Electrodynamics and Radiative Processes I

Lecture 8 – Bremsstrahlung

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IUCAA-NCRA Graduate School

August-September 2018

Date : 4th September 2018

Bremsstrahlung in Astrophysics

Bremsstrahlung is a German word directly describing the process: "Strahlung" means "radiation", and "Bremse" means "break.

- ✓ Electrons in a plasma are accelerated by encounters with massive ions.
- ✓ This is the dominant continuum emission mechanism in thermal plasmas.
- ✓ An important *coolant* for plasmas at high temperature˙

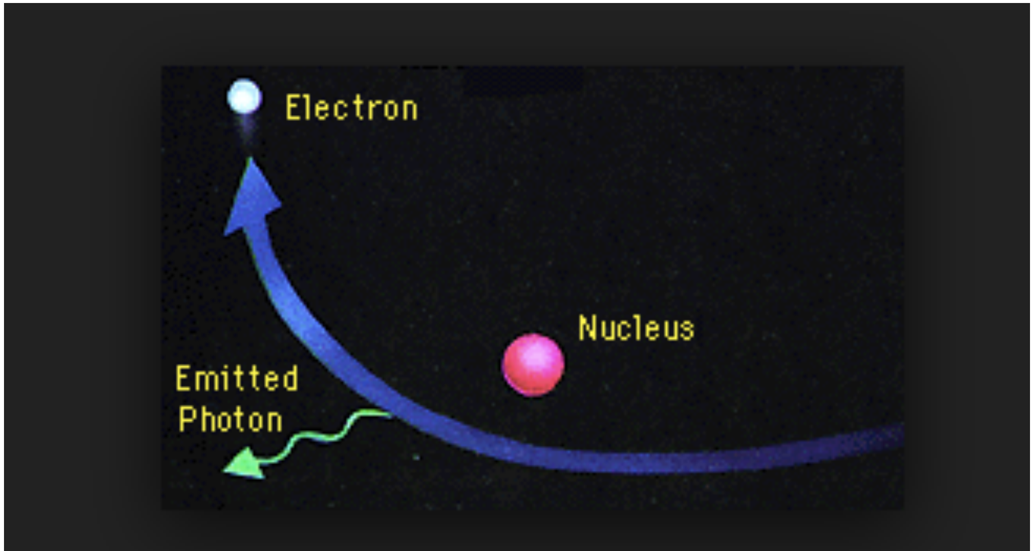
Whenever there is hot ionised gas in the universe it emits bremsstrahlung

Bremsstrahlung in Astrophysics

An incoming free electron can get close to the nucleus of an atom (or other charged particle), the strong electric field of the nucleus will attract the electron, thus changing direction and speed of the electron – accelerating it.

Several subsequent interactions between one and the same electron and different nuclei are possible.

Free-free because the electron is free before and free after.



Bremsstrahlung in Astrophysics

Whenever there is hot ionised gas in the universe it emits bremsstrahlung

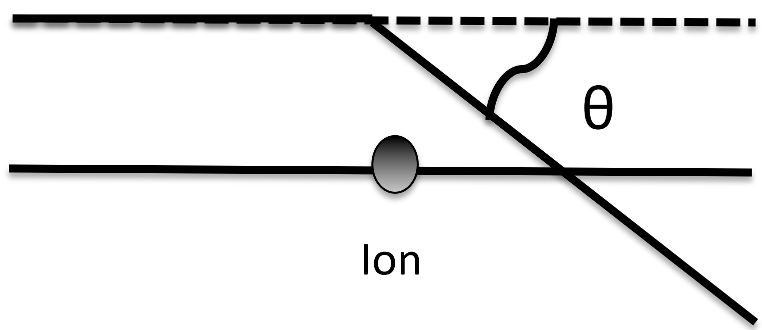
Examples

Radio : HII regions compact regions of hot ionised hydrogen at $T \sim 10^4$ K
Radio emission from ionised winds and jets

X-ray : Binary X-ray sources at $T \sim 10^7$ K
Diffuse X-ray emission from intergalactic region in cluster of galaxies $T \sim 10^8$ K

Bremsstrahlung
"Free-Free Emission"
"Breaking Radiation"

Radiation due to the acceleration of a charge in the coulomb field of another charge Is called bremsstrahlung.



- Bremsstrahlung due to collision of like particles electro-electron or proton-proton is zero in the dipole approximation
- In electron-ion bremsstrahlung the electrons are the primary radiators, since relative accelerations are inversely proportional to masses.

Bremsstrahlung
“Free-Free Emission”
“Breaking Radiation”

Full understanding of the process will require a quantum treatment.

Classical treatment is justified in some regimes (discussed later in the lecture)

Approach
We first state the classical treatment and then quantum results as corrections.

