Memory Effect from Spinning Compact Binaries in Hyperbolic Orbits

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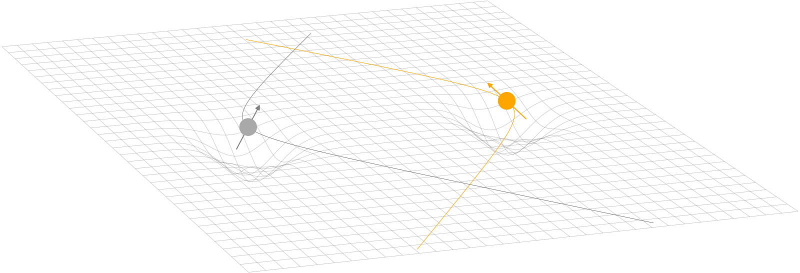

In collaboration with Lorenzo D. Vittori, A. Gopakumar and P. Jetzer, arXiv:1410.6311

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Outline

- We present an efficient prescription to compute post-Newtonian (PN) accurate h_+ & $h_\times$ for spinning compact binaries in hyperbolic orbits.
- It turns out that both h_+ & $h_\times$ exhibit the memory effect with the inclusion of spins.
- In contrast, only $h_\times$ shows the memory effect for GWs from non-spinning compact binaries.
- Why these signals are important for pulsar timing array (PTA) searches?
- Can we detect GW memory with help of ongoing and planned PTAs?

Prescription to compute $h_{+,\times}$ for spinning binaries

- The PN accurate expressions for $h_{+,\times}$ for binaries in general orbits

$$h_+ = \frac{1}{2}\left(p_i p_j - q_i q_j\right) h_{ij}^{\mathrm{TT}}\,, \qquad h_\times = \frac{1}{2}\left(p_i q_j + p_j q_i\right) h_{ij}^{\mathrm{TT}}$$

- The leading order (quadrupolar oder) expressions for $h_{+,\times}$ read

$$h_+\big|_{\mathrm{Q}} = \frac{2G\mu}{c^4 R}\left\{(\mathbf{p}\cdot\mathbf{v})^2 - (\mathbf{q}\cdot\mathbf{v})^2 - z\left[(\mathbf{p}\cdot\mathbf{n})^2 - (\mathbf{q}\cdot\mathbf{n})^2\right]\right\},$$

$$h_\times\big|_{\mathrm{Q}} = \frac{4G\mu}{c^4 R}\left\{(\mathbf{p}\cdot\mathbf{v})(\mathbf{q}\cdot\mathbf{v}) - z\,(\mathbf{p}\cdot\mathbf{n})(\mathbf{q}\cdot\mathbf{n})\right\}$$

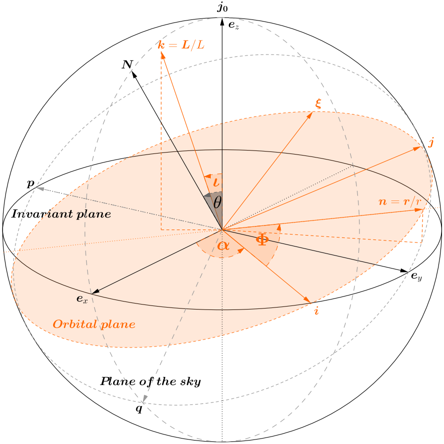
Using $$h_{ij}^{\mathrm{TT}} \to h_{ij}^{\mathrm{TT}}\big|_{\mathrm{Q}} = \frac{4G\mu}{c^4 R}\mathcal{P}_{kmij}(\mathbf{N})\left(\mathrm{v}_{km} - \frac{Gm}{r}n_{km}\right)$$

$\mathbf{p} = \mathbf{N}\times\mathbf{j}_0$, $\quad \mathbf{q} = \mathbf{N}\times\mathbf{p}$, $\quad \mathbf{N} \equiv$ Line-of-sight, $\quad \mathbf{v} = \dot{\mathbf{r}}$, $\quad \mathbf{n} = \mathbf{r}/r$

$\mathbf{J} = \mathbf{L} + \mathbf{S}_1 + \mathbf{S}_2$, $\quad z = \frac{G\,m}{r}$

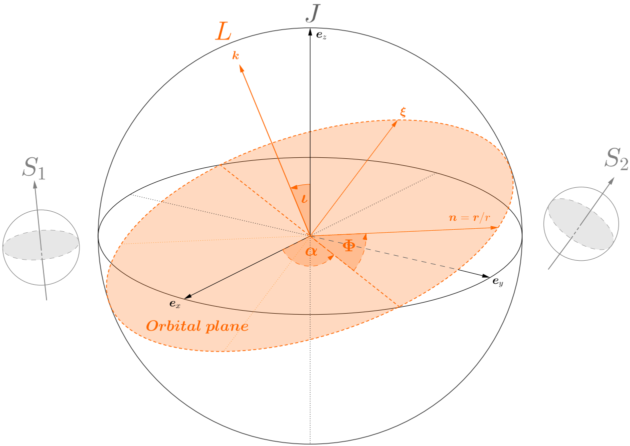
- Need to get the dot products $\Rightarrow$ have to describe the dynamics

The Inertial coordinate System



$$h_{+,\times}(t) = h_{+,\times}\Big(\underbrace{r(t), \dot{r}(t)}_{\text{radial part}}, \underbrace{\Phi(t), \dot{\Phi}(t), \alpha(t), \iota(t)}_{\text{angular part}} \Big)$$

The Precessing Dance



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Radial Part of the Dynamics

- Radial part has Keplerian-type parametric solution

- Hyperbolic Kepler equation:

$$l = \bar{n}(t - t_0) = e_t \sinh v - v \qquad\qquad r = a_r(e_r \cosh v - 1)$$

 can be solved numerically for $v(l)$ through Mikkola's method.

