

Astronomical Techniques II

Lecture 12 - Imaging

Divya Oberoi

IUCAA NCRA Graduate School

div@ncra.tifr.res.in

March-May 2015

Weighting Functions - controlling the beam shape

1 $W(u, v) = \sum_{k=1}^M R_k T_k D_k \delta(u - u_k, v - v_k)$

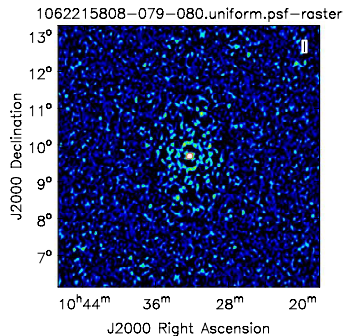
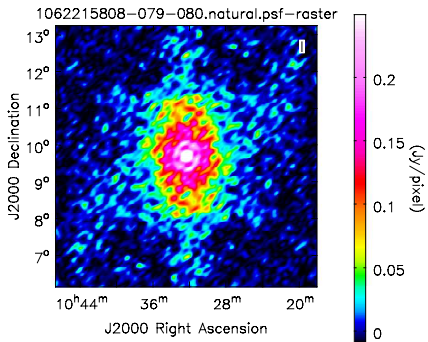
2 $V^W(u, v) = \sum_{k=1}^M R_k T_k D_k \delta(u - u_k, v - v_k) V'(u_k, v_k)$

1 R_k - Reliability $\sim \left(\frac{T_{sys}}{\sqrt{\Delta t \Delta \nu}} \right)^{-1}$

2 T_k - Tapering function

3 D_k - Density weighting function

PSF Weighting Example - MWA



1 Interpolation

2 Convolution

1 Predictable impact on the images

2 Convolve V^W with some C and then sample this convolution at centre of each cell of the *grid*

3 $C = 0$, outside some small bounded region, A_C , support size.

$$4 \quad V^R(u_c, v_c) = \sum_{k=1}^M C(u_c - u_k, v_c - v_k) V^W(u_k, v_k)$$

5 $V^R = R(C \star V^W) = R(C \star (W V^I))$, where

$$R = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - \frac{u}{\Delta u}, k - \frac{v}{\Delta v})$$

Dirty Image

1 $\tilde{I}^D = \mathcal{F}V^R$

2 $\tilde{I}^D = \mathcal{F}R \star [(\mathcal{F}C) (\mathcal{F}V^W)]$

3 $\tilde{I}^D = \mathcal{F}R \star [(\mathcal{F}C) (\mathcal{F}W \star \mathcal{F}V')]$

4 $(\mathcal{F}R)(l, m) = \Delta u \Delta v \prod (l\Delta u, m\Delta v) =$

$$\Delta u \Delta v \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - l\Delta u, k - m\Delta v)$$

1 \tilde{I}^D is periodic in l and m , with a period of $1/\Delta u$ and $1/\Delta v$, respectively

2 Aliasing, due to convolution with the resampling function, $\mathcal{F}R$

Dirty Image

- 1 FFT generates one period of \tilde{I}^D
- 2 To image $N_l \Delta\theta_l$ rad, grid spacing should satisfy $N_l \Delta u = \frac{1}{\Delta\theta_l}$
- 3 $N_l \times N_m$ FFT yields a discretely sampled version of \tilde{I}^D .
- 4 Primary field of view - $|l| < N_l \Delta\theta_l/2$; $|m| < N_m \Delta\theta_m/2$
- 5 $c = \mathcal{FC}$
- 6 $\tilde{I}_c^D(l, m) = \frac{\tilde{I}^D(l, m)}{c(l, m)}$ - Corrected Dirty Image
- 7 $\tilde{B}_c^D(l, m) = \frac{\tilde{B}^D(l, m)}{c(l, m)}$ - Corrected Dirty Beam (PSF)

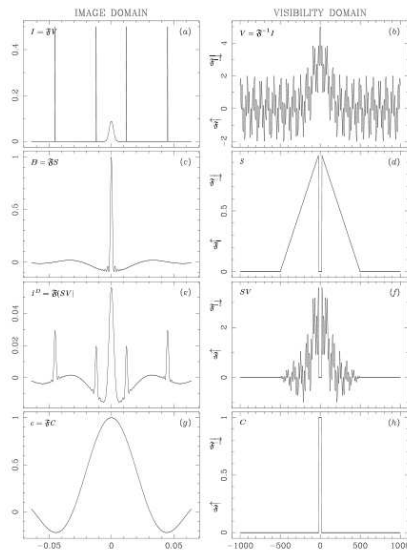
Choice of Gridding Convolution function

1 $C(u, v) = C_1(u)C_2(v)$

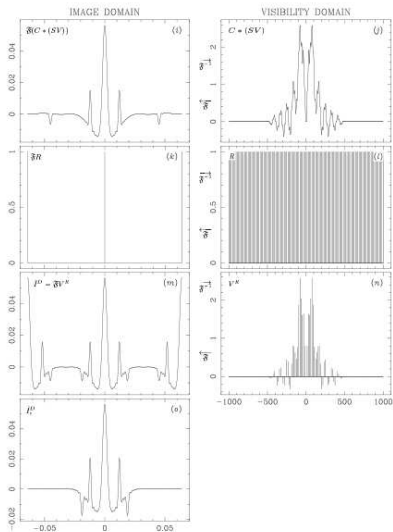
2 Chosen on the basis of energy concentration ratio $\frac{\int_P |c(l)|^2 dl}{\int_{-\infty}^{\infty} |c(l)|^2 dl}$