Approach
We first treat nonrelativistic bremsstrahlung and then consider relativistic corrections.

History of Bremsstrahlung
“Free-Free Emission”
“Breaking Radiation”

1930 : Curl Anderson found that ionisation loss−rate is under estimated for relativistic electrons (though was noted by Tesla in 1880)

Additional energy loss mechanism was associated with radiation of electromagnetic waves because of acceleration of the electron in the electrostatic field of nucleus

Radiation corresponds to transition between unbound states of the electron in the field of the nucleus

1939 calculation on relativistic and non relativistic spectrum by Bathe and Heilter

Improved treatment appropriate to astrophysical situations Blumenthal et al 1970

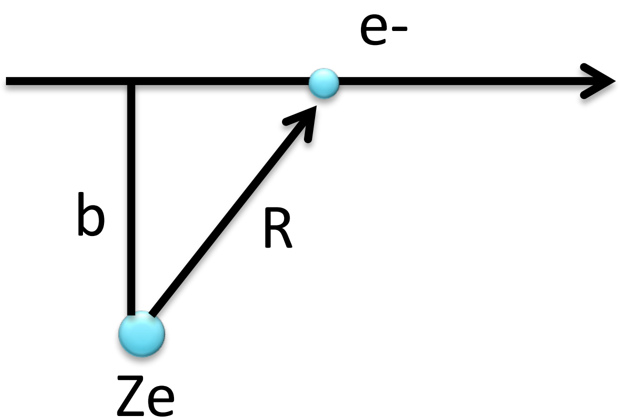
Bremsstrahlung
Layout

(1) Emission from single speed electron
    pick rest frame of ion
    calculate dipole radiation
    correct for quantum effects (Gaunt factor)

(2) Emission from collection of electron
    Thermal bremsstrahlung
    Free-Free Absorption
    Non-thermal bremsstrahlung

(3) Relativistic bremsstrahlung (Virtual Quanta)

Bremsstrahlung
Emission from a single speed electron



Assume: electron moves rapidly and its path is straight line

Consider an electron of charge −e moving past an ion of charge Ze with impact parameter b

Dipole moment **d**= -e **R**

2nd derivative of dipole moment

$$\ddot{\mathbf{d}} = -e\dot{\mathbf{v}}$$

Fourier transform

$$-\omega^2\hat{\mathbf{d}}(\omega) = -\frac{e}{2\pi}\int_{-\infty}^{\infty}\dot{\mathbf{v}}e^{i\omega t}dt.$$

Bremsstrahlung
Emission from a single speed electron

Collision time : time interval over which electron and ion are close enough to interact

$$\tau = \frac{b}{v}$$

$$-\omega^2 \hat{\mathbf{d}}(\omega) = -\frac{e}{2\pi} \int_{-\infty}^{\infty} \dot{\mathbf{v}} e^{i\omega t} dt.$$

Case-1 $\omega\tau >> 1$ the exponential of the integral oscillates rapidly and integral is small
Case-2 $\omega\tau << 1$ exponential is essentially unity

$$\hat{\mathbf{d}}(\omega) \sim \begin{cases} \frac{e}{2\pi\omega^2} \Delta\mathbf{v}, & \omega\tau \ll 1 \\ 0, & \omega\tau \gg 1, \end{cases}$$

Δv change of velocity during collision

Bremsstrahlung
Emission from a single speed electron

Recall Spectrum of dipole radiation

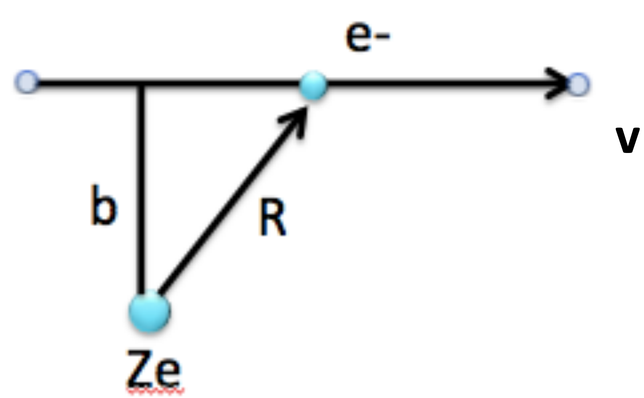
$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{d}(\omega)|^2$$

$$\hat{\mathbf{d}}(\omega) \sim \begin{cases} \dfrac{e}{2\pi\omega^2}\Delta\mathbf{v}, & \omega\tau \ll 1 \\ 0, & \omega\tau \gg 1, \end{cases}$$

So Spectrum of Bremsstrahlung radiation

$$\frac{dW}{d\omega} = \begin{cases} \dfrac{2e^2}{3\pi c^3}|\Delta\mathbf{v}|^2, & \omega\tau \ll 1 \\ 0, & \omega\tau \gg 1. \end{cases}$$

Bremsstrahlung
Emission from a single speed electron



Considering linear path, change in velocity is normal to the path.
Integrate component of acceleration normal to the path.

