Astronomical Techniques II
Lecture 3 - Noise, Temperature and SNR

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Airy Disc vs Beam Shape

- Airy Disc
  - Gives the Point Spread Function (PSF) for an imaging device
  - Independent of the Field-of-View (FoV), which is defined by other aspects (F ratio, magnification)
- Beam Shape
  - Defines the FoV
  - PSF for a non-imaging device
- In *Synthesis Imaging*, the analog of Airy disc is *synthesised beam*, which we will encounter later in this course.

Recap

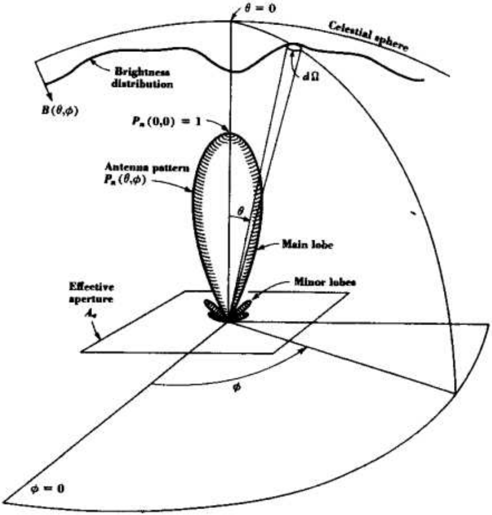


Fig. 3-2. Relation of antenna pattern to celestial sphere with associated coordinates.

Recap

$$W = \int_{\nu} \int_{aperture} \int_{\Omega} B(\theta, \phi, \nu) \;\; cos\theta dA \;\; d\Omega \;\; d\nu \quad W$$

$$w_{\nu} = \int_{aperture} \int_{\Omega} B(\theta, \phi, \nu) \;\; cos\theta dA \;\; d\Omega \quad W \;\; Hz^{-1}$$

$$w_{\nu} = \frac{1}{2} A_{eff} \int_{\Omega} B(\theta, \phi, \nu) \;\; P_n(\theta, \phi, \nu) \;\; d\Omega \quad W \;\; Hz^{-1}$$

For a uniform source of Brightness B_u, this becomes
$w_{\nu} = \frac{1}{2} \; A_{eff} \; B_u \; \Omega_A \quad W \;\; Hz^{-1}$

A question

Consider the following artificial scenario - a telescope has a beam width of 1° and uniform sidelobes -40 dB below the peak of the main lobe for the 2π sr centered on the main lobe and 0 in the remainin 2π. Assume the sky Brightness to be a constant all over the sky and the main lobe response to be a constant across the entire mainlobe.

- What fraction of the total power picked up by such a dish comes from the sidelobes.

Submit your answer in the next class!

Spectral Power - Convolution form

$$w_\nu = \frac{1}{2} A_{eff} \int_\Omega B(\theta, \phi, \nu) \;\; P_n(\theta, \phi, \nu) \;\; Sin\theta \;\; d\theta \;\; d\phi; \;\; W \; Hz^{-1}$$

$$w_\nu = \frac{1}{2} A_{eff} \int_\Omega B(\theta, \phi, \nu) \;\; P_n(\theta - \theta_0, \phi - \phi_0, \nu) \;\; d\Omega \quad W \; Hz^{-1}$$

Cross-correlation form:

$$w_\nu = \frac{1}{2} A_{eff} \int_\Omega B(\Omega, \nu) \;\; P_n(\Omega - \Omega_0, \nu) \;\; d\Omega \quad W \; Hz^{-1}$$

Convolution form:

$$w_\nu = \frac{1}{2} A_{eff} \int_\Omega B(\Omega, \nu) \;\; \tilde{P}_n(\Omega_0 - \Omega, \nu) \;\; d\Omega \quad W \; Hz^{-1}$$

Compact and extended sources

- The telescope measures an integral over the entire beam

$$S_\nu = \int_{Beam} B(\Omega,\nu)\ \tilde{P}_n(\Omega_0 - \Omega,\nu)\ \ d\Omega \quad W\ m^{-2}\ Hz^{-1}$$

