Astronomical Techniques II
Lecture 6 - Coherence and towards a more realistic description of interferometry

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Form of the observed electric field

- $\vec{E}(\vec{R}, t)$
- $\vec{E}_\nu(\vec{R}) = \vec{E}_\nu(\vec{R})e^{-i\omega t}$
- $E_\nu(\vec{r}) = \int\int\int P_\nu(\vec{R}, \vec{r})\vec{E}_\nu(\vec{R})dx\ dy\ dz$
  where $P_\nu(\vec{R}, \vec{r})$ is the *propagator* from $\vec{R}$ to $\vec{r}$.

- Assumption 1 - No polarization (scalar field)
- Assumption 2 - Sources lie on a *Celestial sphere*
    - 3D $\rightarrow$ 2D
- Assumption 3 - There is no additional emission, absorption, scattering inside the Celestial sphere.
    - So we only have to describe the distribution of sources of electric field at this surface.

- $E_\nu(\vec{r}) = \int \mathcal{E}_\nu(\vec{R}) \, \frac{e^{\frac{2\pi i \nu |\vec{R}-\vec{r}|}{c}}}{|\vec{R}-\vec{r}|} \; \mathrm{dS}$

  where dS - surface area element on the celestial sphere.

Spatial Coherence

- $V_\nu(\vec{r_1}, \vec{r_2}) = \langle E_\nu(\vec{r_1})\ E^*_\nu(\vec{r_2})\rangle$
- $V_\nu(\vec{r_1}, \vec{r_2}) = \left\langle \int\int \mathcal{E}_\nu(\vec{R_1})\ \mathcal{E}^*_\nu(\vec{R_2})\ \frac{e^{\frac{2\pi i\nu|\vec{R_1}-\vec{r_1}|}{c}}}{|\vec{R_1}-\vec{r_1}|}\ \frac{e^{\frac{-2\pi i\nu|\vec{R_2}-\vec{r_2}|}{c}}}{|\vec{R_2}-\vec{r_2}|}\ dS_1\ dS_2 \right\rangle$
- Assumption 4 - Emission is spatially incoherent
  - $\left\langle \mathcal{E}_\nu(\vec{R_1})\ \mathcal{E}^*_\nu(\vec{R_2}) \right\rangle = 0$ for $R_1 \neq R_2$

Spatial Coherence Function

- $V_\nu(\vec{r_1}, \vec{r_2}) = \int \left\langle |\mathcal{E}_\nu(\vec{R})|^2 \right\rangle \, |\vec{R}|^2 \, \frac{e^{\frac{2\pi i \nu |\vec{R}-\vec{r_1}|}{c}}}{|\vec{R}-\vec{r_1}|} \, \frac{e^{\frac{-2\pi i \nu |\vec{R}-\vec{r_2}|}{c}}}{|\vec{R}-\vec{r_2}|} \, dS$
- $V_\nu(\vec{r_1}, \vec{r_2}) = \int \mathcal{B}_\nu(\vec{s}) \, e^{\frac{-2\pi i \nu \vec{s}.(\vec{r_1}-\vec{r_2})}{c}} \, d\Omega$

  where $\vec{s} = \frac{\vec{R}}{|\vec{R}|}$; $\quad \mathcal{B}_\nu(\vec{s}) = \left\langle |\mathcal{E}_\nu(\vec{R})|^2 \right\rangle \, |\vec{R}|^2$

  and $d\Omega = |\vec{R}|^2 \, dS$

- Also known as *Spatial Autocorrelation Function*

Fourier inversion for synthesis imaging

- $V_\nu(u, v, w) = \int \int \mathcal{B}_\nu(l, m) \, \frac{e^{-2\pi i(ul+vm+wn)}}{\sqrt{1-l^2-m^2}} \, dl \, dm$
- Components of $\vec{s}$ are $(l, m, \sqrt{1 - l^2 - m^2})$.
- To get to a proper FT relationship - get rid of *wn* term in the exponential (Assumption 5)
  - Let's confine all our measurements to preferred plane such that $\vec{r_1} - \vec{r_2} = \lambda(u, v, w = 0)$.
  - Small field-of-view - $(\sqrt{1 - l^2 - m^2})w \approx -\frac{1}{2}(l^2 + m^2)w << ul + vm$

Effect of the Antenna reception pattern

- $V_\nu(u,v,w) = \int\int \mathcal{B}_\nu(l,m)\, \mathcal{A}_\nu(l,m)\, \frac{e^{-2\pi i(ul+vm)}}{\sqrt{1-l^2-m^2}}\, dl\, dm$

