Astronomical Techniques II
Lecture 13 - Some things not covered in this course

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A formalism for 3-D imaging

1

$$V(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\mathcal{A}(l,m)\, I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm+w(\sqrt{1-l^2-m^2}-1))}\, dl\, dm$$

2

$$V(u,v,w)\, e^{-2\pi i w} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\mathcal{A}(l,m)\, I(l,m)}{\sqrt{1-l^2-m^2}} \delta(n-\sqrt{1-l^2-m^2}) e^{-2\pi i(ul+vm+wn)}\, dl\, dm\, dn$$

1.
$$I^{D(3)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(u,v,w)\; S(u,v,w) e^{-2\pi i w} e^{2\pi i (ul+vm+wn)} du\; dv\; dw$$

2. $I^{D(3)} = I^{(3)} \bigstar B^{D(3)}$ where,

$$I^{(3)}(l,m,n) = \frac{\mathcal{A}(l,m)\; I(l,m)}{\sqrt{1-l^2-m^2}}\; \delta(n - \sqrt{1-l^2-m^2})$$

3D Imaging

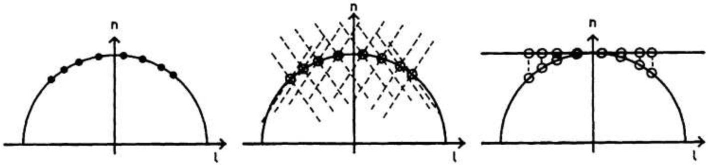


Figure 19–1. The image volume and its relation to the sky brightness. *(Left)* Three-dimensional transformation of the analytic visibility function maps the sky brightness onto a unit sphere. The dots represent these sources. *(Middle)* Convolution with a dirty beam results in sidelobes, shown as dashed lines, throughout the volume above and below the unit sphere. *(Right)* After deconvolution, the images are represented by finite-size "clean beams" on the unit sphere. The two-dimensional image is recovered by projection onto the tangent plane, indicated by vertical dashed lines.

3D Imaging

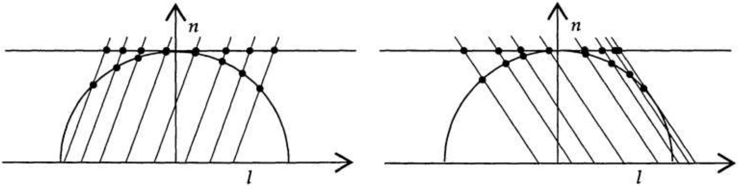


Figure 19–2. The image volume and its relation to a 'standard' two-dimensional image. *(Left)* At a particular time, a 'snapshot' with a two-dimensional array will project the true structure on the unit sphere onto the tangent plane with a 'ray beam', tilted at a particular angle given by the geometry of the array at the time of observation. *(Right)* At a later time, the array geometry has changed due to earth rotation, so the projection is now at a different angle. The apparent positions of the objects which are not located at the tangent point have changed with respect to the earlier observation.

3D Imaging

1. Faceting/Polyhedron imaging
    1. Divide the image into many many facets, each small enough that the small FoV and small w term approximation are sastified within it
    2. CLEAN flux is subtracted from ungridded visibilities
    3. No. of facets depends upon FoV and resolution
    4. 100-1000 times slower than 2D imaging
2. w projection (Cornwell, Golap and Bhatnagar, 2008)
    1. $V(u, v, w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} \mathbf{G}(l, m, w) e^{-2\pi i(ul+vm)}\, dl\, dm,$
    where
    $\mathbf{G}(l, m, w) = e^{-2\pi i(w\sqrt{1-l^2-m^2}-1)}$
    2. $V(u, v, w) = \hat{G}(u, v, w) \bigstar V(u, v, w = 0)$
    3. Order of magnitude faster

Polarization

1. Most non-thermal processes give rise to at least partially polarised emission
2. Polarized emission is an important diagnostic of the conditions in the radio source and in the intervening medium
3.
4. DoF needed to describe the polarization state of radiation
5. A given feed is sensitive to only one of the orthogonal pols (linear or circular)
6. Measure both polarizations and compute all four cross-correlations

