Astronomical Techniques II
Lecture 7 - *u-v* Coverage and Array Design

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Towards van-Cittert-Zernike Theorem

- Correlator Output -

$$r(\vec{D_\lambda}, \vec{s_0}) = \Delta\nu \int_{4\pi} A(\vec{\sigma})\ B(\vec{\sigma})\ cos[2\pi \vec{D_\lambda} \cdot (\vec{s_0} + \vec{\sigma})]\ d\Omega =$$

$$\Delta\nu\ cos(2\pi \vec{D_\lambda} \cdot \vec{s_0}) \int_{4\pi} A(\vec{\sigma})\ B(\vec{\sigma})\ cos(2\pi \vec{D_\lambda} \cdot \vec{\sigma})\ d\Omega -$$

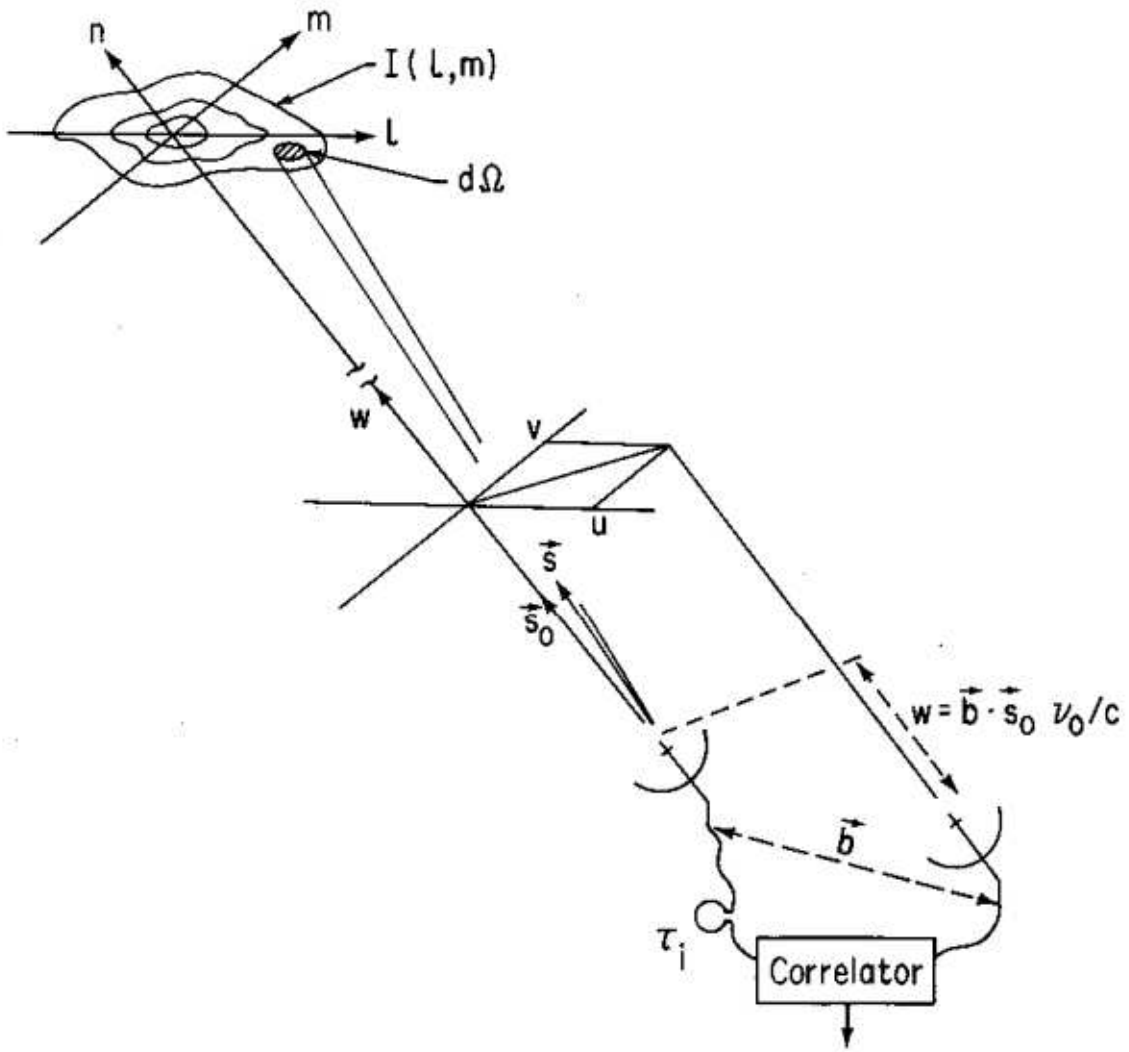
$$\Delta\nu\ sin(2\pi \vec{D_\lambda} \cdot \vec{s_0}) \int_{4\pi} A(\vec{\sigma})\ B(\vec{\sigma})\ sin(2\pi \vec{D_\lambda} \cdot \vec{\sigma})\ d\Omega$$

- Complex visibility -

$$\mathcal{V} = |\mathcal{V}| e^{i\phi_v} = \int_{4\pi} A_N(\vec{\sigma})\ B(\vec{\sigma})\ e^{-i\ 2\pi\ \vec{D_\lambda} \cdot \vec{\sigma}}\ d\Omega$$

- $$\int_{4\pi} A_N(\vec{\sigma})\ B(\vec{\sigma})\ cos(2\pi\ \vec{D_\lambda}\cdot\vec{\sigma})\ d\Omega = |\mathcal{V}|cos(\phi_v)$$
  $$\int_{4\pi} A_N(\vec{\sigma})\ B(\vec{\sigma})\ sin(2\pi\ \vec{D_\lambda}\cdot\vec{\sigma})\ d\Omega = -|\mathcal{V}|sin(\phi_v)$$
- $r(\vec{D_\lambda}, \vec{s_0}) = A_0\ \Delta\nu\ |\mathcal{V}|\ cos(2\pi\vec{D_\lambda}\cdot\vec{\sigma} - \phi_v)$

Geometric Relationship



Geometric Relationship

- $\vec{D_\lambda} \cdot \vec{s_0} = w$
- $\vec{D_\lambda} \cdot \vec{s} = (ul + vm + wn);\ n = \sqrt{1 - l^2 - m^2}$
- $d\Omega = \frac{dl\ dm}{\sqrt{1-l^2-m^2}}$

- $\vec{s} = \vec{s_0} + \vec{\sigma}$
  $\implies \vec{D_\lambda} \cdot \vec{\sigma} = \vec{D_\lambda} \cdot \vec{s} - \vec{D_\lambda} \cdot \vec{s_0}$
- $\vec{D_\lambda} \cdot \vec{s} = ul + vm + w(\sqrt{1 - l^2 - m^2} - 1)$
- $\mathcal{V}(u, v, w) =$
  $$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_N(l, m)\ B(l, m)\ e^{-i2\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]} \frac{dl\ dm}{\sqrt{1-l^2-m^2}}$$

- Thompson, Moran, Swenson - Chap. 3

Small FoV approximation

- $w(\sqrt{1-l^2-m^2}-1) \sim -\frac{1}{2}(l^2+m^2)w << ul+vm$
- $\mathcal{V}(u,v,w) \sim \mathcal{V}(u,v,0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_N(l,m)\ B(l,m)\ e^{-i2\pi[ul+vm]}\frac{dl\ dm}{\sqrt{1-l^2-m^2}}$