- The 1.5PN-accurate solution for $r(t)$ and $\dot{r}(t)$ turns out to be

$$r = \frac{Gm}{c^2}\frac{1}{\bar{\xi}^{2/3}}\left\{e_\mathrm{t} \cosh v - 1 - \bar{\xi}^{2/3}(\ldots) + \bar{\xi}(\ldots)\right\}$$

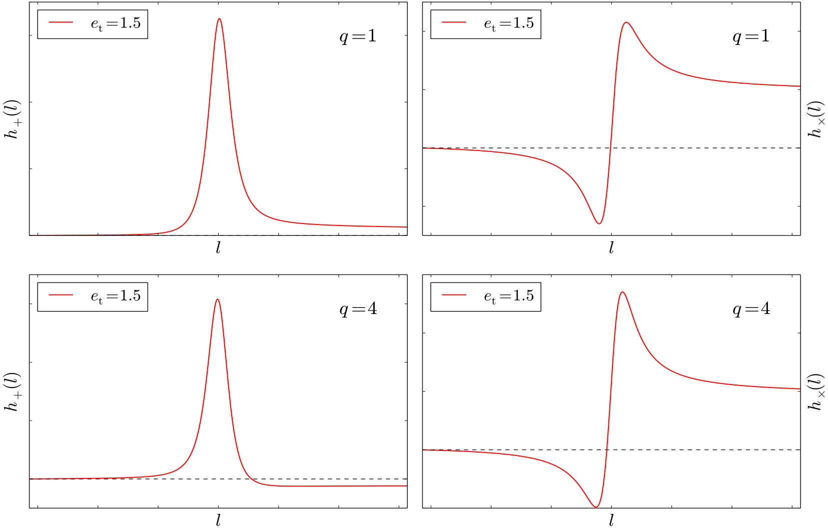
$$\dot{r} = \bar{\xi}^{1/3}\frac{c\, e_\mathrm{t}\, \sinh v}{e_\mathrm{t} \cosh v - 1}\left\{1 - \bar{\xi}^{2/3}\,(\ldots)\right\}$$

$\bar{\xi} = \frac{G\, m\, \bar{n}}{c^3}$

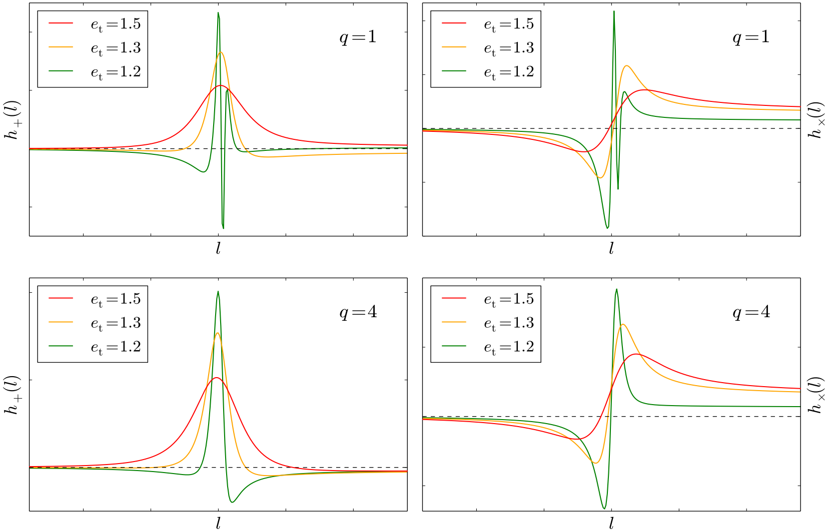
Angular part of the dynamics

- Angular part has no parametric solution
- The evolution of angular variables is obtained through a set of coupled differential equations
  - We need 9 precessional equations: $\dot{\mathbf{L}}$, $\dot{\mathbf{S}}_1$ and $\dot{\mathbf{S}}_2$
  - We need the 1 evolution equation for Φ: $\dot{\Phi}$
  - It also include 2 radiation reaction equations: $\dot{e}_t$ and $\dot{\bar{n}}$
- We numerically solve a set of 12 differential equations and get $\Phi(t), \dot{\Phi}(t), \alpha(t), \iota(t)$ at any time during the interaction