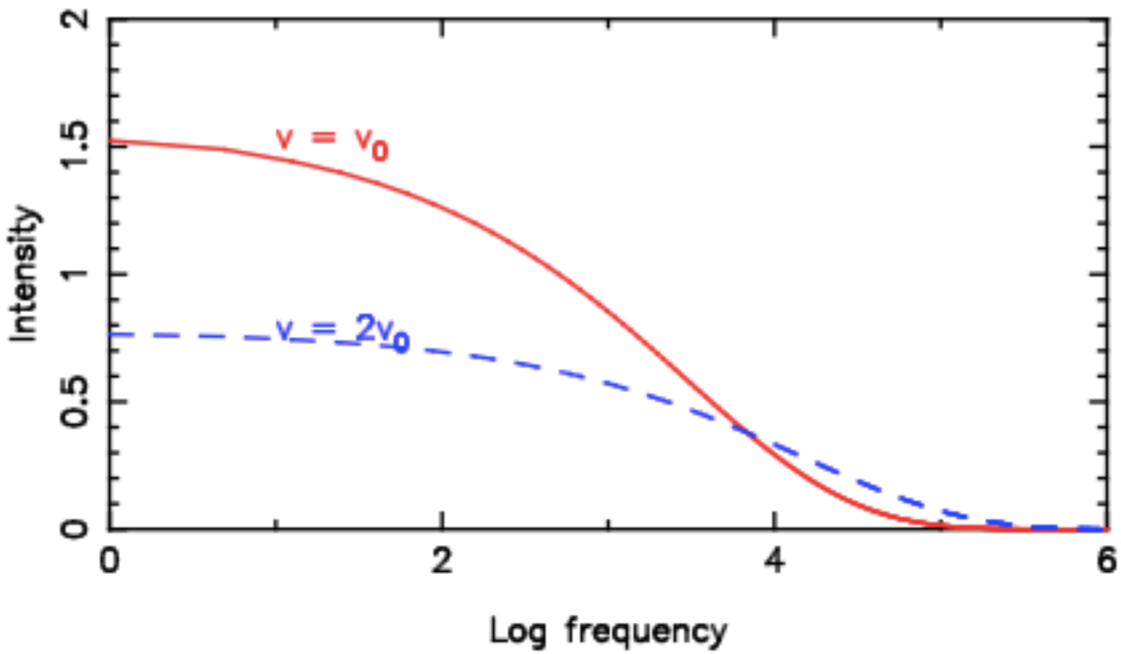
$$\Delta v = \frac{Ze^2}{m} \int_{-\infty}^{\infty} \frac{b\,dt}{(b^2 + v^2 t^2)^{3/2}} = \frac{2Ze^2}{mbv}$$

Bremsstrahlung
Emission from a single speed electron

Thus for small angle scattering spectra of emission from a single collision is

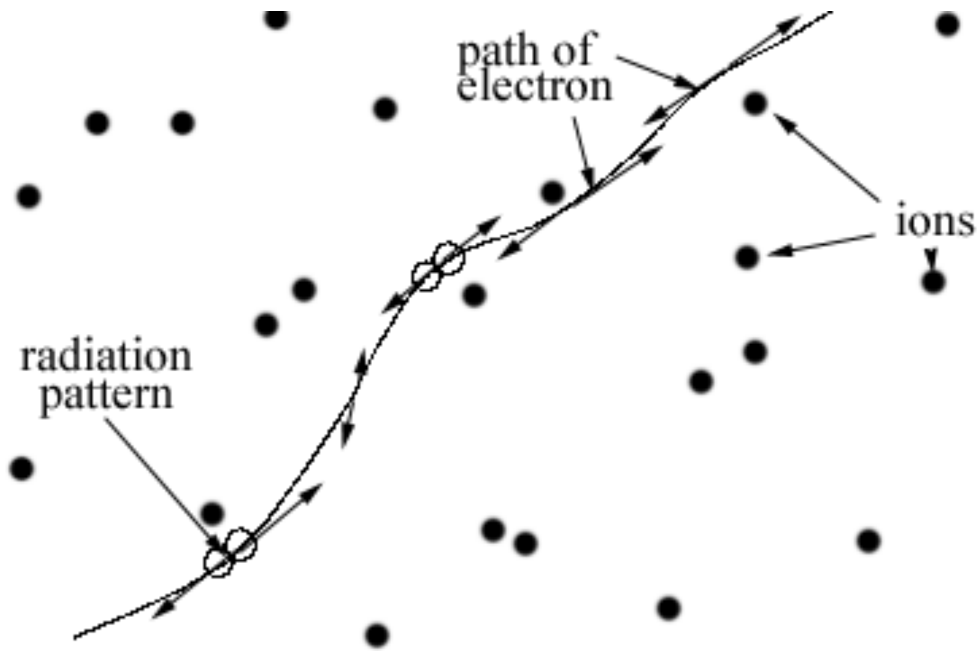
$$\frac{dW(b)}{d\omega} = \begin{cases} \dfrac{8Z^2e^6}{3\pi c^3 m^2 v^2 b^2}, & b \ll v/\omega \\ 0, & b \gg v/\omega. \end{cases}$$

