Masses and Radii of Neutron Stars: Probing Neutron Star Interior

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Plan of My Talk

- Introduction
- Observed Masses of Neutron Stars
- SKA and Relativistic Pulsars
- Exotic forms of matter, Relativistic models
- Simultaneous observations of Mass and Radius
- Conclusions

Neutron stars are one of the densest forms of matter in this observable universe.

Neutron star matter is cold and highly dense. The matter density in the core exceeds by a few times normal nuclear matter density.

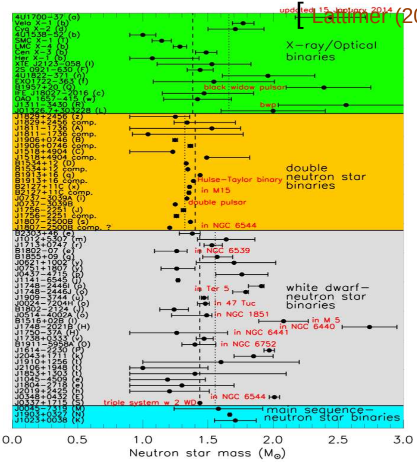
Observations of binary pulsars and isolated neutron stars provide information about masses and radii.

The theoretical mass-radius relationships of compact stars are direct probes of neutron star interior.

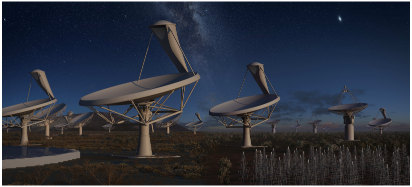
Consequently, the composition and EoS of dense matter in a neutron star interior might be probed.

- Accurately measured highest Neutron Star mass is $2.01\pm0.04\ _{\odot}$. [J. Antoniadis et al., Science 340 (2013)]
- Does exotic matter (hyperon, Bose condensates, quarks) exist in NS?
- Exotic EoS should satisfy the constraint $M_{max}^{theo} > M^{obs}$.

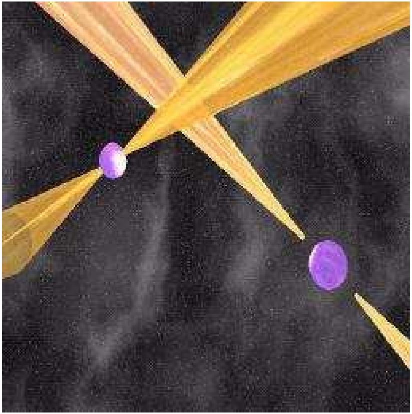
[Lattimer (2014)]



Science Programmes with SKA



- Strong field tests of gravity using pulsars and black holes
- Galaxy evolution, Cosmology and Dark Energy
- The origin and evolution of cosmic magnetism
- Probing the cosmic dawn
- The cradle of life



- First ever observed Double Pulsar System

  Burgay et al. 426(2003) 531

- Keplerian parameters P_{orb} =2.45 h, a_p, e =0.088, ω and T_0 were measured from the pulsar timing data
- Pulsar A has a spin period of 22.7 ms and mass of 1.337 $M_\odot$ whereas those of Pulsar B are 2.8 s and 1.25 $M_\odot$
- Accurate measurements of relativistic corrections to the Keplerian description

Post-Keplerian Orbital Parameters

Besides the normal 5 “Keplerian” parameters (P_{orb}, e, $a\sin(i)/c$, T_0, ω), General Relativity gives:

$$\dot{\omega} = 3\left(\frac{P_b}{2\pi}\right)^{-5/3} (T_\odot M)^{2/3} (1-e^2)^{-1}$$ (Orbital Precession)

$$\gamma = e\left(\frac{P_b}{2\pi}\right)^{1/3} T_\odot^{2/3} M^{-4/3} m_2 (m_1 + 2m_2)$$ (Grav redshift + time dilation)

$$\dot{P}_b = -\frac{192\pi}{5}\left(\frac{P_b}{2\pi}\right)^{-5/3}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)(1-e^2)^{-7/2} T_\odot^{5/3} m_1 m_2 M^{-1/3}$$

$$r = T_\odot m_2$$

$$s = x\left(\frac{P_b}{2\pi}\right)^{-2/3} T_\odot^{-1/3} M^{2/3} m_2^{-1}$$

(Shapiro delay: “range” and “shape”)

where: $T_\odot \equiv GM_\odot/c^3 = 4.925490947\ \mu s$, $M = m_1 + m_2$, and $s \equiv \sin(i)$