Impact of the w term

- Phase error - $\Delta\phi = \pi\ w\ (l^2 + m^2)$

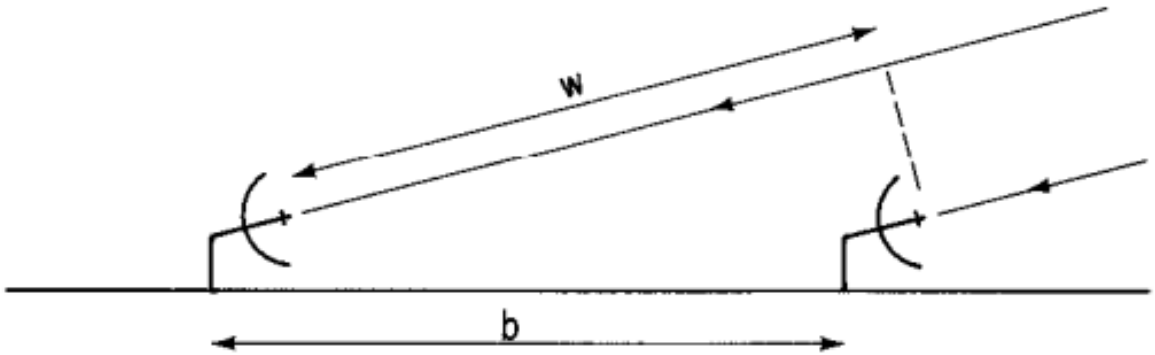


Figure 2-10. Comparison of the w-component and the antenna spacing when the direction of the source is close to that of the baseline. This condition can occur when the source is rising or setting.

- $\frac{1}{\theta_{HPBW}} \sim \frac{b_{max}}{\lambda} \sim w_{max}$; θ_{HPBW}- Synthesised Beam
- $\Delta\phi_{max} \sim \pi \left(\frac{\theta_F}{2}\right)^2 \frac{1}{\theta_{HPBW}}$; θ_F - size of the Map

Earth Rotation Synthesis Geometry

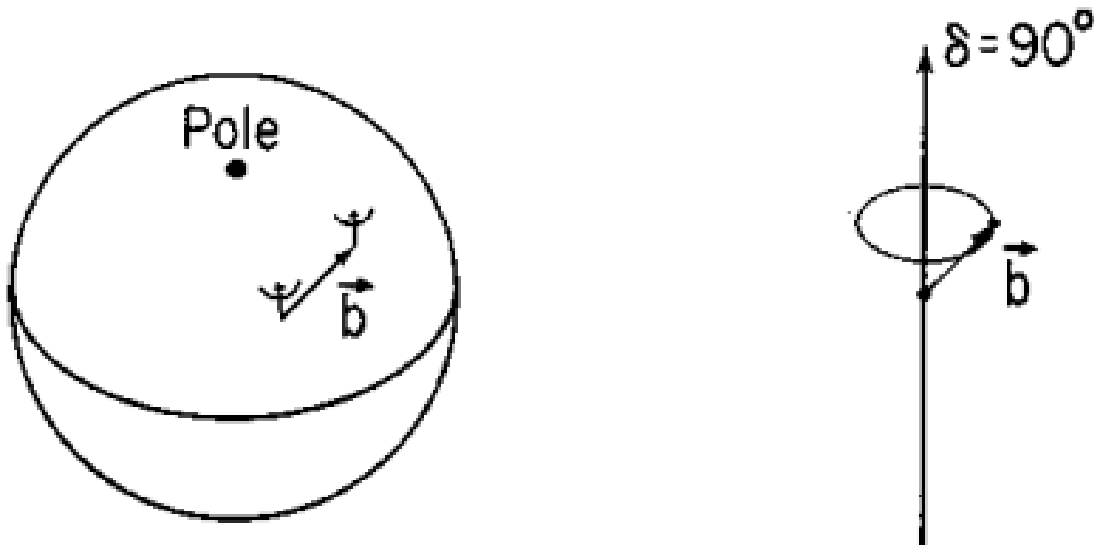


Figure 2-8. As the Earth rotates, the baseline vector b, which represents the spacing of the two antennas, traces out a circular locus in a plane normal to the direction of declination (δ) equal to 90°. If the antennas are in an East–West line on the Earth, then the vector b is normal to the rotation axis.

Coordinate Frame

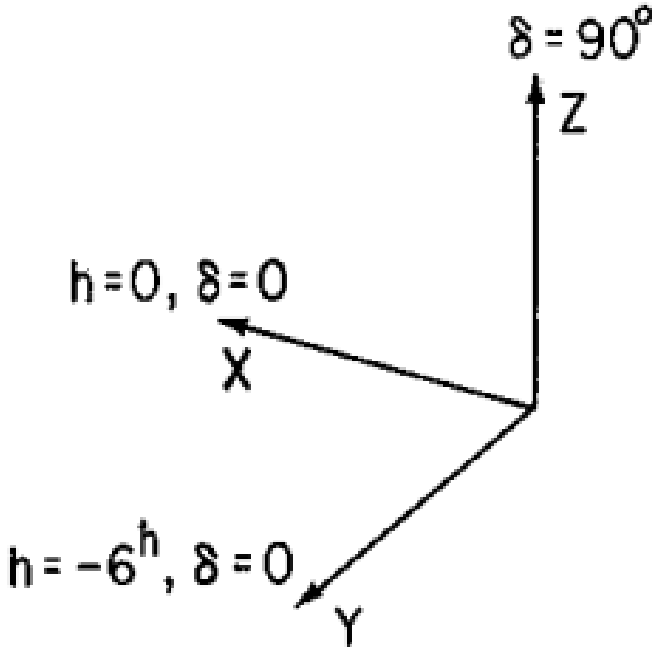


Figure 2-11. Coordinate system for specification of baseline parameters. X is the direction of the meridian at the celestial equator, Y is toward the East, and Z toward the North celestial pole.

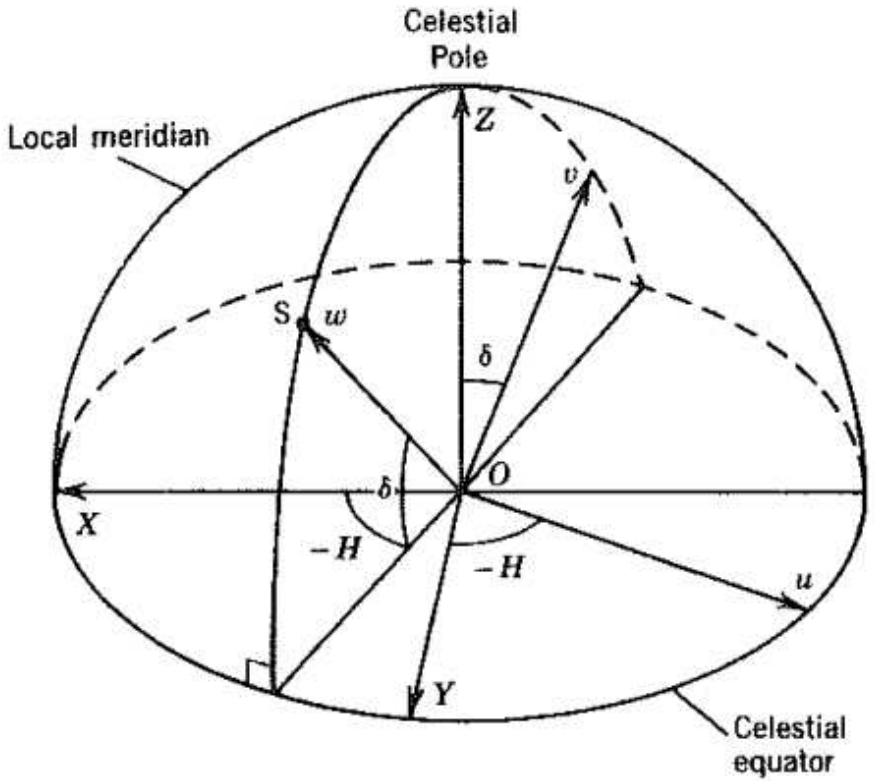


Figure 4.2 Relationships between the (X, Y, Z) and (u, v, w) coordinate systems. The (u, v, w) system is defined for observation in the direction of the point S, which has hour angle and declination H and δ. As shown, S is in the eastern half of the hemisphere and H is therefore negative. The direction cosines in the transformation matrix in Eq. (4.1) follow from the relationships in this diagram. The relationship in Eq. (4.2) can also be derived if we let S represent the direction of the baseline and put the baseline coordinates (h, d) for (H, δ).