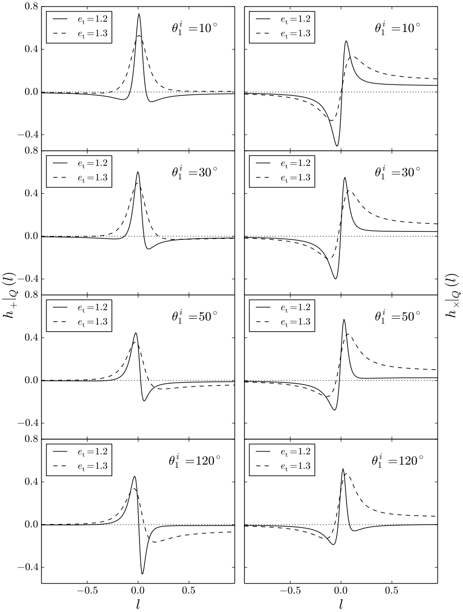
Waveform for Spinning Binaries in Hyperbolic Orbits: Effect of Mass-ratio



Effect of Eccentricity



Effect of spin orientation



Memory effect

- The **Memory effect** we see in the plots is

$$\boxed{\Delta h_{+,\times}^{\text{mem}} = \lim_{t \to +\infty} h_{+,\times}(t) - \lim_{t \to -\infty} h_{+,\times}(t)}$$

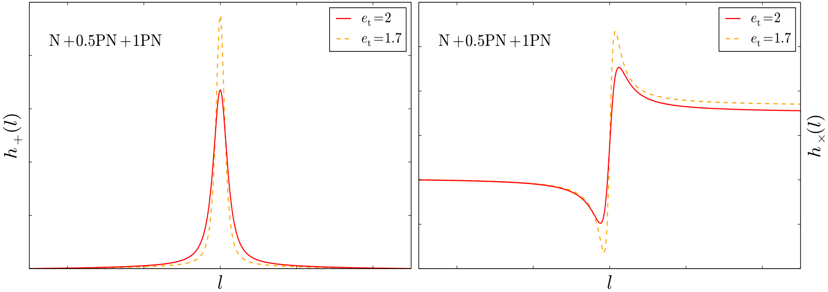
- For **spinning binaries** **both** polpolarizations show memory effect

$$\Delta h_{+}^{\text{mem}} \neq 0 \qquad \text{and} \qquad \Delta h_{\times}^{\text{mem}} \neq 0$$

- However, for non-spinning binaries only cross polarization exhibits memory effect

$$\Delta h_{+}^{\text{mem}} = 0 \qquad \text{and} \qquad \Delta h_{\times}^{\text{mem}} \neq 0$$

Waveform for non-spinning binaries



Non-spinning binary system with masses $m_1 = 8M_\odot$, $m_2 = 13M_\odot$, $r_{\text{min}} = 2 \times 10^9 m$, and $R = 21000$ ly. ($\sim$ Hulse-Taylor pulsar)

$\longrightarrow$ **Only the $\times$-polarization shows a memory!**

Why GW Memory is Interesting?

Two types of GW memory

Linear memory

- Change in the time derivatives of source multipole moments
- Hyperbolic orbits
  Captured, disrupted, mass loss
  GW recoil in binary BH merger

Non-linear memory

- Change in radiative multipole moments
- Mergers of supermassive BH binaries
  Any system that radiates GWs

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- It is non-oscillatory and visually distinctive in the waveform.
- GW with memory lead to permanent deformations of space-time ⇒ detector does not relapse to it's initial configuration
- The non-linear GW memory is observable and could be serve as a test of general relativity.

Detecting GW Memory with Laser Interferometers

- Unfortunately, LIGO-like detectors are not the ideal instruments to detect both linear and non-linear memory effects (M. Favata'09)

- Because the internal forces present in such instruments are expected to bring the test masses back to their original configurations

- eLISA-like instruments has truly freely falling masses and could, in principle, be deformed by the passage of GW with memory

Detecting GW Memory with PTAs

- It may be possible, in principle, to detect non-linear GW memory associated with the merger of SMBH binaries with the help of the ongoing and planned PTAs
(Seto '09; Pshirkov et al. '10; van Haasteren & Levin '10)

- $z \sim 0.1$, $M = 10^8 M_\odot$ mergers may be possibly detectable with 2σ constrains (van Haasteren & Levin '10)
($M = 10^{10} M_\odot$ will be detectable throughout the universe!)

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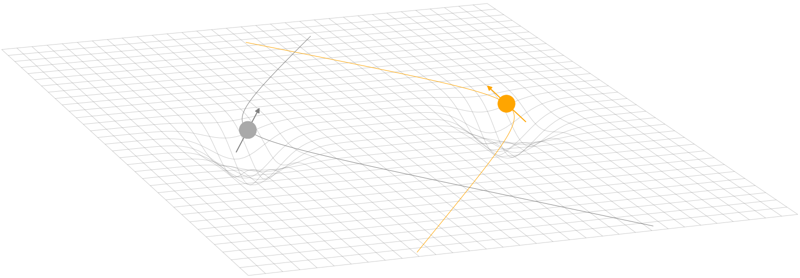
Either scenario will teach us something important about the population of these sources!



Thank you!