Astronomical Techniques II
Lecture 12 - Imaging

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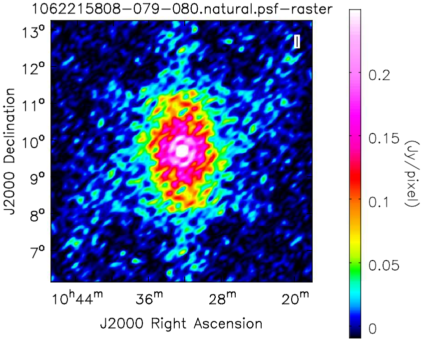
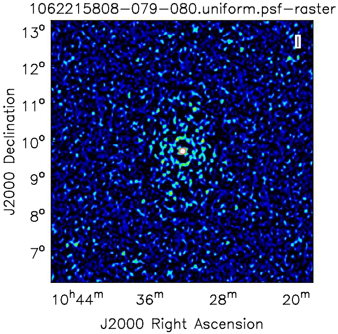
Weighting Functions - controlling the beam shape

1. $W(u,v) = \sum_{k=1}^{M} R_k \; T_k \; D_k \; \delta(u - u_k, v - v_k)$
2. $V^W(u,v) = \sum_{k=1}^{M} R_k \; T_k \; D_k \; \delta(u - u_k, v - v_k) \; V'(u_k, v_k)$
   1. R_k - Reliability $\sim \left(\frac{T_{sys}}{\sqrt{\Delta t \; \Delta \nu}} \right)^{-1}$
   2. T_k - Tapering function
   3. D_k - Density weighting function

PSF Weighting Example - MWA





Gridding

1. Interpolation
2. Convolution
   1. Predictable impact on the images
   2. Convolve V^W with some C and then sample this convolution at centre of each cell of the *grid*
   3. $C = 0$, outside some small bounded region, A_C, support size.
   4. $V^R(u_c, v_c) = \sum_{k=1}^{M} C(u_c - u_k, v_c - v_k)\, V^W(u_k, v_k)$
   5. $V^R = R\left(C \bigstar V^W\right) = R(C \bigstar (W\ V'))$, where

$$R = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - \frac{u}{\Delta u},\ k - \frac{v}{\Delta v})$$

Dirty Image

1. $\tilde{I}^D = \mathcal{F} V^R$
2. $\tilde{I}^D = \mathcal{F} R \bigstar \left[(\mathcal{F} C) \left(\mathcal{F} V^W \right) \right]$
3. $\tilde{I}^D = \mathcal{F} R \bigstar \left[(\mathcal{F} C) \left(\mathcal{F} W \bigstar \mathcal{F} V' \right) \right]$
4. $(\mathcal{F} R)(l, m) = \Delta u \, \Delta v \, \coprod (l \Delta u \, , m \Delta v) =$

   $\Delta u \, \Delta v \, \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - l\Delta u, \; k - m\Delta v)$

   1. $\tilde{I}^D$ is periodic in l and m, with a period of $1/\Delta u$ and $1/\Delta v$, respectively
   2. Aliasing, due to convolution with the resampling function, $\mathcal{F} R$

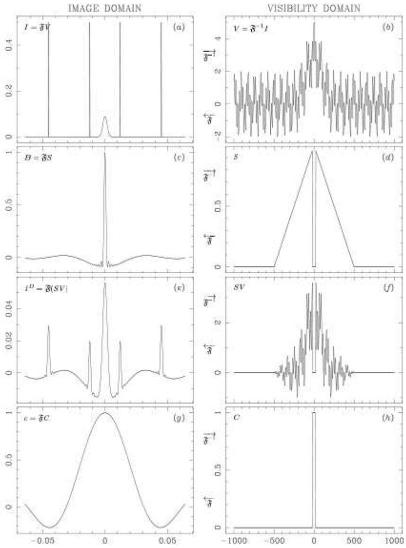
Dirty Image

1. FFT generates one period of $\tilde{I}^D$
2. To image $N_l\ \Delta\theta_l$ rad, grid spacing should satisfy $N_l \Delta u = \dfrac{1}{\Delta\theta_l}$
3. $N_l \times N_m$ FFT yields a discreetly sampled version of $\tilde{I}^D$.
4. Primary field of view - $|l| < N_l\ \Delta\theta_l/2$; $|m| < N_m\ \Delta\theta_m/2$
5. $c = \mathcal{F}C$
6. $\tilde{I}^D_c(l,m) = \dfrac{\tilde{I}^D(l,m)}{c(l,m)}$ - Corrected Dirty Image
7. $\tilde{B}^D_c(l,m) = \dfrac{\tilde{B}^D(l,m)}{c(l,m)}$ - Corrected Dirty Beam (PSF)