Antenna Spacing Coordinates and u, v, w

($\delta = 90^\circ$) for Z may be used as in Figure 2-11. Then if L_X, L_Y, and L_Z are the corresponding coordinate differences for two antennas, the baseline components (u, v, w) are given by

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}, \quad (2\text{-}30)$$

where H_0 and δ_0 are the hour-angle and declination of the phase reference position, and λ is the wavelength corresponding to the center frequency of the receiving system. The elements in the transformation matrix in Equation 2-30 are the direction cosines of the (u, v, w) axes relative to (X, Y, Z) axes: for further details see, e.g., Thompson, Moran and Swenson (1986). By eliminating

Misc. comments

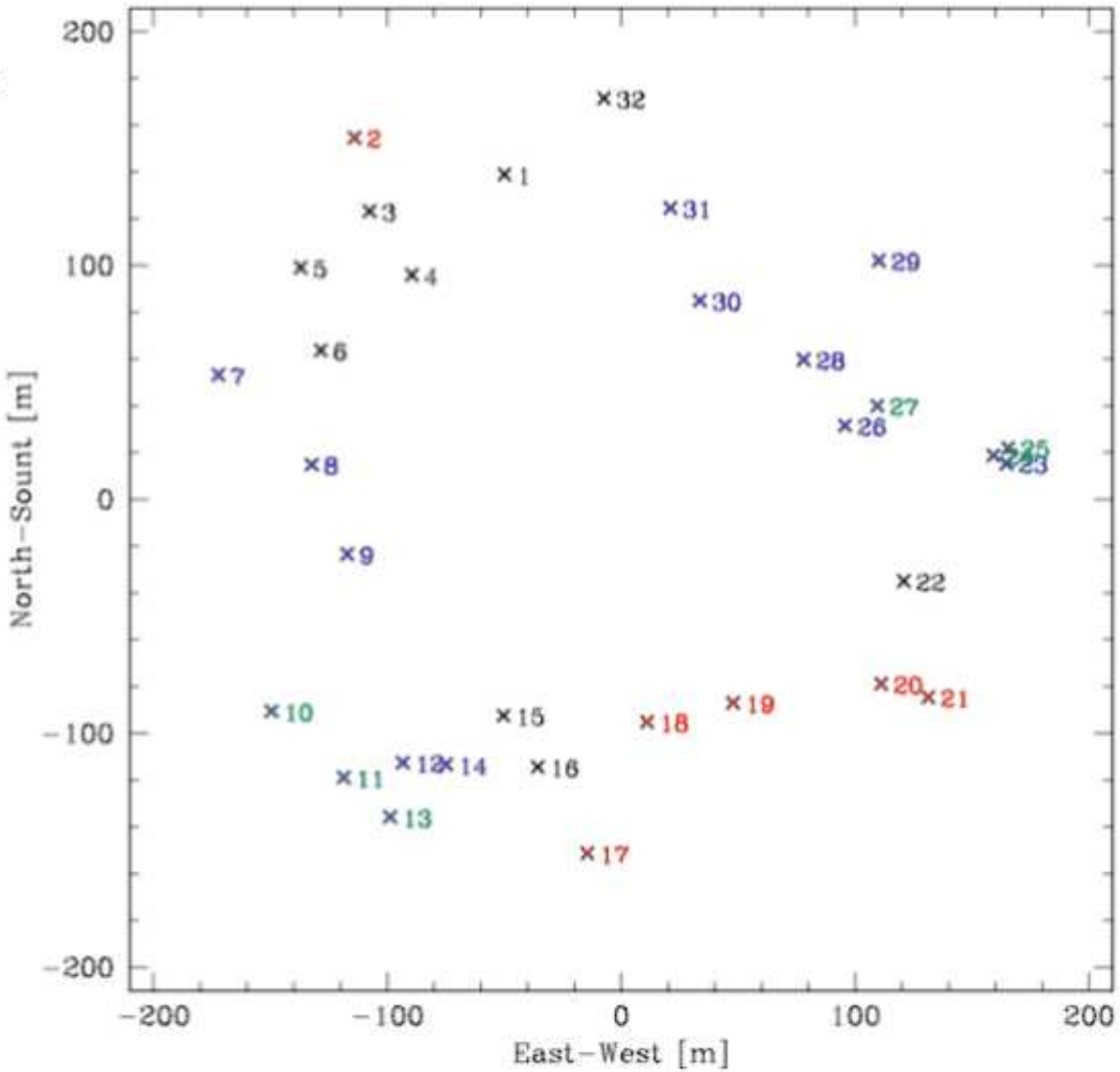
- Fringe Frequency
- Locus of a u,v track
- An East-West baseline
- Source at $\delta_0 = 0^\circ$
- Source at $\delta_0 = 90^\circ$