Bremsstrahlung – single electron accelerated by an ion



Bremsstrahlung
Emission from multiple single speed electron

Bunch of electrons, all with the same speed, v, which interact with a bunch of ions.



Bremsstrahlung
Emission from multiple single speed electron

Total spectrum for a medium with ion density n_i electron density n_e and fixed electron speed v

$$\frac{dW}{d\omega\, dV\, dt} = n_e n_i 2\pi v \int_{b_{\min}}^{\infty} \frac{dW(b)}{d\omega} b\, db$$

Flux of electrons (electrons per unit area per unit time) incident on one ion is $n_e\, v$

The element of area is $2\pi b db$ about a single ion.

b_{min} is minimum value of impact parameter

Bremsstrahlung
Emission from multiple single speed electron

$$\frac{dW}{d\omega\, dV\, dt} = n_e n_i 2\pi v \int_{b_{\min}}^{\infty} \frac{dW(b)}{d\omega}\, b\, db$$

$$\frac{dW(b)}{d\omega} = \begin{cases} \dfrac{8Z^2e^6}{3\pi c^3 m^2 v^2 b^2}, & b \ll v/\omega \\ 0, & b \gg v/\omega. \end{cases}$$

For $b \ll v/\omega$

$$\frac{dW}{d\omega\, dV\, dt} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

Where b_{max} is some value of b beyond which $b \ll v/\omega$ is not applicable and contribution to integral is negligible

$$b_{\max} \cong \frac{v}{\omega}$$

Bremsstrahlung
Emission from multiple single speed electron

Value of b_{min} can be estimated in two ways

First we can take the value at which straight line approximation is no longer valid.

$$b_{\min}^{(1)} = \frac{4Ze^2}{\pi m v^2}$$

Second value for b_{min} is quantum in nature and comes from uncertainty principle

$$b_{\min}^{(2)} = \frac{h}{mv}$$

$$\Delta x \sim b \qquad \Delta p \sim mv$$

Bremsstrahlung
Emission from multiple single speed electron

$b_1^{min} >> b_2^{min}$ a classical description of scattering is valid

This occurs when $1/2\, mv^2 << Z^2 Ry$

where $$Ry = \frac{me^4}{(2\hbar^2)}$$

$b_2^{min} >> b_1^{min}$ quantum treatment required

We choose which ever of these values of b_{min} is the larger for the physical conditions of the problem

Bremsstrahlung
Emission from multiple single speed electron

For any regime the exact results are stated in terms of a correction factor g_{ff}

$$\frac{dW}{d\omega\, dV\, dt} = \frac{16\pi e^6}{3\sqrt{3}\, c^3 m^2 v} n_e n_i Z^2 g_{ff}(v,\omega).$$

