Astronomical Techniques II
Lecture 11 - Sensitivity and Deconvolution

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Noise and Temperature

1. $P = kT\Delta\nu$
2. $P_N = kT_{sys}\Delta\nu G$

   $T_{sys} = T_{bg} + T_{atm} + T_{spill} + T_{loss} + T_{rec}$
   Everything but the target source
3. $P_a = kT_a\Delta\nu G$

   T_a - contribution from the target source
4. $T_{a1} = \frac{\eta_{a1} A_1 S}{2k} = K_1 S$
5. $K = \frac{\eta_a A}{2k}$ $K\ Jy^{-1}$ - Flux collecting ability of an antenna

Sensitivity of a 2 element interferometer

1. $V_1(t) = S_1(t) + n_1(t)$
2. $V_2(t) = S_2(t) + n_2(t)$
3. Assumptions
    1. Point source at phase centre
    2. Appropriate delays and fringe stop
    3. Gaussian white noise
4. Components of the correlated output
    1. Constant (DC) - $S_1(t)\ S_2(t)$ - the object of our measurement
    2. zero mean, time varying output - unavoidable noise

Sensitivity of a 2 element interferometer

1. Ratio of DC component to the RMS of the time varying component
2. Derivation based on
    1. Wiener-Khinchine theorem
3. $$\Delta S = \frac{1}{\sqrt{\Delta t \; \Delta \nu}} \sqrt{S^2 + \frac{S T_{sys}}{K} + \frac{T_{sys}^2}{2K^2}}$$

   where $K = \frac{\eta_a A}{2k}$

Sensitivity of a 2 element interferometer

1. Weak source case $S << \frac{T_{sys}}{K}$

$$\Delta S = \frac{1}{\sqrt{2\Delta t\ \Delta\nu}} \frac{T_{sys}}{K}$$

2. Strong source case $S >> \frac{T_{sys}}{K}$

$$\Delta S = \frac{S}{\sqrt{\Delta t\ \Delta\nu}}$$

Sensitivity of a 2 element complex correlator

1. $S_m = \sqrt{S_R^2 + S_I^2}$

   $\phi_m = tan^{-1} \frac{S_I}{S_R}$

2. Noise distribution for S_m - Rice distribution

   $$P(S_m) = \frac{S_m}{\Delta S^2}\, I_0 \left(\frac{S_m S}{\Delta S^2} \right) e^{\dfrac{-(S_m^2 + S^2)}{s \Delta S^2}}$$

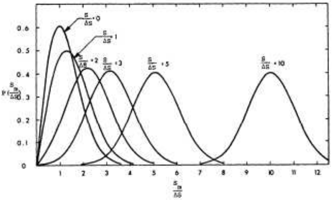
   where I_0 is the modified Bessels function of the first kind, order zero, and S is the true amplitude.

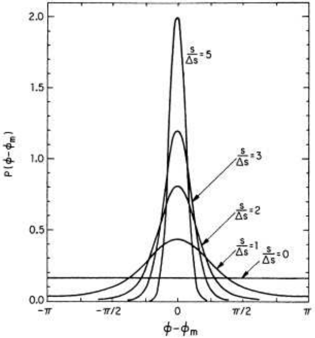
3. Probability distribution for phase error $\phi - \phi_m$, where ϕ is the true phase

   $$P(\phi - \phi_m) = \frac{1}{2\pi} e^{\dfrac{-S^2}{2\,\Delta S^2}} \left(1 + G\sqrt{\pi} e^{G^2} (1 + erfG \right)$$

   where $G(\theta) = \dfrac{S cos\theta}{\sqrt{2}\Delta S}$

Probability distribution of measured amplitude and phase





Sensitivity for a point source

1. $$\Delta I_m = \frac{\sqrt{2} k T_{sys}}{\eta_a \eta_c A \sqrt{N_{base} N_{IF} \Delta T \Delta \nu}}$$
2. $$\eta_c = \frac{Sensitivity\ of\ the\ correlator}{Sensitivity\ of\ a\ perfect\ analog\ correlator}$$

   1 bit - 64%; 2 bit 3 level - 81%

Sensitivity for an extended source

1. $B(l, m)$ - $Jy\ beam^{-1}$
2. $\frac{I\ \Omega_s}{\Delta I_m}$

Effect of the primary beam

1. $I_m(l, m) = I(l, m)\ P(l, m) + N(l, m)$
2. $\dfrac{I_m(l, m)}{P(l, m)} = I(l, m) + \dfrac{N(l, m)}{P(l, m)}$