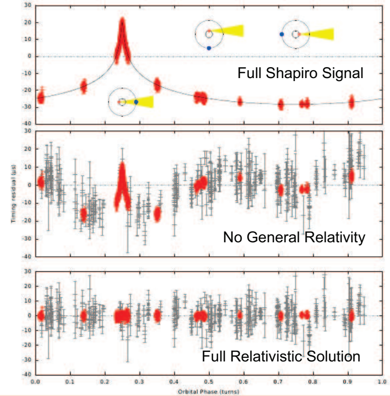
These are only functions of:

- the (precisely!) known Keplerian orbital parameters P_b, e, asin(i)
- the mass of the pulsar m_1 and the mass of the companion m_2

MSP J1614-2230: Incredible Shapiro Delay Signal

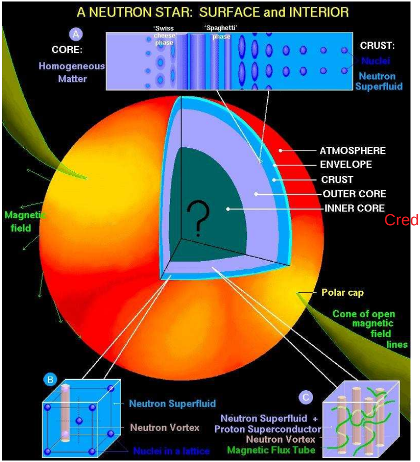
$M_{wd} = 0.500(6)\ M_\odot$
Inclination = 89.17(2) deg!

Mpsr = **1.97(4)** $\mathbf{M_\odot}$!



Demorest et al. 2010, *Nature*, 467, 1081D
see Ozel et al. 2010, ApJL, 724, 1990

Structure of a Neutron Star



Credit: D. Page

- Atmosphere (atoms) $n \leq 10^4\ g/cm^3$
- Outer Crust (free e^- s, lattice of nuclei) $10^4 - 4 \times 10^{11}\ g/cm^3$
- Inner crust (lattice of nuclei with free e^-s and n's)
- Outer core (atomic particle fluid)
- Inner core (exotic subatomic particles) $n \geq 10^{14}\ g/cm^3$

Exotic forms of Matter

Various exotic components of matter such as hyperons, Bose-Einstein Condensates (pion or kaon) & quarks, may appear in the neutron star core. Hyperons

- Hyperons produced at the cost of the nucleons.
$$n + p \longrightarrow p + \Lambda + K^0, \quad n + n \longrightarrow n + \Sigma^- + K^+$$
- Chemical equilibrium in compact star interior through weak processes,
- $p + e^- \longrightarrow \Lambda + \nu_e, \quad n + e^- \longrightarrow \Xi^- + \nu_e$
- Condition for chemical equilibrium
$$\mu_i = b_i \mu_n - q_i \mu_e$$
- Threshold Condition for Hyperons
$\mu_n - q_i \mu_e \geq m_B^* + g_{\omega B}\omega_0 + g_{\rho B}\rho_{03}\tau_3$

Quark Matter

Witten Conjecture: u, d, s quark matter is the ground state of matter (energy/baryon $<$ 939 MeV at finite density).

Ref: E. Witten, Phys. Rev. D30 (1984) 272

Quarks are in chemical equilibrium:

$$d \longrightarrow u + e^- + \bar{\nu}_e, \quad s \longrightarrow u + e^- + \bar{\nu}_e;$$

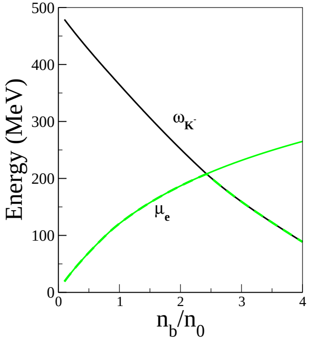
$$\mu_d = \mu_u + \mu_e, \quad \mu_s = \mu_d$$

MIT Bag model: $P \longrightarrow P - B, \quad \& \quad \epsilon \longrightarrow \epsilon + B$

Recently it has been predicted that quark matter might be a color superconductor. Quarks near their Fermi surfaces form Cooper pairs due to the attractive quark-quark interaction in color antisymmetric channel.