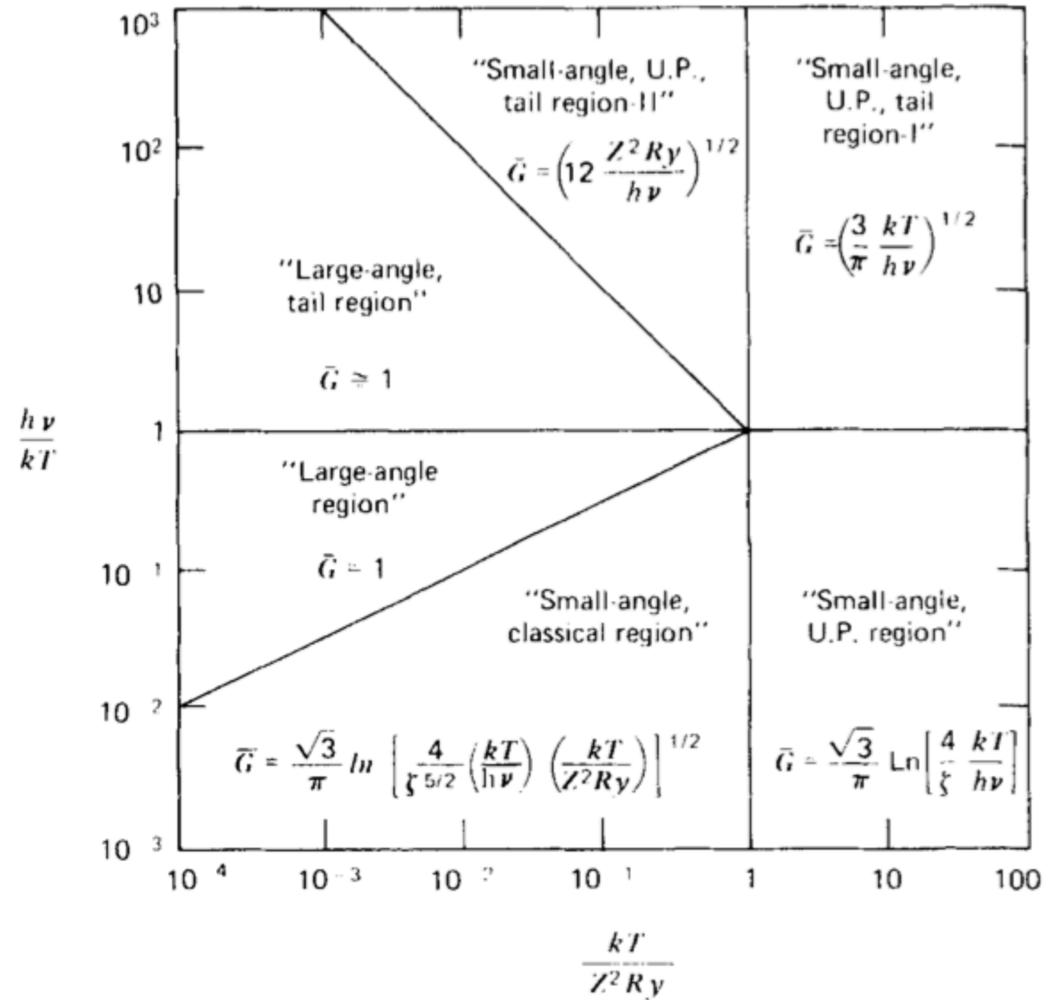
Gaunt factor (correction factor)

$$g_{ff}(v,\omega) = \frac{\sqrt{3}}{\pi} \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

Gaunt factor is a function of energy of the electron and of frequency of emission.

Thermal Bremsstrahlung Emission

For any regime the exact results are stated in terms of a correction factor g_{ff}



Approximate analytical formula for gaunt factor $g_{ff}(\nu,T)$ for thermal bremsstrahlung (Rybicki & Lightman)

Thermal Bremsstrahlung Emission

Use of formulas derived for single velocity of charged particle in their application to thermal bremsstrahlung

↓

We average the derived single speed expression over a range of thermal distribution of speeds

Thermal Bremsstrahlung Emission

The probability dP that a particle has velocity in the range d^3 V

$$dP \propto e^{-E/kT} d^3\mathbf{v} = \exp\left(-\frac{mv^2}{2kT}\right) d^3\mathbf{v}.$$

$$dP \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv \qquad d^3\mathbf{v} = 4\pi v^2 dv$$

Lower limit of electron velocity from the condition $h\nu \leqslant \frac{1}{2} m v^2$

$$v_{\min} \equiv (2h\nu / m)^{1/2}$$

Thermal Bremsstrahlung Emission

Thus the total emission per unit time per unit volume per unit frequency for a range of velocities for ion density n_i and electron density n_e

$$\frac{dW(T,\omega)}{dV\,dt\,d\omega} = \frac{\int_{v_{min}}^{\infty} \frac{dW(v,\omega)}{d\omega\,dV\,dt} v^2 \exp(-mv^2/2kT)\,dv}{\int_0^{\infty} v^2 \exp(-mv^2/2kT)\,dv}$$

Limits of integration:

$0 < v < \alpha$

But at frequency ν the incident velocity must be at least such that $h\nu << (1/2)mv^2$ otherwise a photon of energy $h\nu$ can not be created

This cut off in the lower limit of the integration over electron velocities is called a *Photon discreteness effect*

Thermal Bremsstrahlung Emission

Thus the total emission per unit time per unit volume per unit frequency for a range of velocities for ion density n_i and electron density n_e

$$\frac{dW(T,\omega)}{dV\,dt\,d\omega} = \frac{\int_{v_{min}}^{\infty} \frac{dW(v,\omega)}{d\omega\,dV\,dt} v^2 \exp(-mv^2/2kT)\,dv}{\int_0^{\infty} v^2 \exp(-mv^2/2kT)\,dv}$$

