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# A model of the warped discs in the Galactic centre

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**Abstract.** Young stars in the Galactic centre appear to be organized in two counter–rotating systems orbiting the central black hole. One of these is a strongly warped disc, whereas the other is thicker and highly inclined with respect to the former. We present a minimal model of this gravitationally coupled stellar systems.

Keywords : Galaxy: centre - Galaxy: nucleus - stellar dynamics

## 1. Introduction

It is believed that the Galactic centre has a massive black hole with dense clusters of stars orbiting it (Genzel, Eisenhauer & Gillessen 2010). Among these, there is a population of about 200 young stars (mostly O supergiants and Wolf–Rayet stars), with masses exceeding  $20 M_{\odot}$ , all probably born  $6 \pm 2$  Myr ago (Paumard et al. 2006). Proper motion and radial velocities are available for most of the stars (Bartko et al. 2009). About half the stars between about 0.04 pc and 0.4 pc seem to belong to a rotating disc (the "clockwise system"), which is probably highly warped (Levin & Beloborodov 2003; Lu et al. 2006, 2009). The remaining young stars appear to be members of a counter–rotating population (the "counter–clockwise system") which is thicker, and inclined by about 110° to the clockwise system (Genzel et al. 2003; Paumard et al. 2006; Bartko et al. 2009, 2010). Different types of models for the young stars have been suggested (Nayakshin et al. 2006; Yu, Lu & Lin 2007; Perets et al. 2009; Löckmann, Baumgardt & Kroupa 2009; Šubr, Schovancová & Kroupa 2009; Kocsis & Tremaine 2011). In this work, we present a minimal model of the two gravitationally coupled, counter–rotating systems.

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### 2. A minimal model

It is convenient to use cylindrical polar coordinates  $(R, \varphi, z)$  centered on the black hole and the spherical star cluster. The unperturbed "clockwise system" (henceforth referred to as the "disc") is modeled as a razor-thin circular disc of stars, with centre at R = 0, and disc normal along  $\hat{z}$ . The equatorial plane of the molecular torus is the z = 0 plane, and its symmetry axis is along  $\hat{z}$ . Stars in the disc move on circular orbits under the combined gravitational influence of the central black hole, the spherical star cluster, and the molecular torus. The "anti-clockwise system" (henceforth referred to as the "ring") is modeled as a single circular ring, with centre at R = 0, that is inclined to the plane of the unperturbed disc. The ring precesses due to the gravitational torques exerted on it by the disc and the molecular torus. This "unperturbed" configuration of the coupled system is shown schematically in Fig. (1a) — the black hole and the torus are not displayed.

The disc deforms in response to the gravity of the inclined, precessing ring. Our interest is in the warping of the disc, and we imagine that a disc particle bobs up and down while executing steady circular motion in the plane. The deformed disc can then be described by a height function,  $Z(R, \varphi, t)$ . For small oscillations Z obeys

$$\frac{D^2 Z}{Dt^2} \equiv \left(\frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \varphi}\right)^2 Z = -v^2 Z + F(R,\varphi,t), \qquad (1)$$

where we have dropped the vertical force due to the self–gravity of the deformed disc. *F* is the vertical force (per unit mass) due to the precessing ring.  $\Omega(R)$  and  $\nu(R)$  are the circular and vertical frequencies due to the black hole, the spherical star cluster and the torus:  $\Omega^2 = \Omega_{BH}^2 + \Omega_{SC}^2 + \Omega_{MT}^2$ , and  $\nu^2 = \nu_{BH}^2 + \nu_{SC}^2 + \nu_{MT}^2$ . Since the equation for *Z* is linear and the forcing is periodic in time, we seek Fourier amplitudes defined by,  $Z(R, \varphi, t) = \text{Re}\{\sum_m \tilde{Z}_m(R) e^{i[m(\varphi - \omega_p t)]}\}$  and  $F(R, \varphi, t) = \text{Re}\{\sum_m \tilde{F}_m(R) e^{i[m(\varphi - \omega_p t)]}\}$ . Then Eqn. (1) gives,

$$\left[\nu(R)^2 - m^2(\Omega(R) - \omega_p)^2\right] \tilde{Z}_m(R) = \tilde{F}_m(R).$$
<sup>(2)</sup>