Bose-Einstein condensates

Kaplan and Nelson first demonstrated that the Bose condensate of K^- mesons could be a possibility in heavy ion collisions and neutron stars. The processes responsible for *p*-wave pion /*s*-wave kaon condensate



- $n \to p + \pi^-$; $n \to p + K^-$
- $e^- \to \pi^- + \nu_e$; $e^- \to K^- + \nu_e$

  • Threshold conditions:
- For K^- $\omega_{K^-} = \mu_e$.
- For π^- $\omega_{\pi^-} = \mu_e$.

A.B. Migdal, A.I Cevnoutsan, I.N. Mishustin, PLB83 (1979) 158

H.A. Bethe and G.E. Brown, ApJ445 (1995) L129

N.K. Glendenning and J. Schaffner-Bielich, PRL81 (1998) 4564

S. Banik, D.B., PRC64 (2001) 055805

Many-body theories of dense matter in Neutron Stars

Neutron star matter is a many-body system

- Two classes of models: non-relativistic and relativistic models
  i) Microscopic models :
- Brueckner Hartree-Fock and Dirac-Brueckner-Hartree-Fock theories (R. Brockmann and R. Machleidt, PRC42 (1990) 1965)
- Variational many-body approach (A. Akmal, V. Pandharipande and D.G. Ravenhall, PRC58 (1998) 1804)
  ii) Effective Field theory approach:
- Density functional theory (R.J. Furnstahl, Lect. Notes Phys. 641 (2004) 1)
- Chiral perturbation theory (K. Hebeler, PRL105 (2010) 161102)
  iii) Phenomenological theories:
- Effective two-body interactions (Skyrme interactions)
- Relativistic Mean Field (RMF) models (J. D. Walecka, Adv. Nucl. Phys. 16 (1986) 1)

Model: Finite Temperature Equation of State

$$
\begin{aligned}
\mathcal{L}_B &= \sum_B \bar{\Psi}_B \left(i\gamma_\mu\partial^\mu - m_B^* - g_{\omega B}\gamma_\mu\omega^\mu - g_{\rho B}\gamma_\mu \mathrm{t}_B \cdot \boldsymbol{\rho}^\mu\right)\Psi_B \\
&+ \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
&- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu + \mathcal{L}_{YY}\ .
\end{aligned}
$$

The thermodynamic potential per unit volume for nucleons is

$$
\begin{aligned}
\frac{\Omega_N}{V} &= \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{2}m_\rho^2\rho_{03}^2 \\
&- 2T\sum_{i=n,p}\int\frac{d^3k}{(2\pi)^3}[ln(1+e^{-\beta(E^*-\nu_i)}) + ln(1+e^{-\beta(E^*+\nu_i)})]\ .
\end{aligned}
$$

The thermodynamic potential per unit volume for nucleons is given by

$$\frac{\Omega_B}{V} = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{4}g_4\omega_0^4 - \frac{1}{2}m_\rho^2\rho_{03}^2$$
$$-2T\sum_B\int\frac{d^3k}{(2\pi)^3}[ln(1+e^{-\beta(E^*-\nu_B)}) + ln(1+e^{-\beta(E^*+\nu_B)})] \,.$$

Here, $\beta = 1/T$ and $E^* = \sqrt{(k^2+m_B^{*2})}$.

$$P_B = -\Omega_B/V.$$

The energy density is given by,

$$\epsilon_B = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{3}{4}g_4\omega_0^4 + \frac{1}{2}m_\rho^2\rho_{03}^2$$
$$+2\sum_B\int\frac{d^3k}{(2\pi)^3}E^*\left(\frac{1}{e^{\beta(E^*-\nu_B)}+1} + \frac{1}{e^{\beta(E^*+\nu_B)}+1}\right) \,.$$