Polarization Measurements

1. Significantly harder than total intensity
    1. For the vast majority of sources, fractional polarization is quite low - pushed into low SNR regimes
    2. Number of DoF for imaging increase by a factor of 4 - less well constrained
    3. Calibration issues
        1. Instrumental
        2. Propagation
        3. Need for polarization calibrator
        4. calibration has a strong direction dependence (absolute, as well as within the fov)1
        5. Alt-Az mounts

The Hamaker-Bregman-Sault Measurement Equation

1. Hamaker, Bregman and Sault - 1996-1998
2. Jones Matrix
   1. $E_0\, cos(\omega t + \phi) = E_0 e^{i\phi}$
   2. $\begin{pmatrix} E'_R \\ E'_L \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} E_R \\ E_L \end{pmatrix}$
   3. $J_{gain} = \begin{pmatrix} g_R & 0 \\ 0 & g_L \end{pmatrix}$
   4. $J_{leakage} = \begin{pmatrix} 1 & D_R \\ -D_L & 1 \end{pmatrix}$
   5. $J_{rotation} = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix}$
   6. $J_{overall} = J_{gain}\, J_{leakage}\, J_{rotation} \ldots$

Jones matrices

1. Js are different for each antenna, and are usually time and frequency dependent
2. Provide a framework to represent propagation of signal path up to the correlator
3. Complicated systems can be handled gracefully
4. Provides an approach which allows individual effects to be modelled in different physically relevant manners
5. Matrix formulation - well suited for computational scalability and efficiency

Jone matrices - Polarimetric Equivalent

1. $V'_{i,j} = g_i \; g_j^* \; V_{i,j}$
2. $\mathbf{A} \bigotimes \mathbf{B} = a_{i,j}\mathbf{B}$

   $$(\mathbf{A}_i\mathbf{B}_i) \bigotimes (\mathbf{A}_j\mathbf{B}_j) = (\mathbf{A}_i \bigotimes \mathbf{A}_j)(\mathbf{B}_i \bigotimes \mathbf{B}_j)$$
3. Inputs to the correlator - $E'_i = \mathbf{J}_i \; E_i$
4. Outputs of the correlator - $E'_i \bigotimes E'^*_j$

   $$(\mathbf{J}_i E_i) \bigotimes (\mathbf{J}_j E_j)^* = (\mathbf{J}_i \bigotimes \mathbf{J}_j^*)(E_i \bigotimes E_j^*)$$
5. $E'_i \bigotimes E'^*_j = \begin{pmatrix} E_{R,i}E^*_{R,j} \\ E_{R,i}E^*_{L,j} \\ E_{L,i}E^*_{R,j} \\ E_{L,i}E^*_{L,j} \end{pmatrix}$

1. $< E'_i \bigotimes E'^*_j >= \begin{pmatrix} V_{RR,ij} \\ V_{RL,ij} \\ V_{LR,ij} \\ V_{LL,ij} \end{pmatrix}$
2. $V'_{ij} = (\mathbf{J}_i \bigotimes \mathbf{J}^*_j) V_{ij}$ - V_{ij} - Coherency vector
3. Calibration requires estimating the different J_is and applying the inverse matrix to the measured Coherency vector

Relationship with Stokes vectors

1. $V_S = \begin{pmatrix} V_I \\ V_Q \\ V_U \\ V_V \end{pmatrix}$ - Stokes Visibility Vector
2. $V_{ij} = \mathbf{S}\ V_{S,ij}$
3. $V'_{ij} = (\mathbf{J}_i \bigotimes \mathbf{J}_j)\ \mathbf{S}\ V_{S,ij}$
4. $\mathbf{S}_{circ} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$ $\mathbf{S}_{linear} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{pmatrix}$