Deconvolution

1. $V'(u,v) = \int\int I(l,m) e^{-2\pi i(ul+vm)} dl\, dm$
2. Direct inversion not possible
3. Model with a finite number of parameters
4. $\hat{I}(p\Delta l, q\Delta m)$
5. $\hat{V}(u,v) = \sum_{p=1}^{N_l} \sum_{q=1}^{N_m} \hat{I}(p\Delta l, q\Delta m) e^{-2\pi i(pu\Delta l + qv\Delta m)}$

1. Range of features which can be captured by the data
   1. $\mathcal{O}(1/max(u,v))$
   2. $\mathcal{O}(1/min(u,v))$
2. Choice of $\Delta l, \Delta m$ and N_l, N_m, must allow these scales to be represented
   1. $\Delta l \leq \frac{1}{2u_{max}}; \; \Delta m \leq \frac{1}{2v_{max}}$
   2. $N_l \Delta l \geq \frac{1}{u_{min}}; \; N_m \Delta m \geq \frac{1}{v_{min}}$
3. Degrees of Freedom - $N_l \times N_m$

1. $V(u_i, v_i) = \hat{V}(u_i, v_i) + \epsilon(u_i, v_i)$
2. $V(u, v) = W(u, v) \left(\hat{V}(u, v) + \epsilon(u, v) \right)$
3. $W(u, v) = \sum_i W_i \, \delta(u - u_i, v - v_i)$
4. $I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \, \hat{I}_{p',q'} + E_{p,q}$ where

   $I^D_{p,q} = \sum_i W(u_i, v_i) \; Re \left(V(u_i, v_i) e^{2\pi i} (pu_i \Delta l + qv_i \Delta m) \right)$ and

   $B_{p,q} = \sum_i W(u_i, v_i) Re \left(e^{2\pi i (pu_i \Delta l + 1 v_i \Delta m)} \right)$

Principal Solution and Invisible Distributions

1. If some spatial frequencies allowed in the model are not present in the data, changing their amplitudes in the model will have no effect on the fit to the data
2. Z - the invisible intensity distribution, then $B \bigstar Z = 0$
3. If I is a solution to the convolution eqn, $I + \alpha Z$ is also a solution
4. The solution which has 0 amplitude at all unsampled spatial frequencies - principal solution
5. The problem of imaging - principal solution + a plausible invisible distribution

The need for *a-priori* information

1. Limitations of the Principal solution
   1. Changes with data available
   2. Sidelobes of order 1-10%
   3. Is it a point source or is it a source shaped like the dirty beam
2. *A-priori* information
   1. Positivity (Stokes I must be positive)
   2. Nature of sources (do not have sidelobes extending to infinity)
   3. Information of the PSF

The CLEAN Algorithm

1. Represent the sky as a collection of point sources in an otherwise empty field of view
2. Iterative procedure to find the positions and strengths of these point sources
3. Deconvolved image - Superposition of point source convolved with a CLEAN beam and the residual noise

The Hogbom Clean

1. Find the location and the strength of the brightest point in I^D - S_i at (l_i, m_i) and add it to the accumulated point source model $\hat{I}_{p,q}$.
2. $I^D - \left(B^D(l + l_i, m + m_i) \times S_i \times \gamma\right)$, where $\gamma << 1$, usually 0.1
3. Iterate till remaining peaks are below some user specified threshold
4. Convolve $\hat{I}_{p,q}$ with a *restoring beam* - an idealised beam, usually an elliptical Gaussian fit to the central part of the B^D
5. Add the residuals to the restored image - *CLEAN image*

The Clark Clean

1. CLEAN involves a lot of shifting, scaling and convolutions
2. Minor cycle
    1. Choose a beam patch (include highest exterior sidelobes)
    2. Select bright points from I^D as before
    3. Perform *Hogbom* clean using the beam patch and the selected point sources
3. Major cycle
    1. Point source model built up in the minor cycle is FFTed, weighted and sampled appropriately and FFTed back to the image domain. This is subtracted from the I^D.
    2. Errors introduced due to the use of the beam patch in the minor cycles are corrected at the major cycle stage

The Cotton-Schwab Clean

1. The major cycle is performed on *ungridded* visibilities
    1. Avoids aliasing and gridding errors

1. Some miscellaneous comments about Clean
    1. Use of clean *boxes*
    2. No. of iterations vs loop gain (γ)
    3. The problem of short spacings
    4. The choice of restoring beam
    5. Clean instabilities
    6. Multi-resolution clean
    7. Sources lying on pixel boundaries