Design of Arrays

- Redundancy
- Sampling in the u-v plane
- Weighting

MWA Prototype: array configuration

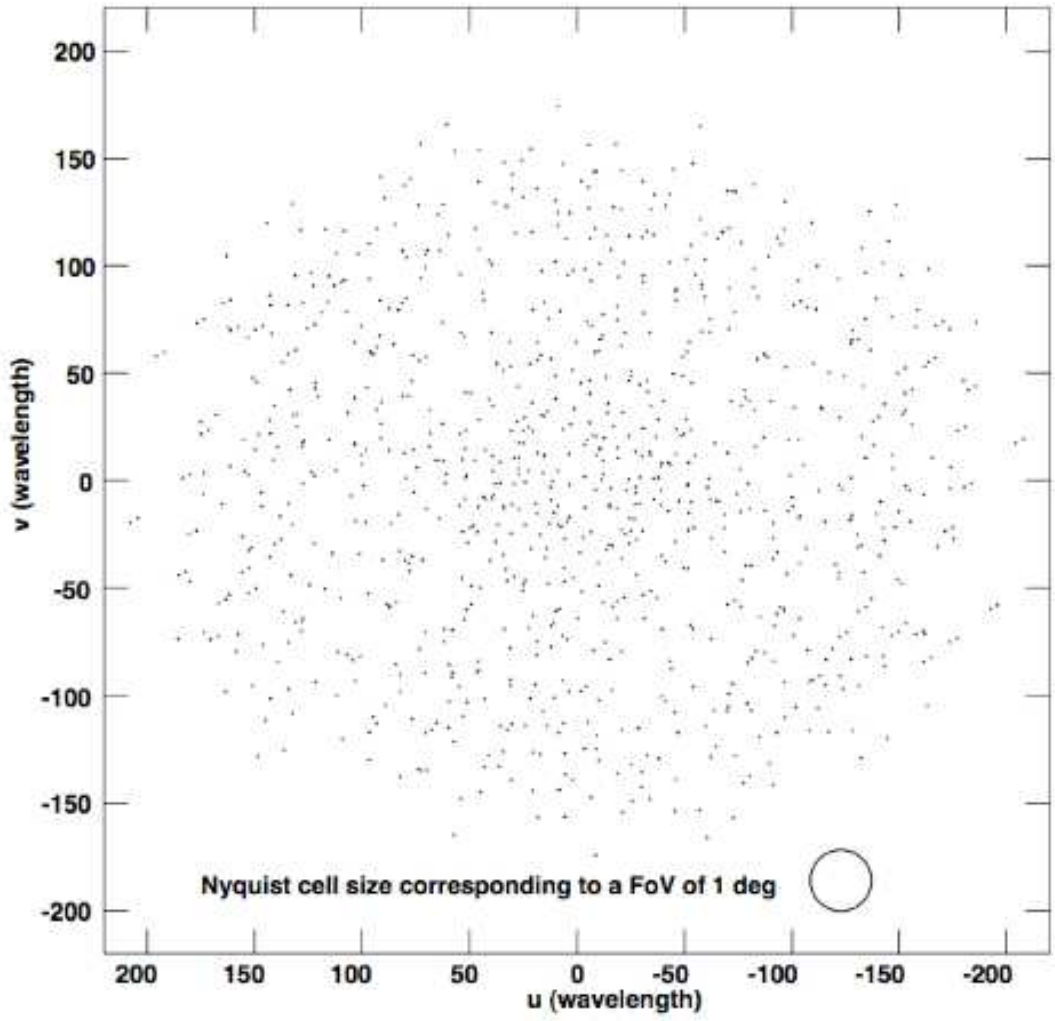
Locations of Antenna Tiles in 32T Array



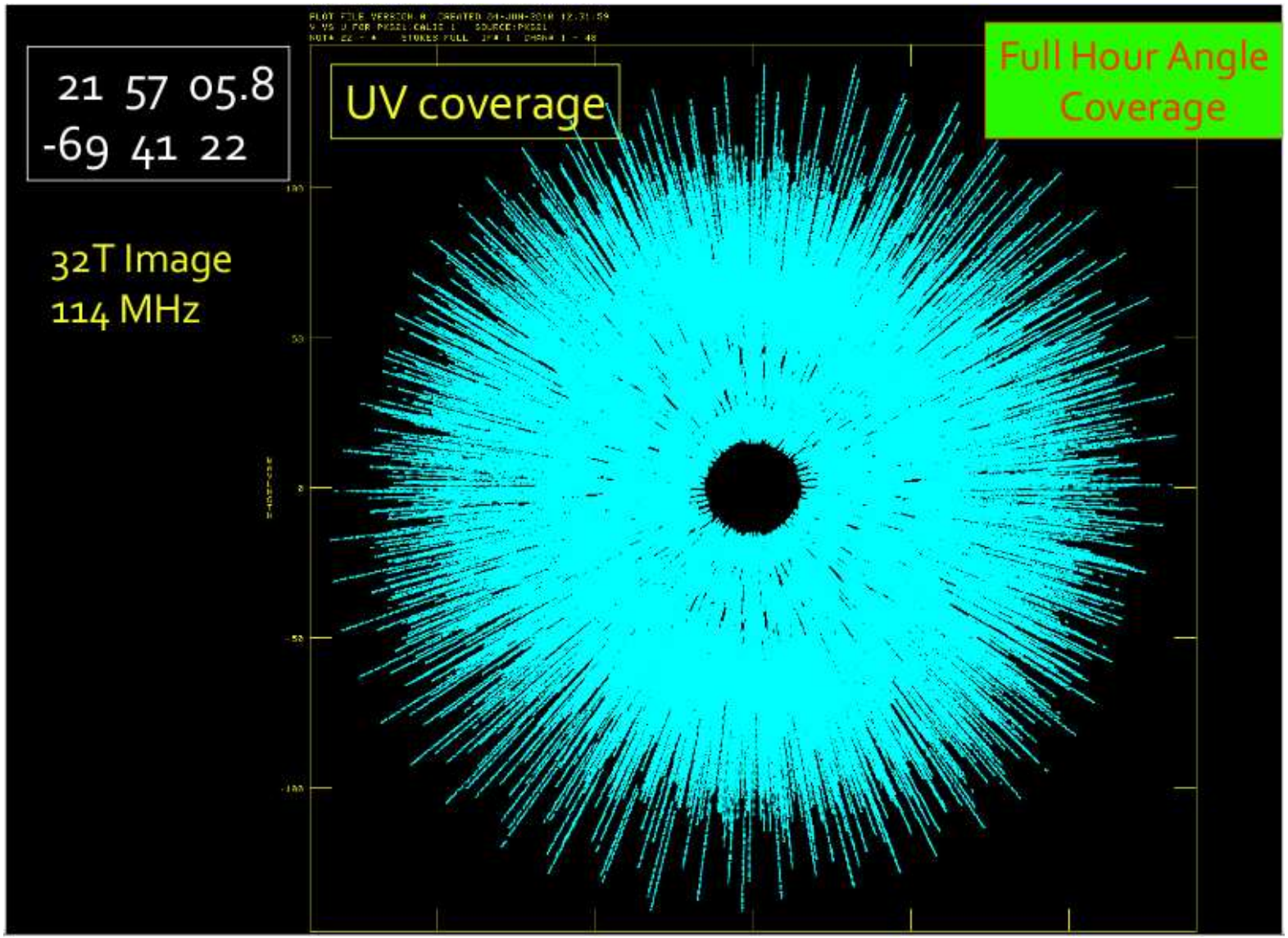
MWA Prototype: instantaneous uv coverage