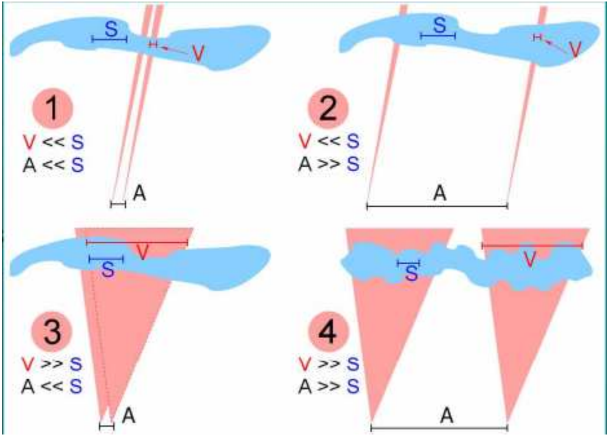
Direction Dependent Effects

1. $V'_{ij} = (\mathbf{J}_{VIS,i} \bigotimes \mathbf{J}^*_{VIS,j}) \sum_k (\mathbf{J}_{SKY,i}(\theta_k, \phi_k) \bigotimes \mathbf{J}^*_{SKY,j}(\theta_k, \phi_k))\mathbf{S}\, V_{S,ij,k}$
2. Assumptions
   1. Sky can be represented as a colletion of point sources
   2. Position *dependent* effects preceed position *independent* effects
3. Jones matrices formalism - not valid in presence of depolarization (analog of loss of coherence)
4. Generalized to include this via Mueller matrices

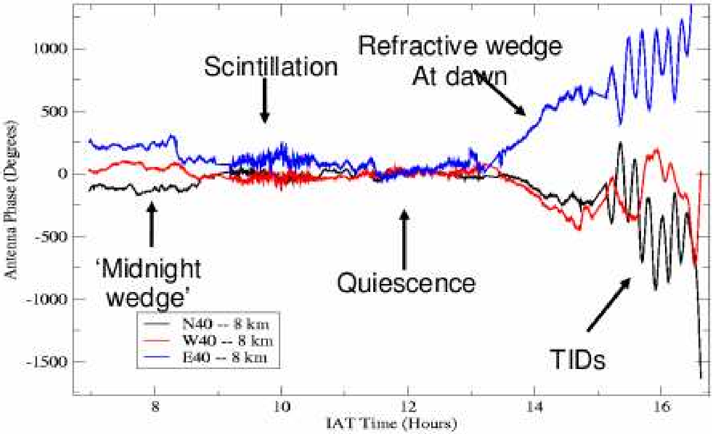
Propagation Effects

1. Media
   1. Interplanetary Medium
   2. Ionosphere
2. Impacts
   1. Refraction
   2. Absorption
   3. Change in polarization properties of the radiation

Ionospheric Propagation Effects



Ionospheric Propagation Effects



Spectral Line Observations

1. SNR considerations for a single spectral channel rather than the full bandwidth
2. Spectral baseline subtraction

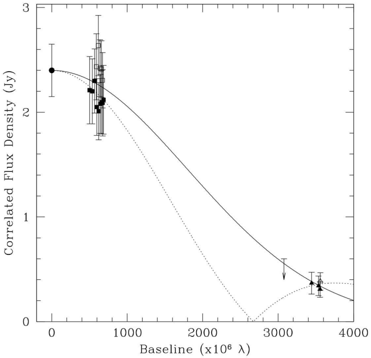
VLBI

1. Highest resolution observations at any wavelength $60\mu arcsec$
2. About the size of an orange on the Moon

Table 1-1. Terms of a VLBI Geometric Model [a]

Item	Approx max Magnitude [b]	Time scale
Zero order geometry.	6000 km	1 day
Nutation	$\sim 20''$	< 18.6 yr
Precession	~ 0.5 arcmin/yr	years
Annual aberration	$20''$	1 year
Retarded baseline	20 m	1 day
Gravitational delay	4 mas @ 90° from sun	1 year
Tectonic motion	10 cm/yr	years
Solid Earth Tide	50 cm	12 hr
Pole Tide	2 cm	$\sim$1 yr
Ocean Loading	2 cm	12 hr
Atmospheric Loading	2 cm	weeks
Post-glacial Rebound	several mm/yr	years
Polar motion	$0.5''$	~ 1.2 years
UT1 (Earth rotation)	Random at several mas	Various
Ionosphere	~ 2 m at 2 GHz	seconds to years
Dry Troposphere	2.3 m at zenith	hours to days
Wet Troposphere	$0 - 30$ cm at zenith	seconds to seasonal
Antenna structure	<10 m. 1cm thermal	—
Parallactic angle	0.5 turn	hours
Station clocks	few microsec	hours
Source structure	5 cm	years

VLBI - Looking for the Black Hole at the center of our Galaxy



1.3 mm VLBI - SMT, JCMT, CARMA The solid curve is the circular Gaussian fit with FWHM of 43 micro arcseconds. Dashed line is for annular ring model of inner radius 35 micro arcseconds and outer radius of 80 micro arcseconds (Doeleman et al, 2008, Nature)