

Astronomical Techniques II

Lecture 5 - Coherence and towards a more realistic description of interferometry

Divya Oberoi

IUCAA NCRA Graduate School

div@ncra.tifr.res.in

March-May 2014

Form of the observed electric field

- $\vec{E}(\vec{R}, t)$
- $\vec{E}_\nu(\vec{R}) = \vec{E}_\nu(\vec{R})e^{-i\omega t}$
- $E_\nu(\vec{r}) = \int \int \int P_\nu(\vec{R}, \vec{r}) \vec{E}_\nu(\vec{R}) dx dy dz$
where $P_\nu(\vec{R}, \vec{r})$ is the *propagator* from \vec{R} to \vec{r} .

- Assumption 1 - No polarization (scalar field)
- Assumption 2 - Sources lie on a *Celestial sphere*
 - 3D \rightarrow 2D
- Assumption 3 - There is no additional emission, absorption, scattering inside the Celestial sphere.
 - So we only have to describe the distribution of sources of electric field at this surface.

- $$E_\nu(\vec{r}) = \int \mathcal{E}_\nu(\vec{R}) \frac{e^{\frac{2\pi i\nu|\vec{R}-\vec{r}|}{c}}}{|\vec{R}-\vec{r}|} dS$$

where dS - surface area element on the celestial sphere.

Spatial Coherence

- $V_\nu(\vec{r}_1, \vec{r}_2) = \langle E_\nu(\vec{r}_1) E_\nu^*(\vec{r}_2) \rangle$
- $V_\nu(\vec{r}_1, \vec{r}_2) = \int \int \mathcal{E}_\nu(\vec{R}_1) \mathcal{E}_\nu^*(\vec{R}_2) \frac{e^{\frac{2\pi i \nu |\vec{R}_1 - \vec{r}_1|}{c}}}{|\vec{R}_1 - \vec{r}_1|} \frac{e^{\frac{2\pi i \nu |\vec{R}_2 - \vec{r}_2|}{c}}}{|\vec{R}_2 - \vec{r}_2|} dS_1 dS_2$
- Assumption 4 - Emission is spatially incoherent
 - $\langle \mathcal{E}_\nu(\vec{R}_1) \mathcal{E}_\nu^*(\vec{R}_2) \rangle = 0$ for $R_1 \neq R_2$

- $V_\nu(\vec{r}_1, \vec{r}_2) = \int |\mathcal{E}_\nu(\vec{R})|^2 |\vec{R}|^2 \frac{e^{\frac{2\pi i\nu|\vec{R}-\vec{r}_1|}{c}}}{|\vec{R}-\vec{r}_1|} \frac{e^{\frac{2\pi i\nu|\vec{R}-\vec{r}_2|}{c}}}{|\vec{R}-\vec{r}_2|} dS$
- $V_\nu(\vec{r}_1, \vec{r}_2) = \int \mathcal{B}_\nu(\vec{s}) e^{\frac{-2\pi i\nu\vec{s}\cdot(\vec{r}_1-\vec{r}_2)}{c}} d\Omega$

Coherence: The physical picture

- Temporal Coherence

- $\tau_c \times \Delta\nu = 1$

- Spatial Coherence

- $u_c \times \Delta\theta = 1$

Response of an interferometer

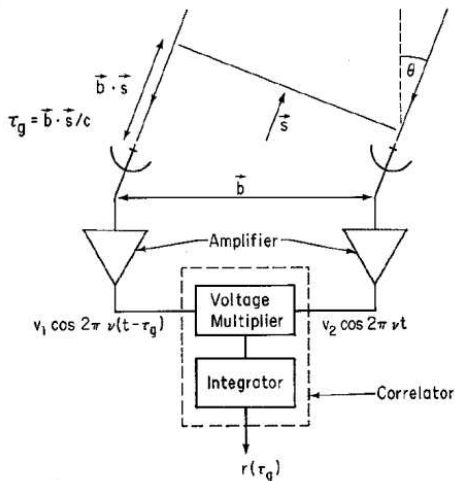


Figure 2-1. Simplified schematic diagram of a two-element interferometer.

Response of an interferometer

- Geometric delay - $\tau_g = \frac{\vec{b} \cdot \vec{s}}{c}$
- Correlator output - $r(\tau_g) = \langle V_1(t) V_2(t) \rangle$
- $V_1 = v_1 \cos 2\pi\nu(t - \tau_g)$; $V_2 = v_2 \cos 2\pi\nu t$;
- $r(\tau_g) = v_1 v_2 \cos 2\pi\nu\tau_g$

Response to a Brightness distribution

- $dr = A(\vec{s}) B(\vec{s}) \Delta\nu \Delta\Omega \cos 2\pi\nu\tau_g$

- $r(\tau_g) = \int_{\Omega} A(\vec{s}) B(\vec{s}) \Delta\nu \cos 2\pi\nu\tau_g d\Omega$

- $\vec{s} = \vec{s}_0 + \vec{\sigma}$

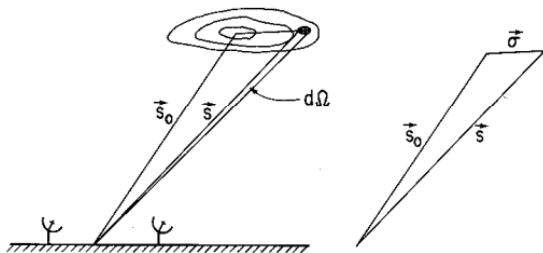


Figure 2-2. Position vectors used in deriving the interferometer response to a source. The source is represented by the contours of radio brightness $I(\mathbf{s})$ on the sky.

- ...

- $V = |V| e^{i\phi_V} = \int_{\Omega} A_N(\vec{\sigma}) B(\vec{\sigma}) e^{-2\pi i \nu \vec{b} \cdot \vec{\sigma} / c} d\Omega$

- ...

- $r = A_0 \Delta\nu |V| \cos 2\pi\nu \frac{\vec{b} \cdot \vec{\sigma}}{c} - \phi_V$

Effect of bandwidth

- $dr = A_0 |V| \cos(2\pi\nu\tau_g - \phi_V) d\nu$

- $r = A_0 |V| \int_{\nu_0 - \Delta\nu/2}^{\nu_0 + \Delta\nu/2} \cos(2\pi\nu\tau_g - \phi_V) d\nu$

- $r = A_0 |V| \frac{\sin \pi \Delta\nu \tau_g}{\pi \Delta\nu \tau_g} \cos(2\pi\nu\tau_g - \phi_V)$

- Delay tracking - automated compensation for τ_g
- Frequency Conversion (mixing) - bringing the signal to an easier to handle (lower) frequency
- Complex Correlator

References

- Chap. 1 and 2, Synthesis Imaging in Radio Astronomy, ASPC Conf. Series Vol 6
- Chap. 2 and 4, Low Frequency Radio Astronomy
- Chap. 2 and 3, Interferometry and Synthesis in Radio Astronomy