Instantaneous 32T uv-coverage

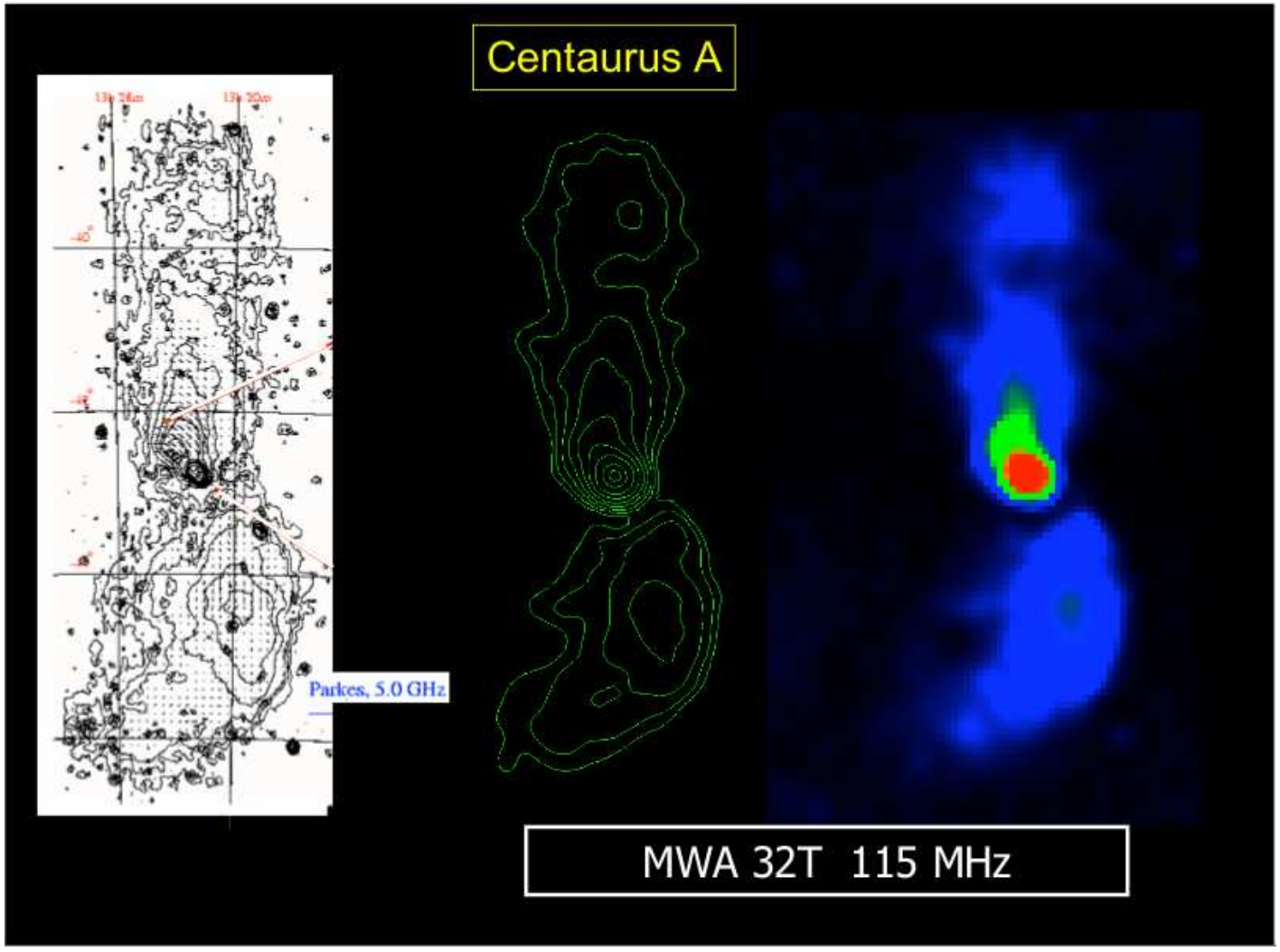
Oberoi, Matthews, et al



MWA Prototype: Rotation+Frequency synthesis



MWA Prototype: Centaurus A