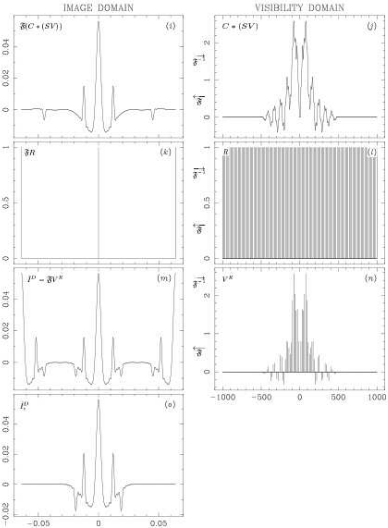
Choice of Gridding Convolution function

1. $C(u, v) = C_1(u)C_2(v)$
2. Chosen on the basis of energy concentration ratio $\frac{\int_P |c(l)|^2 \, dl}{\int_{-\infty}^{\infty} |c(l)|^2 \, dl}$
   Prolate spheroidal wave function

Imaging Process



Imaging Process



Deconvolution

1. $V'(u,v) = \int\int I(l,m) e^{-2\pi i(ul+vm)} dl\, dm$
2. Direct inversion not possible
3. Model with a finite number of parameters
4. $\hat{I}(p\Delta l, q\Delta m)$
5. $\hat{V}(u,v) = \sum_{p=1}^{N_l} \sum_{q=1}^{N_m} \hat{I}(p\Delta l, q\Delta m) e^{-2\pi i(pu\Delta l + qv\Delta m)}$

1. Range of features which can be captured by the data
    1. $\mathcal{O}(1/max(u,v))$
    2. $\mathcal{O}(1/min(u,v))$

2. Choice of $\Delta l, \Delta m$ and N_l, N_m, must allow these scales to be represented
    1. $\Delta l \leq \frac{1}{2u_{max}}$; $\Delta m \leq \frac{1}{2v_{max}}$
    2. $N_l \Delta l \geq \frac{1}{u_{min}}$; $N_m \Delta m \geq \frac{1}{v_{min}}$
3. Degrees of Freedom - $N_l \times N_m$

1. $V(u_i, v_i) = \hat{V}(u_i, v_i) + \epsilon(u_i, v_i)$
2. $V(u, v) = W(u, v)\left(\hat{V}(u, v) + \epsilon(u, v)\right)$
3. $W(u, v) = \sum_i W_i\ \delta(u - u_i, v - v_i)$
4. $I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'}\ \hat{I}_{p',q'} + E_{p,q}$ where

   $I^D_{p,q} = \sum_i W(u_i, v_i)\ Re\left(V(u_i, v_i)e^{2\pi i}(pu_i\Delta l + qv_i\Delta m)\right)$ and

   $B_{p,q} = \sum_i W(u_i, v_i) Re\left(e^{2\pi i(pu_i\Delta l + 1v_i\Delta m)}\right)$

Principal Solution and Invisible Distributions

1. If some spatial frequencies allowed in the model are not present in the data, changing their amplitudes in the model will have no effect on the fit to the data
2. Z - the invisible intensity distribution, then $B \bigstar Z = 0$
3. If I is a solution to the convolution eqn, $I + \alpha Z$ is also a solution
4. The solution which has 0 amplitude at all unsampled spatial frequencies - principal solution
5. The problem of imaging - principal solution + a plausible invisible distribution

The need for *a-priori* information

1. Limitations of the Principal solution
    1. Changes with data available
    2. Sidelobes of order 0.1-10%
    3. Is it a point source or is it a source shaped like the dirty beam
2. *A-priori* information
    1. Positivity (Stokes I must be positive)
    2. Nature of sources (do not have sidelobes extending to infinity)
    3. Information of the PSF

The CLEAN Algorithm

1. Represent the sky as a collection of point sources in an otherwise empty field of view
2. Iterative procedure to find the positions and strengths of these point sources
3. Deconvolved image - Superposition of all point sources found convolved with a CLEAN beam with the residual noise added back

The Hogbom Clean (1974)

1. Find the location and the strength of the brightest point in I^D - S_i at (l_i, m_i) and add it to the accumulated point source model $\hat{I}_{p,q}$.
2. $I^D - \left(B^D(l - l_i, m - m_i) \times S_i \times \gamma\right)$, where $\gamma \leq 1$, usually 0.1
3. Iterate till remaining peaks are below some user specified threshold
4. Convolve $\hat{I}_{p,q}$ with a *restoring beam* - an idealised beam, usually an elliptical Gaussian fit to the central part of the B^D
5. Add the residuals to the restored image - *CLEAN image*

The Clark Clean (1980)

1. CLEAN involves a lot of shifting, scaling and convolutions
2. Minor cycle
   1. Choose a beam patch (include highest exterior sidelobes)
   2. Select bright points from I^D as before
   3. Perform *Hogbom* clean using the beam patch and the selected point sources
3. Major cycle
   1. Point source model built up in the minor cycle is FFTed, weighted and sampled appropriately and FFTed back to the image domain. This is subtracted from the I^D.
   2. Errors introduced due to the use of the beam patch in the minor cycles are corrected at the major cycle stage

The Cotton-Schwab Clean (1984)

1. The major cycle is performed on *ungridded* visibilities
    1. Avoids aliasing and gridding errors
2. Able to image and clean many separate but proximate fields simultaneously

1. Some miscellaneous comments about Clean
    1. Use of clean *boxes*
    2. No. of iterations vs loop gain (γ)
    3. The problem of short spacings
    4. The choice of restoring beam
    5. Clean instabilities
    6. Multi-resolution clean
    7. Sources lying on pixel boundaries