Recap the total emission per unit time and per unit volume and per unit frequency for single velocity electrons with electron density n_e considering ion density n_i

$$\frac{dW}{d\omega\,dV\,dt} = \frac{16\pi e^6}{3\sqrt{3}\,c^3 m^2 v} n_e n_i Z^2 g_{ff}(v,\omega).$$

$$\frac{dW}{dV\,dt\,d\nu} = \frac{2^5\pi e^6}{3mc^3}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i e^{-h\nu/kT}\bar{g}_{ff}$$

Thermal Bremsstrahlung Emission

Total emission per unit time per unit volume per unit frequency for a range of velocities for ion density n_i and electron density n_e

$$\frac{dW}{dV\,dt\,d\nu} = \frac{2^5 \pi e^6}{3mc^3}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i e^{-h\nu/kT} \bar{g}_{ff}$$

In C.G.S. units we have Free-free emission coefficient
i.e. total emission per unit volume per unit frequency

$$\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$$

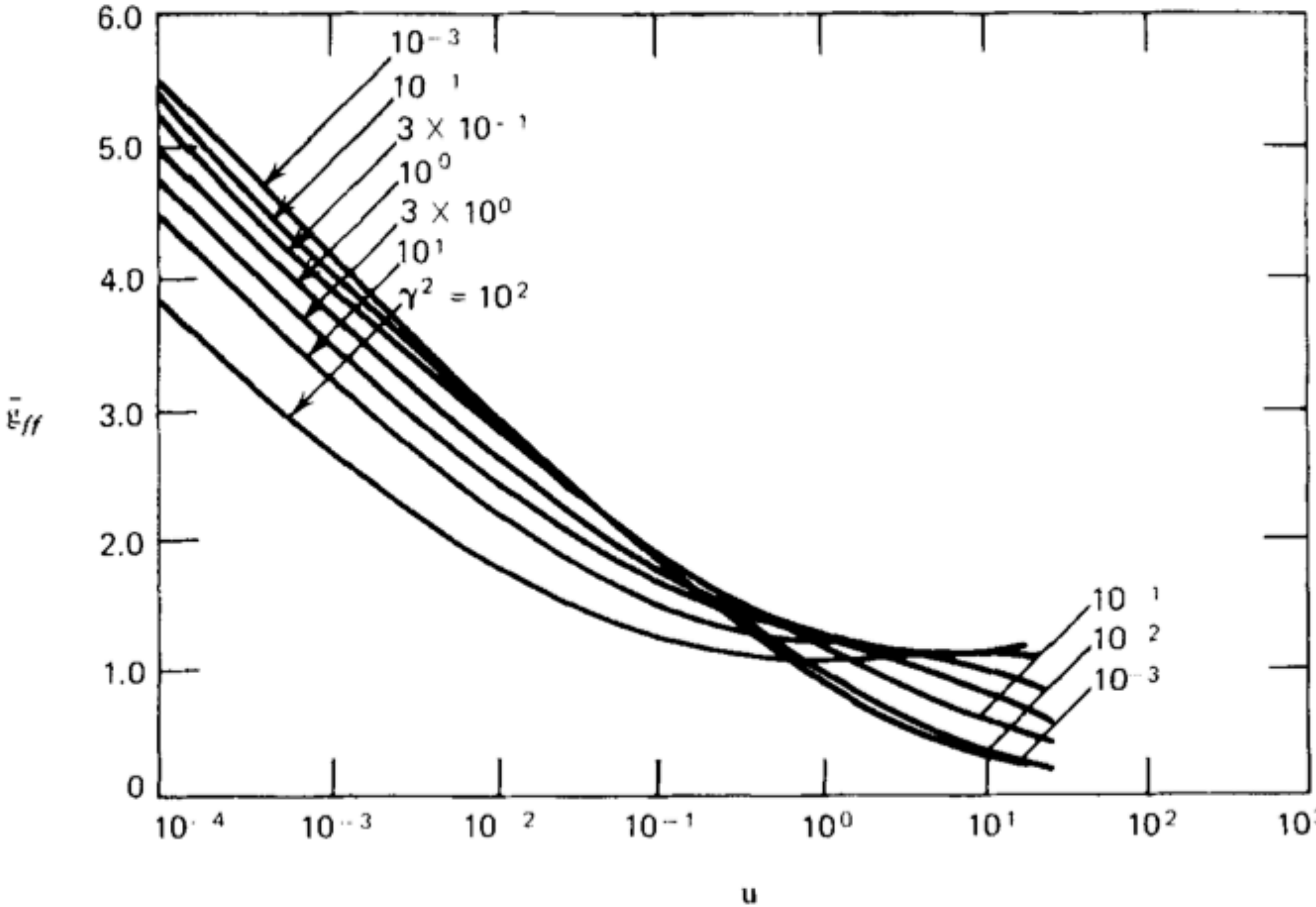
Unit erg s^{-1} cm^{-3} Hz^{-1}

$dW/dV\,dt\,d\omega \propto v^{-1}$

$\langle v \rangle \propto T^{1/2}$

Velocity averaged Gaunt factor

Numerical values of gaunt factor



Numerical values of gaunt factor $g_{ff}(\nu,T)$. Frequency variable $u=4.8 \times 10^{11}\nu/T$