- Compact - much smaller than the main lobe
  - Assuming there is no other source in the beam, the S_ν equals the spectral flux density of the source
- Extended - comparable or larger than the main lobe
  - The measured S_ν underestimates the true spectral flux density of the source.
  - Correct for $\tilde{P}_n$
  - Use multiple pointings if needed

Convolution implies *smoothing*

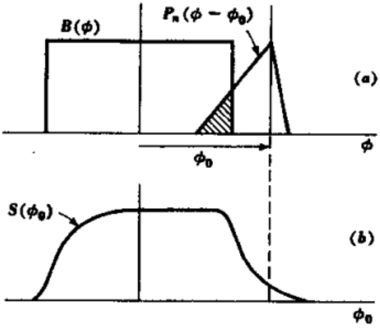


Fig. 3-7. Example of a uniform source distribution scanned by an antenna with an asymmetric pattern of triangular shape.

What will the sidelobes do?

A Blackbody and Planck's Law

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \;\; W\ m^{-2}\ sr^{-1}\ Hz^{-1}$$

$$B_\lambda = \frac{2hc^3}{\lambda^5}\frac{1}{e^{hc/kT\lambda}-1} \;\; W\ m^{-2}\ sr^{-1}\ m^{-1}$$

Planck's Law

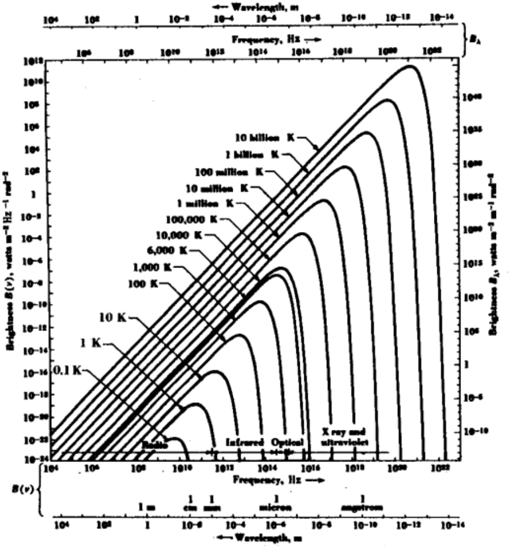


Fig. 3-14. Planck-radiation-law curves with frequency increasing to the right.

Planck's and Rayleigh-Jeans Law

$$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$B_\nu = \frac{2kT\nu^2}{c^2} \frac{\frac{h\nu}{kT}}{e^{h\nu/kT} - 1}$$

Limit $h\nu << kT$
Rayleigh-Jeans Law

$$B_\nu = \frac{2kT}{\lambda^2} \;\; W \; m^{-2} \; sr^{-1} \; Hz^{-1}$$

Power received at a detector

$$dW = B(\theta, \phi, \nu) \;\; cos\theta dA \;\; d\Omega \;\; d\nu$$

dW - W
$B(\theta, \phi) - W \; m^{-2} \; sr^{-1} \; Hz^{-1}$

Practical quantitative definition

$$B(\theta, \phi, \nu) = \frac{dW}{d\Omega \; cos\theta dA \;\; d\nu} = \frac{2kT}{\lambda^2}$$

- Intrinsic property of the source
- Independent of the distance from the source (ONLY for a resolved object)
- Can be thought of as energy *received* at the detector OR as energy *emitted* by the source.

Spectral flux density and Temperature

$$S = \frac{2kT_a\Omega_s}{\lambda^2}$$

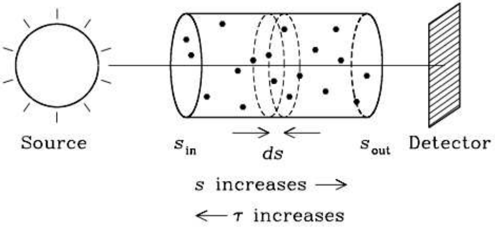
$$S_{True} = \frac{2k}{\lambda^2}\int_{\Omega_s} T(\Omega)\, d\Omega$$

$$S_{Measured} = \frac{2k}{\lambda^2}\int_{\Omega_{beam}} T(\Omega)\, \tilde{P}_n(\Omega_0 - \Omega)\, d\Omega$$

Brightness Temperature (T_B)