(S. Banik et al., Phys.Rev.C78:065804,2008)

- (Anti)kaon-baryon interaction is treated in the same footing as the baryon-baryon interaction. The Lagrangian density for (anti)kaons in the minimal coupling scheme is

$$\mathcal{L}_K = D^*_\mu \bar{K} D^\mu K - m_K^{*2} \bar{K} K \ ,$$

where $D_\mu = \partial_\mu + i g_{\omega K} \omega_\mu + i g_{\rho K} \mathrm{t}_K \cdot \boldsymbol{\rho}_\mu$ and the effective mass of (anti)kaons is $m_K^* = m_K - g_{\sigma K} \sigma$.

- The equation of motion for (anti)kaons is

$$(D_\mu D^\mu + m_K^*) K = 0$$

The thermodynamic potential for antikaons is given by,

$$\frac{\Omega_K}{V} = T \int \frac{d^3p}{(2\pi)^3} [ln(1 - e^{-\beta(\omega_{K^-} - \mu)}) + ln(1 - e^{-\beta(\omega_{K^+} + \mu)})] \ .$$

The net (anti)kaon number density is given by

$$n_K = n_K^C + n_K^T \ , \tag{0}$$

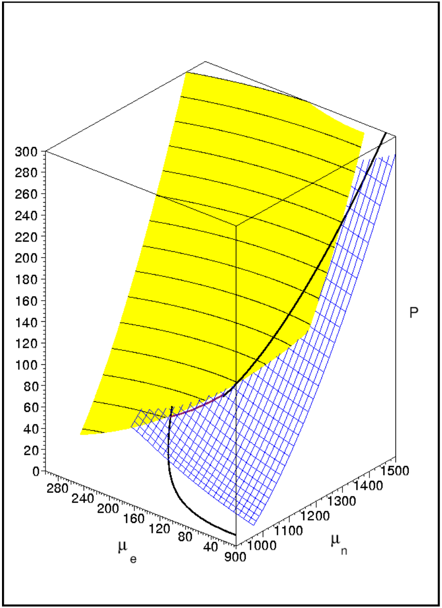
where the thermal (anti)kaon density is given by,

$$n_K^T = \int \frac{d^3p}{(2\pi)^3} \left(\frac{1}{e^{\beta(\omega_{K^-} - \mu)} - 1} - \frac{1}{e^{\beta(\omega_{K^+} + \mu)} - 1} \right) \ . \tag{1}$$

The energy density of (anti)kaons is given by

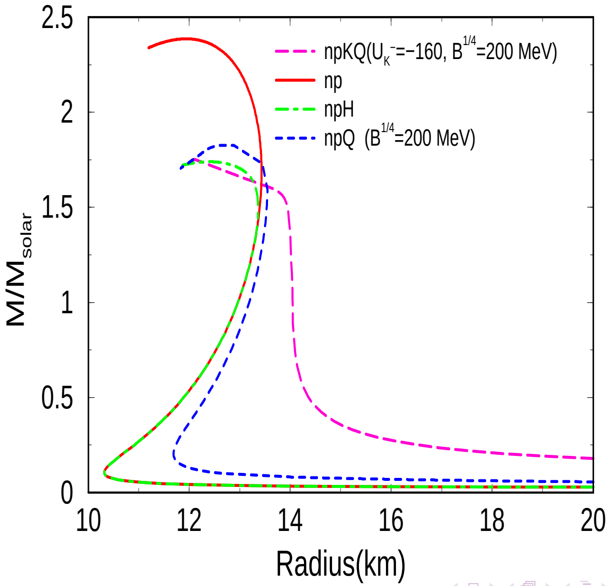
$$\begin{aligned} \epsilon_K &= m_K^* n_K^C + \left(g_{\omega K} \omega_0 + \frac{1}{2} g_{\rho K} \rho_{03} \right) n_K^T \\ &+ \int \frac{d^3p}{(2\pi)^3} \left(\frac{\omega_{K^-}}{e^{\beta(\omega_{K^-} - \mu)} - 1} + \frac{\omega_{K^+}}{e^{\beta(\omega_{K^+} + \mu)} - 1} \right) \ . \end{aligned}$$

The pressure due to thermal (anti)kaons $P_K = -\Omega_K / V$.



