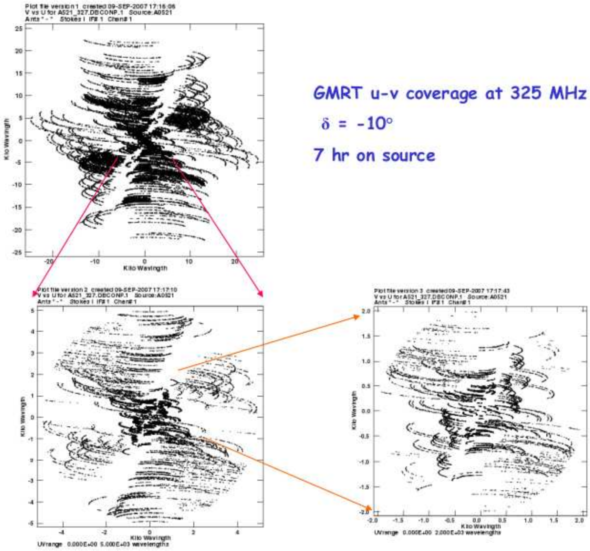
# LOFAR



## uv Coverage

John McKean - LOFAR - Manchester 10

# GMRT

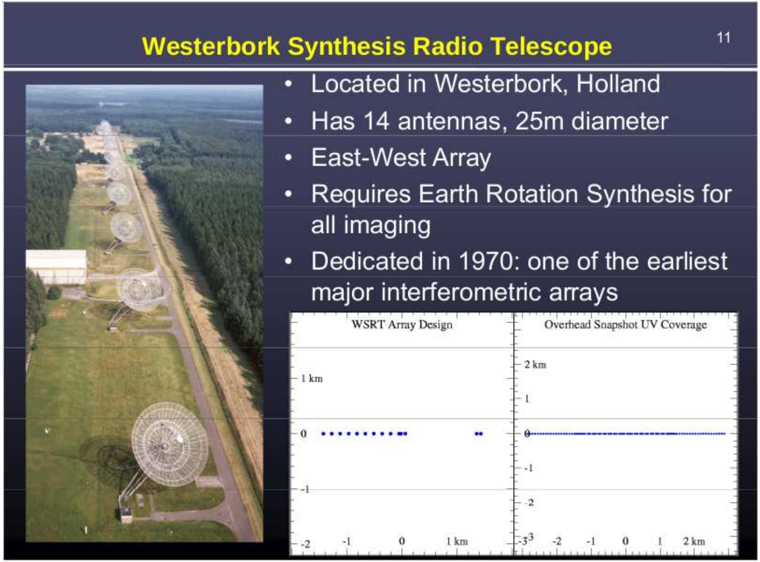


**GMRT u-v coverage at 325 MHz**

**$\delta$ = -10°**

**7 hr on source**

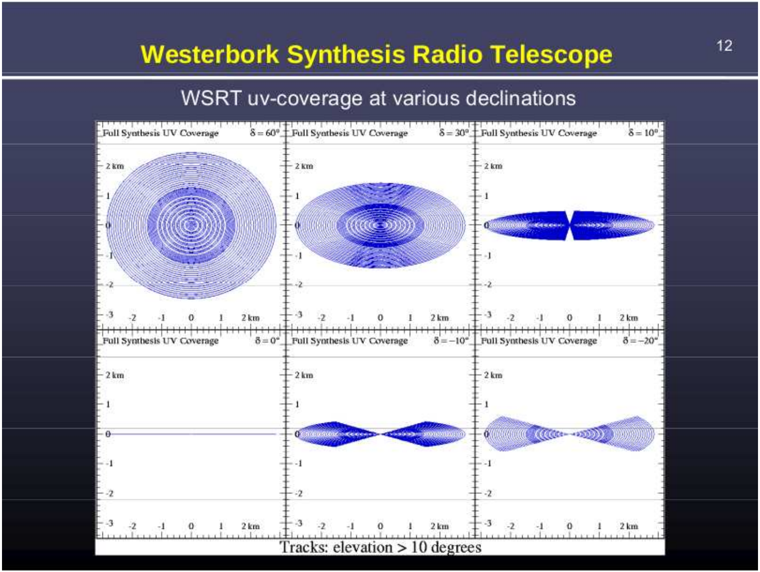
# Wesrterbork



## Westerbork Synthesis Radio Telescope

- Located in Westerbork, Holland
- Has 14 antennas, 25m diameter
- East-West Array
- Requires Earth Rotation Synthesis for all imaging
- Dedicated in 1970: one of the earliest major interferometric arrays

# Wesrterbork



## Westerbork Synthesis Radio Telescope

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WSRT uv-coverage at various declinations

Tracks: elevation > 10 degrees

# References

- Chap. 1 and 2, Synthesis Imaging in Radio Astronomy, ASPC Conf. Series Vol 6
- Chap. 2 and 4, Low Frequency Radio Astronomy
- Chap. 2 and 3, Interferometry and Synthesis in Radio Astronomy