Thermal Bremsstrahlung Emission

$$\frac{dW}{dV\,dt\,d\nu} = \frac{2^5 \pi e^6}{3mc^3}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i e^{-h\nu/kT} \bar{g}_{ff}$$

Integrate over frequency

$$\frac{dW}{dt\,dV} = \left(\frac{2\pi kT}{3m}\right)^{1/2} \frac{2^5 \pi e^6}{3hmc^3} Z^2 n_e n_i \bar{g}_B$$

$\bar{g}_B(T)$ → Frequency average of the velocity averaged Gaunt factor value ranges from 1.1 to 1.5

Numerically, total emission per unit volume per unit time in C.G.S. unit

$$\epsilon^{ff} \equiv \frac{dW}{dt\,dV} = 1.4 \times 10^{-27} T^{1/2} n_e n_i Z^2 \bar{g}_B$$

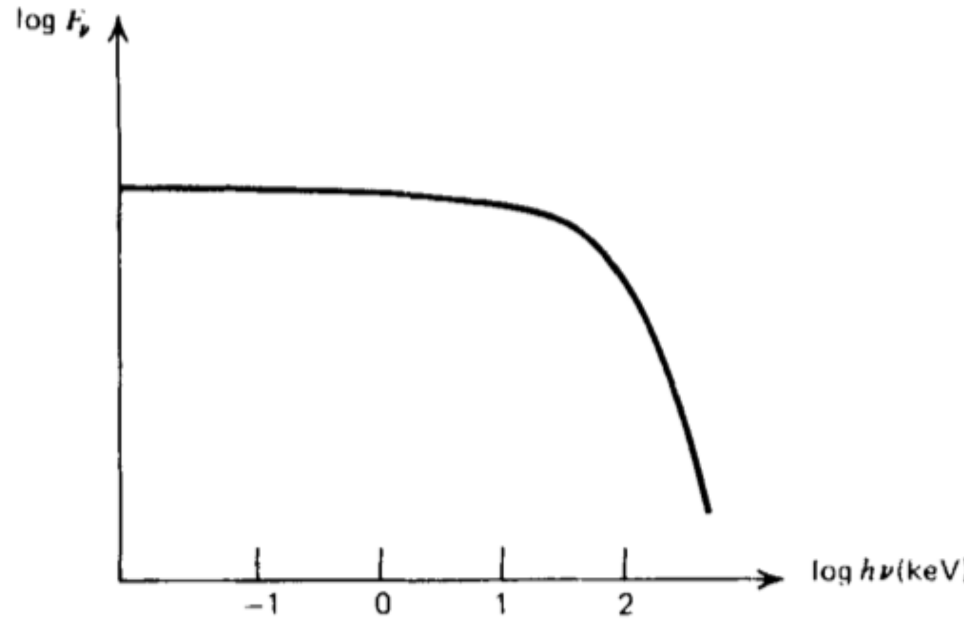
↓

Unit erg s^{-1} cm^{-3}

Thermal Bremsstrahlung Emissivity

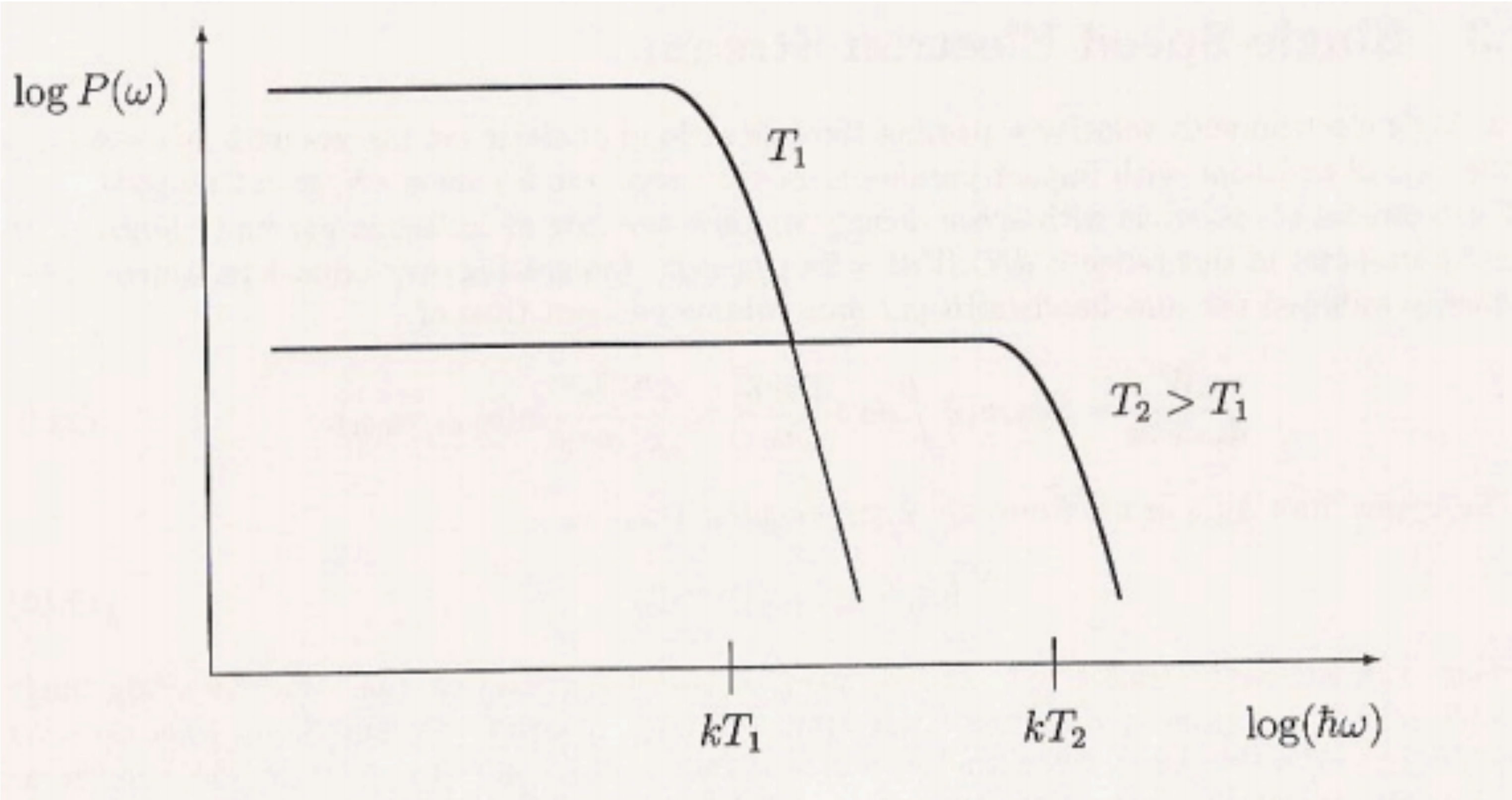
Rather flat spectrum in the log-log plot
Up to a cut off at about hv=KT

True for optically thin sources,
not considering absorption of
photons by free-free absorption



- ✓ ε_ν^{ff} is ~ constant with hv at low frequencies
- ✓ ε_ν^{ff} falls off exponentially at hv~kT

Thermal Bremsstrahlung spectra



Spectra for thermal bremsstrahlung at two different temperatures (though same density)

Thermal Bremsstrahlung Recap

Consider a charged particle at a specific impact parameter(b) and velocity(v).