Ben McKinley and Frank Briggs et al. (2013)

ASKAP array configuration

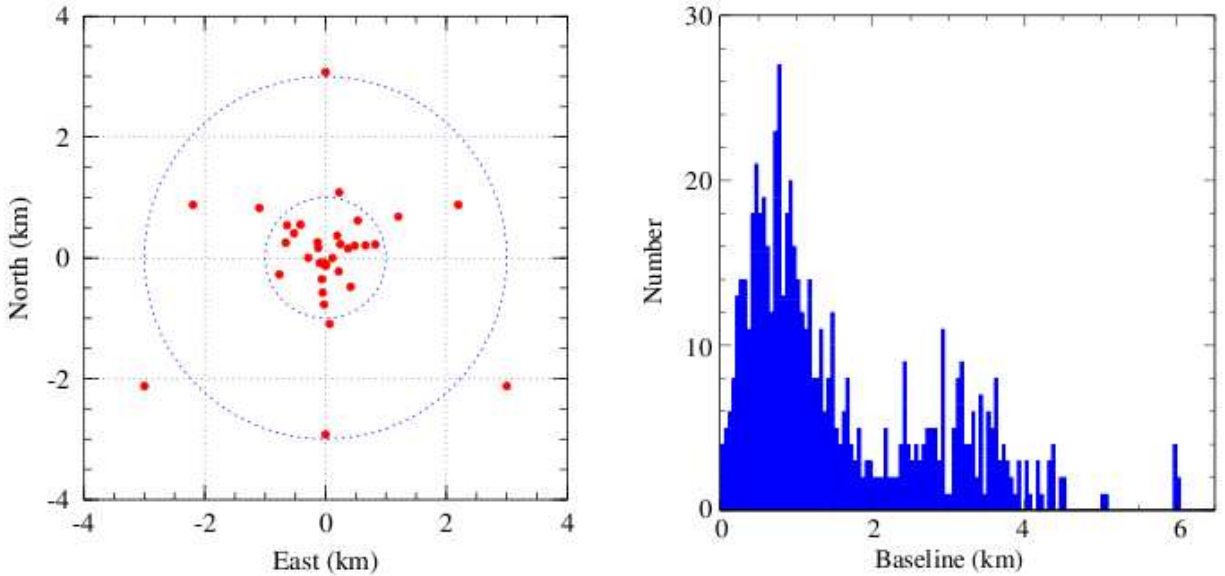
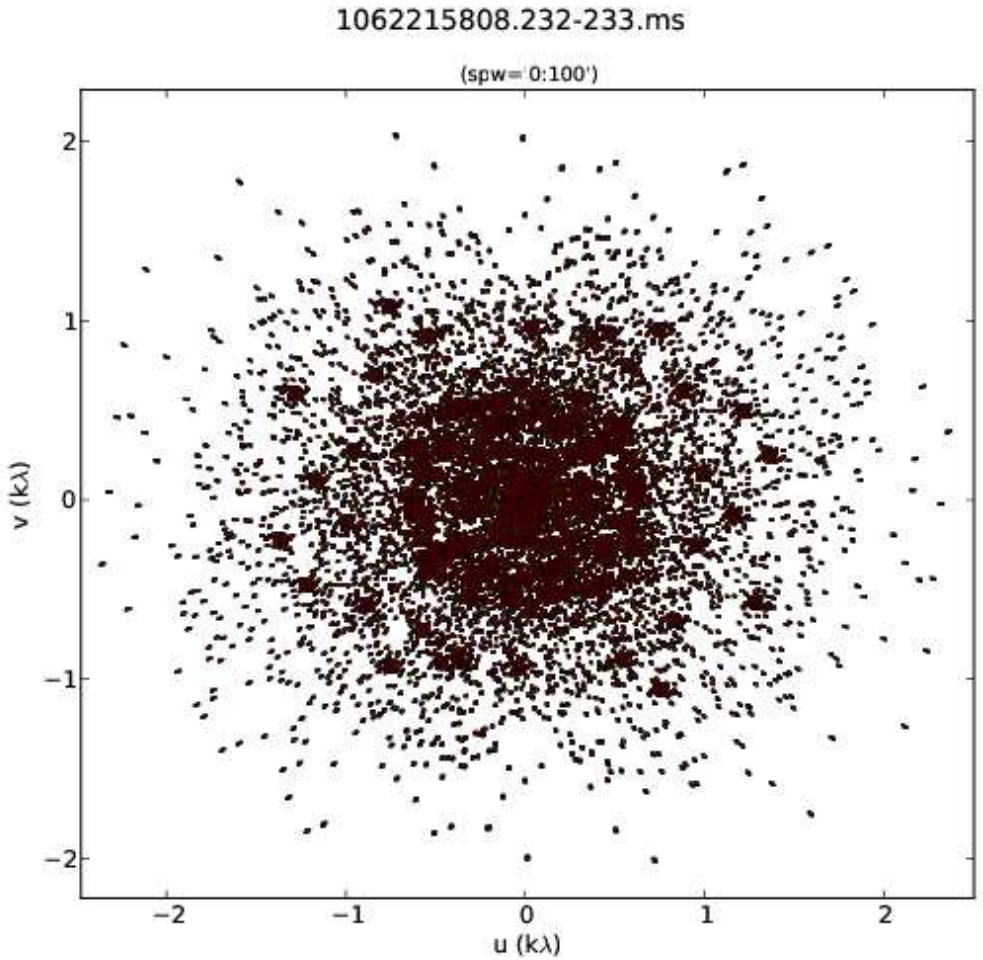
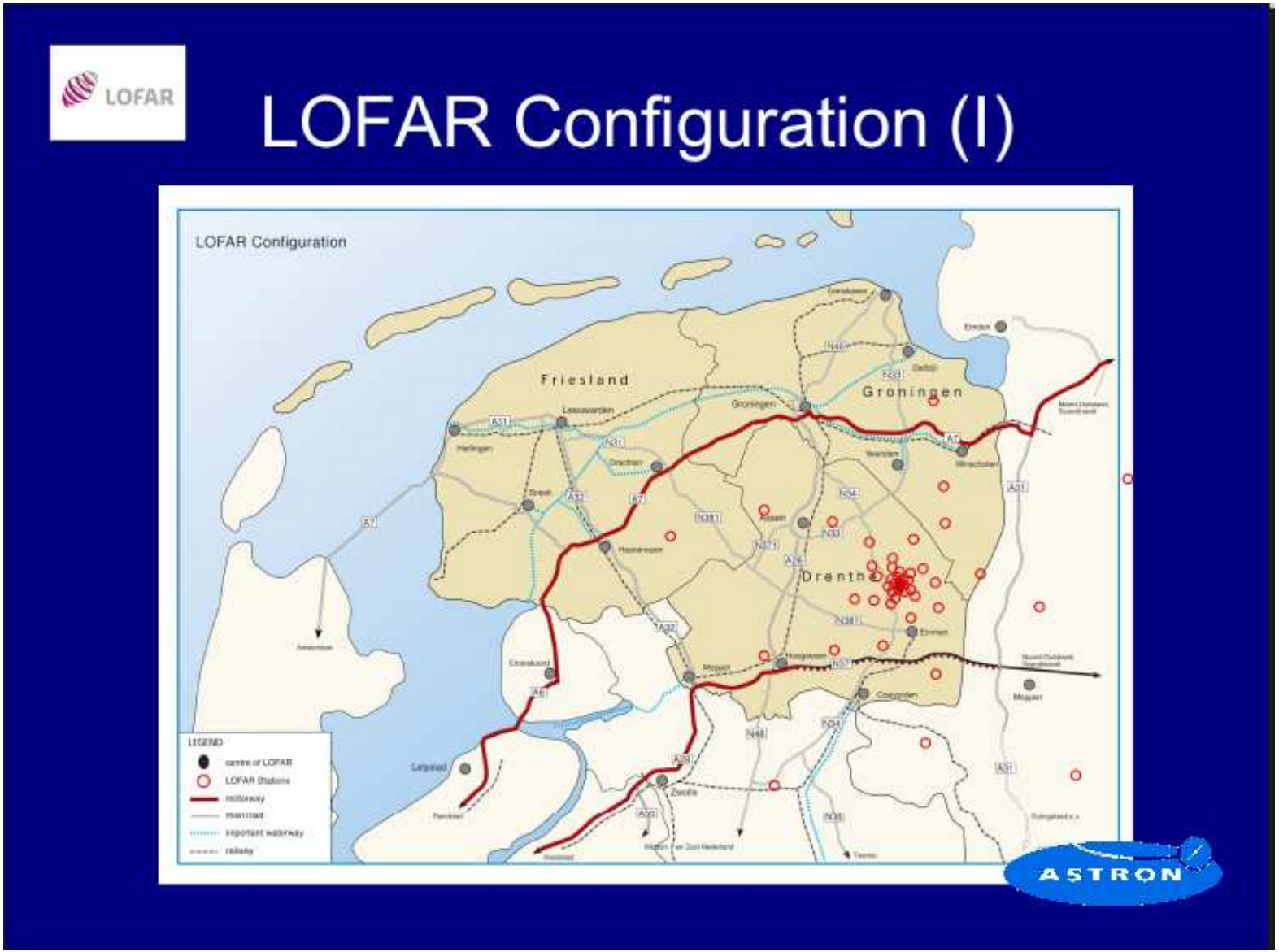