Prolate spheroidal wave function

Imaging Process



Imaging Process



Deconvolution

- 1 $V'(u, v) = \int \int I(l, m) e^{-2\pi i(ul+vm)} dl dm$
- 2 Direct inversion not possible
- 3 Model with a finite number of parameters
- 4 $\hat{I}(p\Delta l, q\Delta m)$
- 5 $\hat{V}(u, v) = \sum_{p=1}^{N_l} \sum_{q=1}^{N_m} \hat{I}(p\Delta l, q\Delta m) e^{-2\pi i(pu\Delta l + qv\Delta m)}$

1 Range of features which can be captured by the data

1 $\mathcal{O}(1/\max(u, v))$

2 $\mathcal{O}(1/\min(u, v))$

2 Choice of Δl , Δm and N_l , N_m , must allow these scales to be represented

1 $\Delta l \leq \frac{1}{2u_{\max}}$; $\Delta m \leq \frac{1}{2v_{\max}}$

2 $N_l \Delta l \geq \frac{1}{u_{\min}}$; $N_m \Delta m \geq \frac{1}{v_{\min}}$

3 Degrees of Freedom - $N_l \times N_m$

1 $V(u_i, v_i) = \hat{V}(u_i, v_i) + \epsilon(u_i, v_i)$

2 $V(u, v) = W(u, v) \left(\hat{V}(u, v) + \epsilon(u, v) \right)$

3 $W(u, v) = \sum_i W_i \delta(u - u_i, v - v_i)$

4 $I_{p,q}^D = \sum_{p',q'} B_{p-p',q-q'} \hat{I}_{p',q'} + E_{p,q}$ where

$$I_{p,q}^D = \sum_i W(u_i, v_i) \operatorname{Re} \left(V(u_i, v_i) e^{2\pi i (pu_i \Delta l + qv_i \Delta m)} \right) \text{ and}$$

$$B_{p,q} = \sum_i W(u_i, v_i) \operatorname{Re} \left(e^{2\pi i (pu_i \Delta l + 1v_i \Delta m)} \right)$$

Principal Solution and Invisible Distributions

- 1 If some spatial frequencies allowed in the model are not present in the data, changing their amplitudes in the model will have no effect on the fit to the data
- 2 Z - the invisible intensity distribution, then $B \star Z = 0$
- 3 If I is a solution to the convolution eqn, $I + \alpha Z$ is also a solution
- 4 The solution which has 0 amplitude at all unsampled spatial frequencies - principal solution
- 5 The problem of imaging - principal solution + a plausible invisible distribution

The need for *a-priori* information

1 Limitations of the Principal solution

- 1 Changes with data available
- 2 Sidelobes of order 0.1-10%
- 3 Is it a point source or is it a source shaped like the dirty beam

2 *A-priori* information

- 1 Positivity (Stokes I must be positive)
- 2 Nature of sources (do not have sidelobes extending to infinity)
- 3 Information of the PSF

The CLEAN Algorithm

- 1 Represent the sky as a collection of point sources in an otherwise empty field of view
- 2 Iterative procedure to find the positions and strengths of these point sources
- 3 Deconvolved image - Superposition of all point sources found convolved with a CLEAN beam and the residual noise

The Hogbom Clean (1974)

- 1 Find the location and the strength of the brightest point in I^D - S_i at (l_i, m_i) and add it to the accumulated point source model $\hat{I}_{p,q}$.
- 2 $I^D - (B^D(l - l_i, m - m_i) \times S_i \times \gamma)$, where $\gamma \leq 1$, usually 0.1
- 3 Iterate till remaining peaks are below some user specified threshold
- 4 Convolve $\hat{I}_{p,q}$ with a *restoring beam* - an idealised beam, usually an elliptical Gaussian fit to the central part of the B^D
- 5 Add the residuals to the restored image - *CLEAN image*

The Clark Clean (1980)

- 1** CLEAN involves a lot of shifting, scaling and convolutions
- 2** Minor cycle
 - 1** Choose a beam patch (include highest exterior sidelobes)
 - 2** Select bright points from I^D as before
 - 3** Perform *Hogbom* clean using the beam patch and the selected point sources
- 3** Major cycle
 - 1** Point source model built up in the minor cycle is FFTed, weighted and sampled appropriately and FFTed back to the image domain. This is subtracted from the I^D .
 - 2** Errors introduced due to the use of the beam patch in the minor cycles are corrected at the major cycle stage

The Cotton-Schwab Clean (1984)

- 1 The major cycle is performed on *ungridded* visibilities
 - 1 Avoids aliasing and gridding errors
- 1 Some miscellaneous comments about Clean
 - 1 Use of clean *boxes*
 - 2 No. of iterations vs loop gain (γ)
 - 3 The problem of short spacings
 - 4 The choice of restoring beam
 - 5 Clean instabilities
 - 6 Multi-resolution clean
 - 7 Sources lying on pixel boundaries