Credit: S. Banik

Mass-Radius Relationship



Role of Symmetry Energy

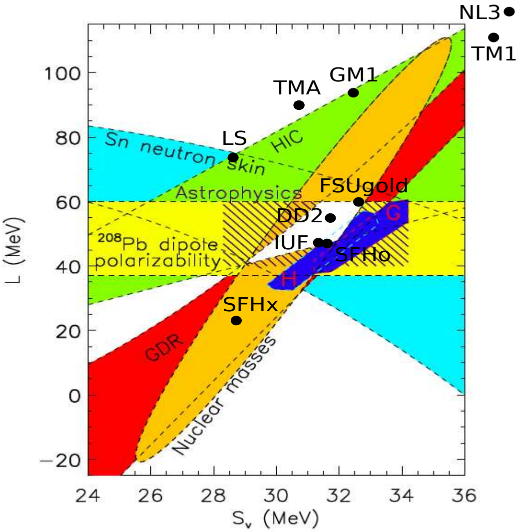
The energy per nucleon in asymmetric matter may be written as

$$E(\rho, \beta) = E(\rho, \beta = 0) + \beta^2 E_{sym}(\rho),$$

where $\beta = \frac{(\rho_n - \rho_p)}{\rho}$ is the asymmetry parameter.
The symmetry energy is an essential ingredient in understanding dense matter. The expression of nuclear symmetry energy follows from

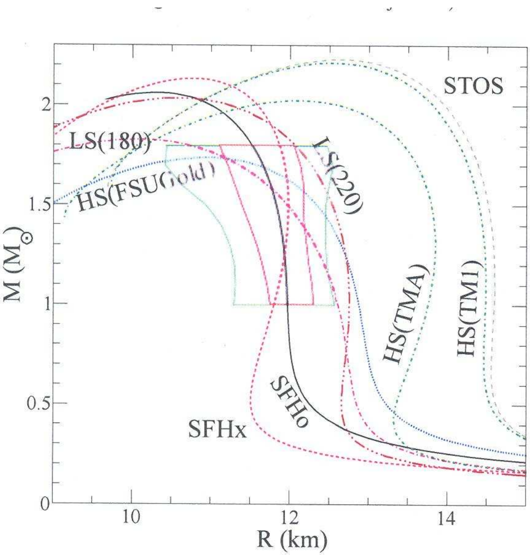
$$\mu_n - \mu_p = 4\beta E_{sym}(\rho).$$

with $\mu_n = \frac{\partial \epsilon}{\partial \rho_n}$ and $\mu_p = \frac{\partial \epsilon}{\partial \rho_p}$.



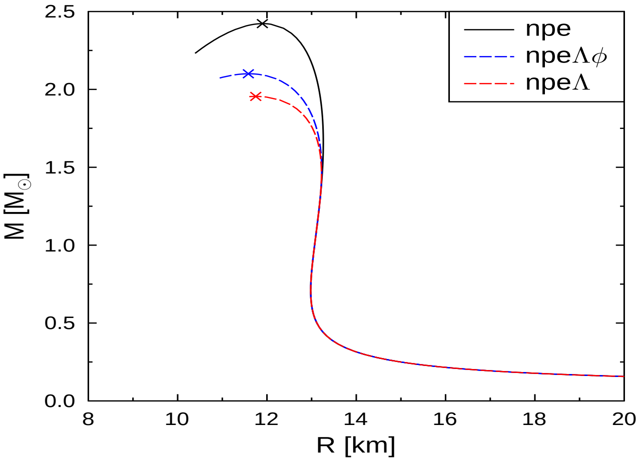
J. M. Lattimer and Y. Lim, ApJ 771, 51 (2013)

Mass-Radius of Neutron Stars From Supernova Models

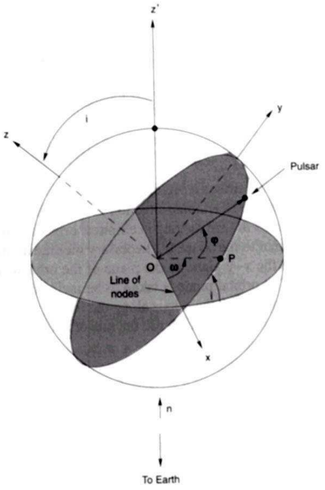


Hyperon EoS is compatible with a 2 $M_\odot$ Neutron Star.