Figure 2: Left: Layout of the 36 antennas of the initial ASKAP configuration (red dots). The blue circles have diameters of 2 and 6 km, respectively. Right: Histogram of telescope baseline lengths for the initial ASKAP configuration.

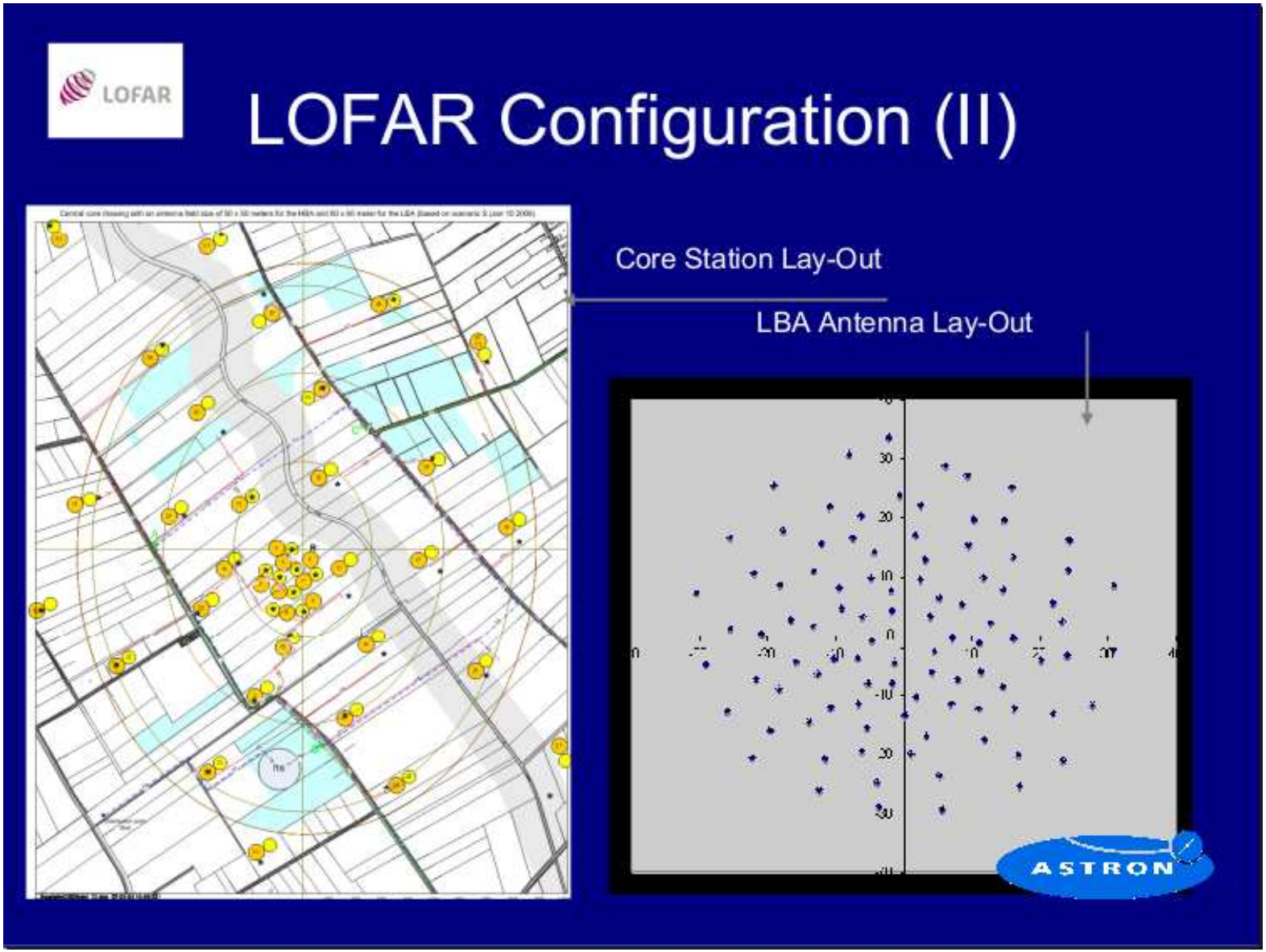
MWA uvcoverage



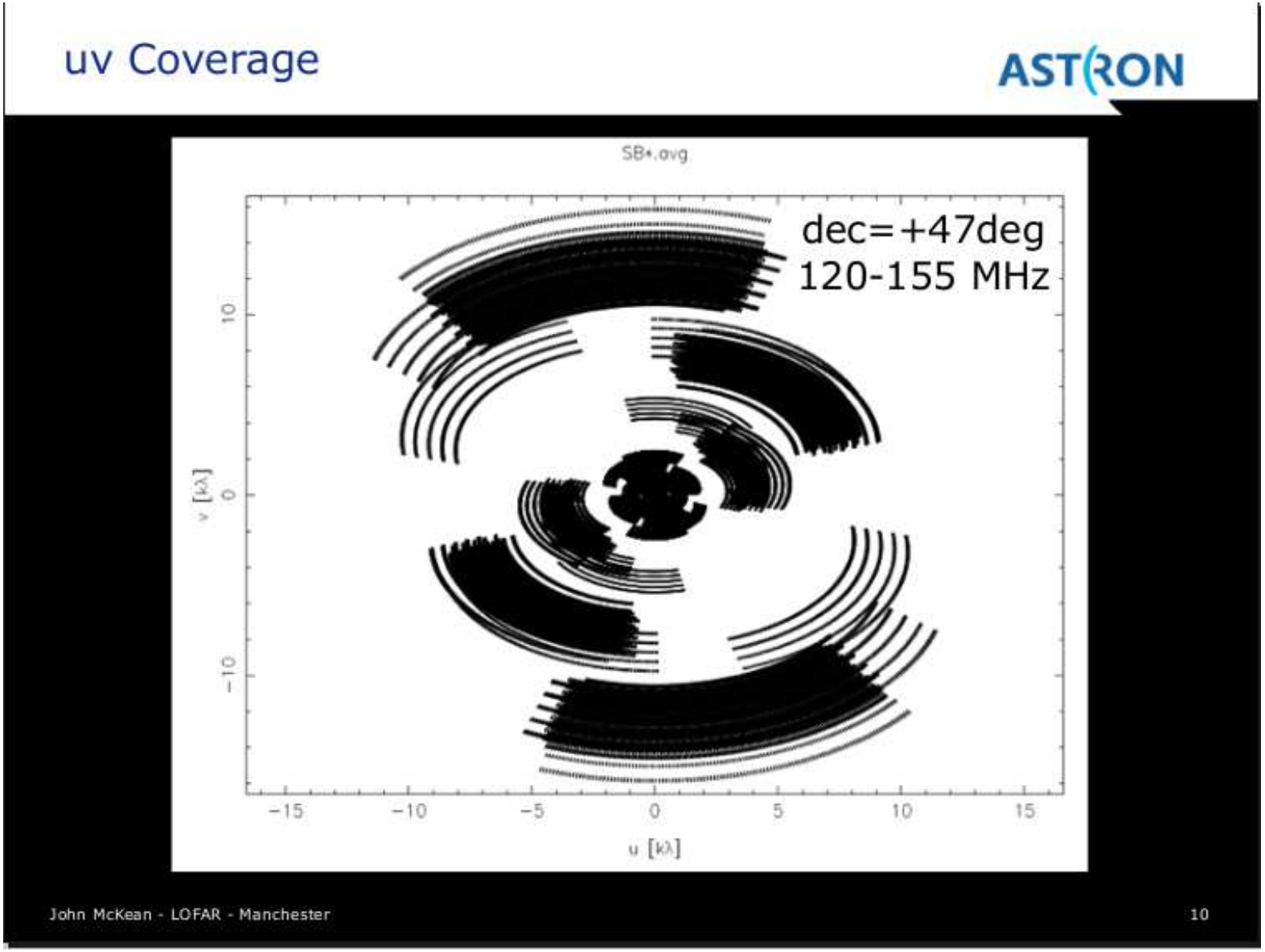
LOFAR


LOFAR
LOFAR Configuration (I)
LOFAR Configuration
Friesland
Groningen
Drenthe
ASTRON