Coherence: The physical picture

- Temporal Coherence
  - $\tau_c \times \Delta\nu = 1$
- Spatial Coherence
  - $u_c \times \Delta\theta = 1$

Response of an interferometer

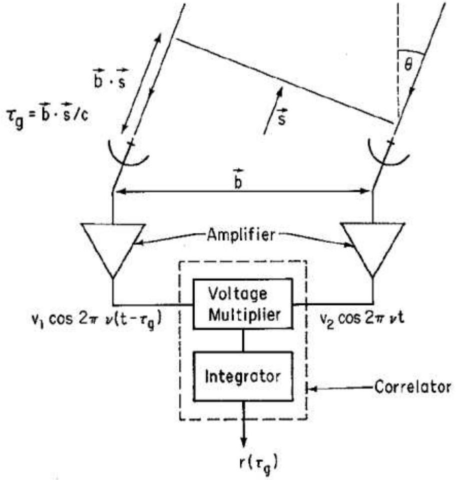

Figure 2-1. Simplified schematic diagram of a two-element interferometer.

Response of an interferometer

- Geometric delay - $\tau_g = \frac{\vec{b}.\vec{s}}{c}$
- Correlator output - $r(\tau_g) =< V_1(t)\ V_2(t) >$
- $V_1 = v_1\ cos\ 2\pi\nu(t - \tau_g)$; $V_1 = v_1\ cos\ 2\pi\nu t$;
- $r(\tau_g) = v_1 v_2\ cos\ 2\pi\nu\tau_g$

Response to a Brightness distribution

- $dr = A(\vec{s})\ B(\vec{s})\ \Delta\nu\ \Delta\Omega\ cos\ 2\pi\nu\tau_g$
- $r(\tau_g) = \int_\Omega A(\vec{s})\ B(\vec{s})\ \Delta\nu\ cos\ 2\pi\nu\tau_g\ d\Omega$
- $r(\tau_g) = \Delta\nu\ \int_\Omega A(\vec{s})\ B(\vec{s})\ cos\ \frac{2\pi\nu\vec{b}\cdot\vec{s}}{c}\ d\Omega$

Phase Tracking Center

- $\vec{s} = \vec{s_0} + \vec{\sigma}$

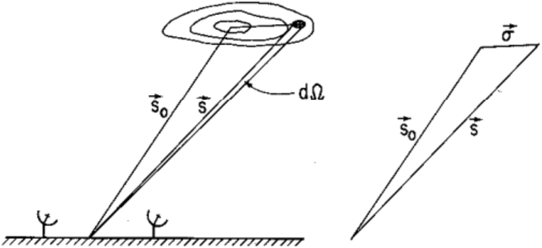

Figure 2-2. Position vectors used in deriving the interferometer response to a source. The source is represented by the contours of radio brightness $I(\mathbf{s})$ on the sky.

- $r(\sigma) = \Delta\nu \int_{\Omega} A(\vec{s})\ B(\vec{s})\ cos\ \frac{2\pi\nu\vec{b}\cdot(\vec{s_0}+\vec{\sigma})}{c}\ d\Omega$
- ...

Visibility

- $V = |V| \, e^{i\phi_V} = \int_\Omega A_N(\vec{\sigma}) \, B(\vec{\sigma}) \, e^{-2\pi i \, \nu \, \vec{b}.\vec{\sigma}/c} \, d\Omega$
- $A_N(\vec{\sigma}) = A(\vec{\sigma})/A_0$
- ...
- $r = A_0 \, \Delta\nu \, |V| \, cos \, (2\pi\nu \frac{\vec{b}.\vec{\sigma}}{c} - \phi_V)$

Effect of bandwidth

- $dr = A_0 \, |V| \, cos(2\pi\nu\tau_g - \phi_V) \, d\nu$
- $r = A_0 \, |V| \, \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} cos(2\pi\nu\tau_g - \phi_V) \, d\nu$
- $r = A_0 \, |V| \, \frac{sin \, \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \, cos(2\pi\nu_0\tau_g - \phi_V)$

Misc.

- Delay tracking - automated compensation for τ_g
- Frequency Conversion (mixing) - bringing the signal to an easier to handle (lower) frequency
- Complex Correlator

References

- Chap. 1 and 2, Synthesis Imaging in Radio Astronomy, ASPC Conf. Series Vol 6
- Chap. 2 and 4, Low Frequency Radio Astronomy
- Chap. 2 and 3, Interferometry and Synthesis in Radio Astronomy