- Associates a unique temperature with the power received at any given frequency, or the Brightness of a source.
- A property of the source.
- Represents the temperature of a Blackbody which would have given out exactly as much radiation at that frequency
- For thermal radiation from an optically thick source - same as the physical temperature of the body emitting the radiation.
- For non-thermal radiation - eq. radiation temperature

Optical Depth and Radiative Transfer



$$T_{Observed} = T_{Source} e^{-\tau_M} + T_{Medium}(1 - e^{-\tau_M})$$

- $\tau_M = 0 \; ; >> 1 \; ; \sim 1$
- $T_{Source} = T_{Medium}$

Temperature and Noise

Spectral power density measured per unit bandwidth at the terminals of a resistance R at temperature T (Nyquist, 1928)
$w = kT \;\; W \; Hz^{-1}$

What does the spectrum of noise power look like?

Antenna Temperature

The temperature of antenna radiation resistance

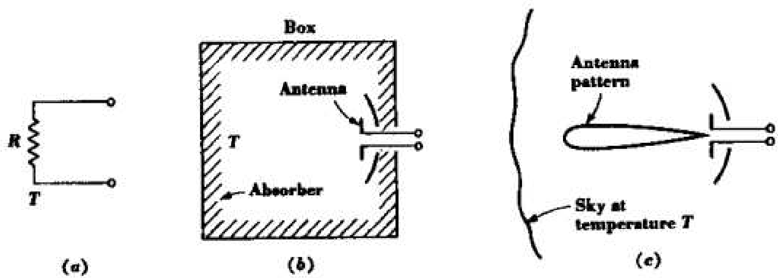


Fig. 3-24. (a) Resistor at temperature T; (b) antenna in an absorbing box at temperature T; and (c) antenna observing sky of temperature T. The same noise power is available at the terminals in all three cases.

Antenna Temperature (T_A)

load $\rightarrow$ lossless antenna of radiation resistance R, the impedence as seen at the terminals is unchanged.
The noise spectral power received by the antenna

$$w = \frac{1}{2}\ A_{eff} \int_{\Omega} B(\Omega)\ \tilde{P}_n(\Omega_0 - \Omega)\ d\Omega = kT_A$$

$$w = \frac{k\ A_{eff}}{\lambda^2} \int_{\Omega} T(\Omega)\ \tilde{P}_n(\Omega_0 - \Omega)\ d\Omega\ \ W\ Hz^{-1}$$

$$w = \frac{kA_{eff}}{\lambda^2}\ T\ d\Omega$$

But $\lambda^2 = A_{eff}\ d\Omega \implies w = kT \implies T_A = T$.

Antenna Temperature

$$T_A = \frac{A_{eff}}{\lambda^2} \int_\Omega T(\Omega)\ \tilde{P}_n(\Omega_0 - \Omega)\ d\Omega$$

$$T_A = \frac{1}{\Omega_A} \int_\Omega T(\Omega)\ \tilde{P}_n(\Omega_0 - \Omega)\ d\Omega$$

The compact source and extended source cases.

Noise and Signal

- Signal - T_{Ant} - what comes from the sky
- Noise - everything else
  - Receiver - T_{Rec}
  - Spillover - T_{Spill}
  - Leakage - T_{Leak}
  - Loss - T_{Loss}
  - Radio Frequency Interference (RFI)

$$T_{Sys} = T_{Ant} + T_{Rec} + T_{Spill} + T_{Leak} + T_{Loss}$$

- The *signal* has the same characteristics as *noise*
- One is looking for an increase of T_{Ant} over a background of T_{Sys}.

Minimum Detectable Signal

$$\Delta T_{min} = \frac{T_{Sys}}{\sqrt{\Delta\nu\Delta\tau}}$$

$$\Delta B_{min} = \frac{2k}{\lambda^2}\frac{T_{Sys}}{\sqrt{\Delta\nu\Delta\tau}}$$

$$\Delta S_{min} = \frac{2k}{A_{Eff}}\frac{T_{Sys}}{\sqrt{\Delta\nu\Delta\tau}}$$

- In theory, can be reduced to an arbitrarily small number by increasing the denominator.
- In practice:
  - Limited Δt - source evolution, source visibility, system stability, TAC, human effort
  - Limited $\Delta\nu$ - spectral signature (emission/absorption lines, EoR), source evolution, instrumental capability, technical challenges