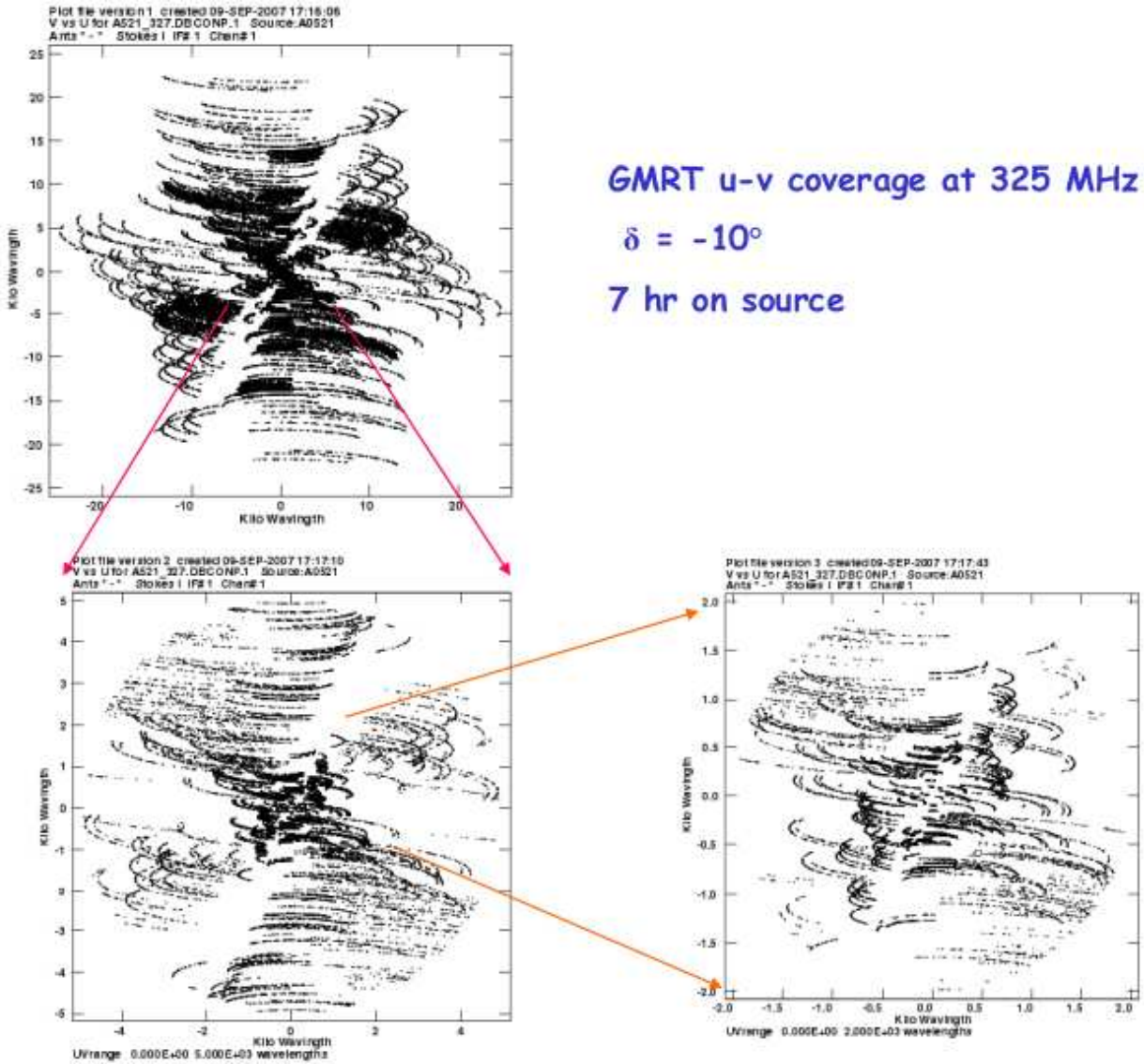
LOFAR


LOFAR
LOFAR Configuration (II)
Core Station Lay-Out
LBA Antenna Lay-Out
ASTRON

LOFAR


uv Coverage
ASTRON
SB*.avg
dec=+47deg
120-155 MHz
v [kλ]
u [kλ]
10
0
−10
−15
−10
−5
0
5
10
15
John McKean - LOFAR - Manchester
10

GMRT



GMRT u-v coverage at 325 MHz

δ = -10°

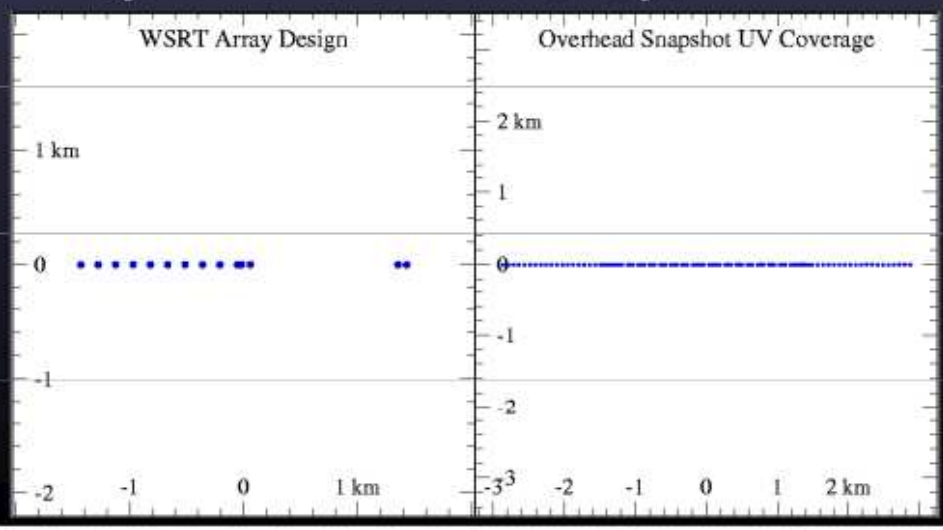
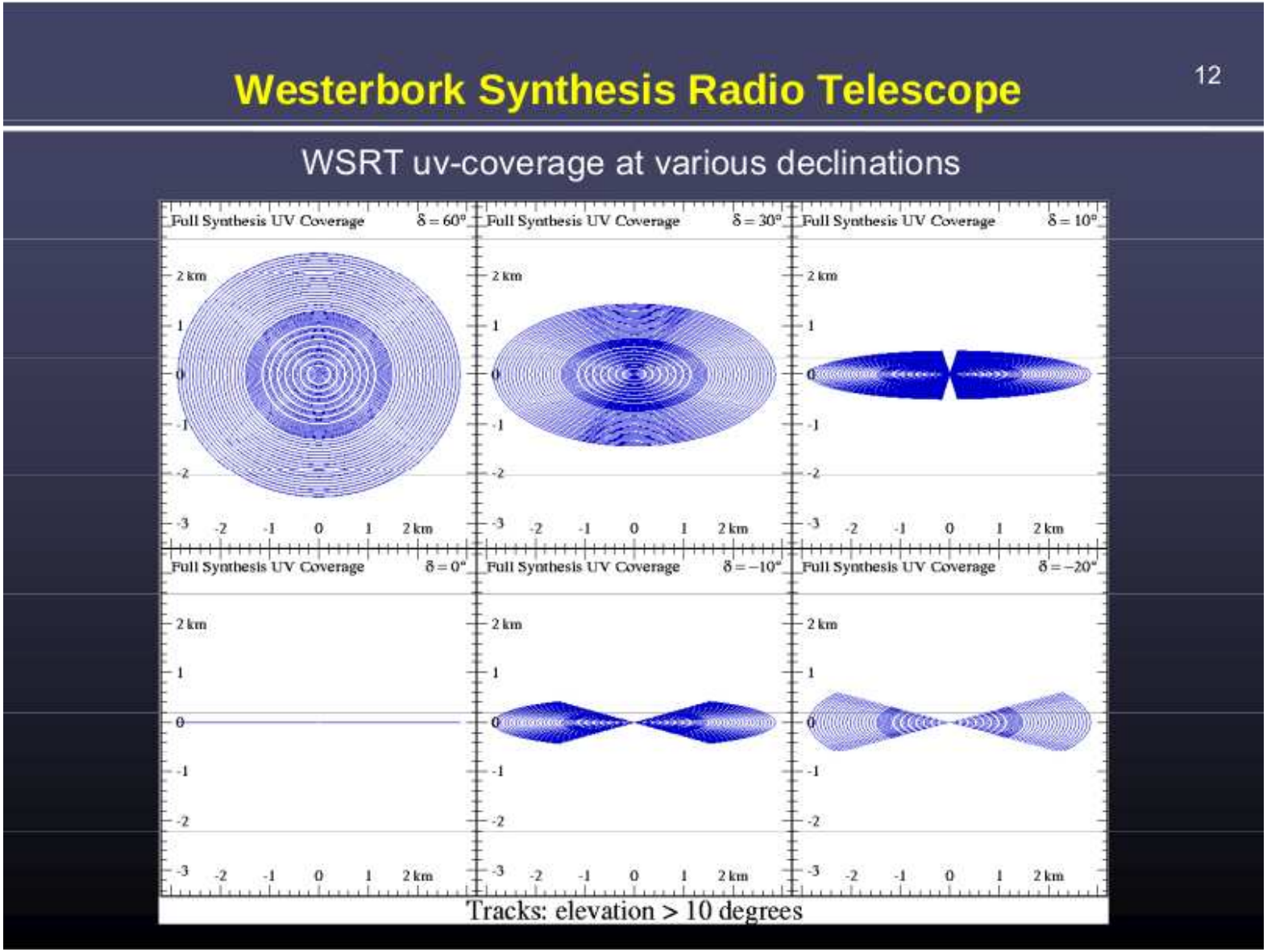
7 hr on source

Wesrterbork

Westerbork Synthesis Radio Telescope

11

- Located in Westerbork, Holland
- Has 14 antennas, 25m diameter
- East-West Array
- Requires Earth Rotation Synthesis for all imaging
- Dedicated in 1970: one of the earliest major interferometric arrays



Wesrterbork