When a charged particle accelerates it emits radiation.

Acceleration is a function of b, v and Z.

Acceleration as a function of time intensity spectrum via Fourier Transform.

Integrate (exact details tricky – gives rise to the Gaunt Factor $\overline{g_{ff}}$, which is a function of ν,T,Z).

Include term for collision rate (depends on number densities n_e and n_i). Integrate over v .

Assume plasma in thermal equilibrium →Maxwellian distribution of v .

Simple Examples

Hydrogen Plasma

A common case is that of an optically thin hydrogen plasma, so n_e=n_i & Z=1

$$I_\nu \propto \int n_e^2\, T^{-1/2}\, dl$$

$$\boxed{\int n_e^2\, dl}$$ → Emission Measure

Unit $cm^{-6}pc$

Simple Examples

Cooling Time $T_{cool} = \dfrac{\text{Energy content of a gas}}{\text{Rate at which energy is being radiated}}$

For fully ionised pure hydrogen gas $\varepsilon_{ff} = 1.7 \times 10^{-27} T^{1/2} n_e^2$

$$T_{cool} = 7900 \, \frac{T^{1/2}}{n_e} \, years$$

For HII region $n_e = 10^2 - 10^3\ cm^{-3}$, $T = 10^3 - 10^4$ K, T_{cool} ~100-1000 years

For Galaxy clusters $n_e = 10^{-3}\ cm^{-3}$, $T = 10^8$ K, T_{cool} ~10^{10} years

End of Lecture 8

Next Lecture : 6th August