S. Banik, M. Hempel, D.B. ApJS 214(2014)22

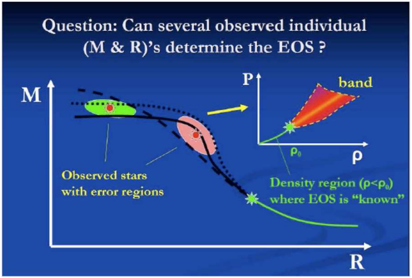


Spin-Orbit Coupling in PSR J0737-3039A



- Precession of the orbital plane about the direction of the total angular momentum
- The amplitude of timing change in the expected arrival of pulses from pulsar A
  $\delta t_0 = \frac{a}{c} \frac{I_A}{cM_A a^2} \frac{P}{P_A} sin\theta_A cosi, \, i = 90^0$
  Lattimer and Schutz, ApJ629 (2005)
- The advance of periastron:
  $k^{tot} = \dot{\omega}_{1PN} + \dot{\omega}_{2PN} + \dot{\omega}_{SO} =$
  $\frac{3\beta_0^2}{1-e_T} \left[1 + f_0\beta_0^2 - g_s^A \beta_0 \beta_s^A - g_s^B \beta_0 \beta_s^B\right]$
  $\beta_0 = (GM\frac{2\pi}{P})^{1/3}/c,$
  $\beta_s = \frac{2\pi c}{G} \frac{1}{P_A} \frac{I_A}{m^2}$
- Moment of Inertia, $I \propto MR^2$, constrains EoS.

Inverting TOV equation using observed Masses and Radii



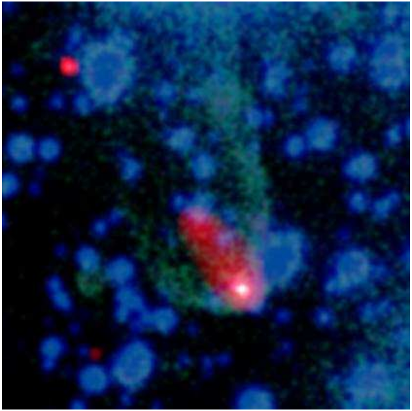
Credit: M. Prakash

- Simultaneous measurements of masses and radii as well as knowledge of the known EoS for $\rho < \rho_0 (= 2.7 \times 10^{14} g/cm^3)$ are needed to deconstruct the EoS (Lindblom, ApJ398 (1992)).
- The EoS below ρ_0 is very well constrained

$$\frac{dr^2}{dh} = -2r^2 \frac{r-2m}{m+4\pi r^3 P}$$
$$\frac{dm}{dh} = -4\pi r^3 \rho \frac{r-2m}{m+4\pi r^3 P},$$

where $dh = dp/(p+\rho(p))$

Black Widow Pulsar (B1957+20): Challenges Ahead



Credit: CXC/NASA

- This system has both pulsar timing and optical light curve information
- A 1.6 ms pulsar in a nearly circular 9.17 h orbit about its companion of 0.03 $M_\odot$
- The pulsar is eclipsed for about 50-60 minutes in each orbit
- The pulsar is eating up its companion
- The likely value of the pulsar mass from observations and modeling is $2.4 \pm 0.4 M_\odot$

Conclusions

- Relativistic binary pulsar system is an excellent laboratory for relativistic gravity
- The high precision timing observations of the double pulsar system offers the possibility of determining the moment of inertia of neutron stars.
- The spin-orbit coupling contribution to the periastron advance ($\dot{\omega}$) is the most promising way to determine the moment of inertia.
- Substantial advancement in the timing precision for the double pulsar system is expected to come from the Square Kilometer Array
- Consequently simultaneous measurements of mass and radius of same neutron star might be possible and this should yield to the equation of state of neutron star matter in a model independent fashion.

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