The parameters appropriate to the Galactic centre are as follows (Genzel et al. 2010; Kocsis & Tremaine 2011). The mass of the central black hole is assumed to be  $M_{\rm BH} = 4 \times 10^6 \,\rm M_{\odot}$ . The density of the spherical star cluster (SC) density is taken as

$$\rho_{\rm SC}(R) = 10^6 \left(\frac{0.4\,{\rm pc}}{R}\right)^{3/2} \,{\rm M}_\odot\,{\rm pc}^{-3}\,. \tag{3}$$

We model the disc surface density as a single power law between an inner radius  $R_{in} = 0.04 \text{ pc}$ , and an outer radius  $R_{out} = 0.4 \text{ pc}$ :

$$\Sigma_{\rm disc}(R) = 4.4 \times 10^3 \left(\frac{0.4 \,\mathrm{pc}}{R}\right)^{3/2} \,\mathrm{M}_{\odot} \,\mathrm{pc}^{-2} \,.$$
 (4)



**Figure 1.** (a) Unperturbed configuration showing flat disc plus counter–rotating ring. (b) Warped disc

The perturbing ring has mass  $M_{\text{ring}} = 4 \times 10^3 \,\text{M}_{\odot}$ , and radius  $a = 0.6 \,\text{pc}$ . The mass of the molecular torus is assumed to be  $M_{\text{MT}} = 10^6 \,\text{M}_{\odot}$ . For the purpose of computing gravitational forces, we approximate it as a thin ring of radius  $R_{\text{MT}} = 1.6 \,\text{pc}$ . Note that the disc and the ring lie well inside the torus. The material in the disc and the ring rotate in such a way that the angle between their angular momenta is  $I = 110^{\circ}$ . The gravitational torques on the ring, due to the combined action of the unperturbed disc and torus, cause the ring to precess around the vertical axis of the disc, with frequency

$$\omega_p \approx -\frac{3\pi}{2} \frac{\Omega_{\text{ring}} \cos(I)}{a^2 M_{\text{BH}}} \int_{R_{\text{in}}}^{R_{\text{out}}} \Sigma_{\text{disc}}(R) R^3 dR - \frac{3}{4} \Omega_{\text{ring}} \frac{M_{\text{MT}}}{M_{\text{BH}}} \frac{a^3}{R_{\text{MT}}^3} \cos(I) \,.$$
(5)

Evaluating various quantities occurring in Eqn. (2), we note that (i)  $\tilde{F}_1(R) \gg \tilde{F}_m(R)$  for  $m \neq 1$ , so we need focus only on the m = 1 mode; (ii) The gravitational potentials of the black hole and star cluster are spherically symmetric:  $v_{\rm BH} = \Omega_{\rm BH}$  and  $v_{\rm SC} = \Omega_{\rm SC}$ . Therefore  $v^2 - \Omega^2 = v_{\rm MT}^2 - \Omega_{\rm MT}^2$ . Note that, because the disc lies inside the molecular torus, the quantity  $\Omega_{\rm MT}^2$  is negative; (iii) It turns out that  $\omega_p \approx 2 \times 10^{-3} \Omega_{\rm out}$ , where  $\Omega_{\rm out}$  is the circular frequency at the outer edge of the disc. Since  $\omega_p$  is much smaller than  $\Omega$ , we can drop  $\omega_p^2$  when compared with  $2\omega_p\Omega$ . Then Eqn. (2) simplifies to

$$\tilde{Z}_1(R) \simeq \frac{\tilde{F}_1(R)}{v_{\rm MT}^2(R) - \Omega_{\rm MT}^2(R) + 2\omega_p \Omega(R)}.$$
(6)

In Fig. (1b), where we show a surface plot of the warped disc, the line to the ascending node points toward the negative Y axis; we find that the disc is warped in the sense suggested by the gravitational pull due to the ring.

#### 3. Conclusions

We have presented a minimal model of the counter-rotating warped discs at the Galactic centre, the main virtue of which is its simplicity. Some of the limitations of our model are the following. Whereas the warp amplitude is large, our formalism is valid only in the linear regime; moreover, the vertical force of self-gravity due to the warped disc has been ignored. The main testable consequence of our model is the relative alignment of the "warp" and the "ring". Preliminary investigations show that self-gravity retains this alignment; however, the sign of the vertical displacement of the warp is sensitive to the parameters of the model.

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