



Astrophysical magnetohydrodynamics

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Abstract. Over the course of roughly a decade, from the late 1950s through the early 1960s, Chandrasekhar made fundamental contributions to basic plasma physics, and the effect of magnetic fields on the dynamics of astrophysical plasmas. This paper reviews recent progress and outstanding problems in *Astrophysical magnetohydrodynamics*, the application of MHD to astrophysical systems, with particular emphasis on the role of Chandra's early contributions to the field. Specific topics discussed include magnetic field amplification by dynamo processes inside stars, the magnetorotational instability and angular momentum transport in accretion disks, MHD turbulence in the interstellar medium of galaxies, and kinetic MHD effects in weakly collisional plasmas. Chandra's contributions in all of these areas endure.

Keywords : MHD – turbulence – accretion disks

1. Introduction

It was a great honour and privilege to speak at the Chandrasekhar Centennial Symposium on the topic of 'Astrophysical Magnetohydrodynamics', especially since there were so many eminent members of the audience whom I would have liked to hear speak on the same topic! The goals of my talk were to provide a summary of recent progress in magnetohydrodynamics (MHD) as applied to a wide variety of astrophysical systems, and to highlight Chandra's early contributions to these topics. The goals of this paper are the same.

Unfortunately, by the time I was a graduate student in the late 1980s, Chandra was no longer working on plasma physics, and therefore I never had the opportunity to meet him personally. However, he still had an enormous impact on me, as on most graduate students, through his books.

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In particular, his books on radiative transfer (Chandrasekhar 1950), hydrodynamic and hydro-magnetic stability (Chandrasekhar 1961), and ellipsoidal figures of equilibrium (Chandrasekhar 1969), all still available as Dover reprints, are as relevant today as they were back then.

Throughout the 1950s and 1960s, Chandra wrote many papers on MHD and plasma physics, following four general themes:

1. the statistical properties of turbulence,
2. problems in astrophysical MHD,
3. basic plasma physics, and
4. hydrodynamic and MHD instabilities.

Chandrasekhar (1989a,b), volumes 3 and 4 of his selected papers, contain his most important work in these areas. Rather than highlighting individual papers or results, instead I have organized this paper around astrophysical objects of increasing scale. Thus, after a brief introduction to some general concepts in MHD, I will discuss evidence for the importance of magnetic fields first in *stars*, then in *accretion disks*, then in *galaxies*, and finally on the largest scale in *clusters of galaxies*. Each topic will be organized into a separate section.

Finally, it is useful to highlight what Chandra himself wrote about astrophysical MHD back in 1957: “It is clear we are very far from an adequate characterization of cosmic magnetic fields” (Chandrasekhar 1957). Obviously we have come very far since 1957, but in some cases it is clear we still have very far to go.

2. Some elementary MHD

Before discussing results, it is worthwhile to summarize some basic physics of MHD. In a highly collisional plasma with perfect conductivity, the equations of motion are essentially the Euler equations of gas dynamics, supplemented with Maxwell’s equations to describe the evolution of the magnetic field (in particular, Faraday’s Law). The result, usually referred to as the equations of ideal MHD, is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho \mathbf{v}] = 0, \quad (1)$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot [\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B} + \mathbf{P}^*] = 0, \quad (2)$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + P^*) \mathbf{v} - \mathbf{B}(\mathbf{B} \cdot \mathbf{v})] = 0, \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \quad (4)$$

where \mathbf{P}^* is a diagonal tensor with components $P^* = P + B^2/2$ (with P the gas pressure), E is the total energy density

$$E = \frac{P}{\gamma - 1} + \frac{1}{2}\rho v^2 + \frac{B^2}{2}, \quad (5)$$

and $B^2 = \mathbf{B} \cdot \mathbf{B}$. The other symbols have their usual meaning. These equations are written in units such that the magnetic permeability $\mu = 1$. An equation of state appropriate to an ideal gas, $P = (\gamma - 1)e$ (where γ is the ratio of specific heats, and e is the internal energy density), has been assumed in writing equation 5. These equations are valid only for non-relativistic flows, and for phenomena at frequencies much less than the plasma frequency. As we shall see in sect 6 there are many interesting frontiers to explore as some of the assumptions underlying the equations of ideal MHD are relaxed, for example in low collisionality plasmas.

Restricting ourselves to one dimensional flow for the moment, it is useful to rewrite the equations of motion in a compact form

$$\frac{\partial \mathbf{U}}{\partial t} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} \frac{\partial \mathbf{U}}{\partial x} \quad (6)$$

where the components of the vectors \mathbf{U} and \mathbf{F} are the conserved variables and their fluxes, respectively, that is

$$\mathbf{U} = \begin{bmatrix} \rho \\ M_x \\ M_y \\ M_z \\ E \\ B_y \\ B_z \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho v_x \\ \rho v_x^2 + P + B^2/2 - B_x^2 \\ \rho v_x v_y - B_x B_y \\ \rho v_x v_z - B_x B_z \\ (E + P^*)v_x - (\mathbf{B} \cdot \mathbf{v})B_x \\ B_y v_x - B_x v_y \\ B_z v_x - B_x v_z \end{bmatrix}, \quad (7)$$

Note that equation 6 defines a system of nonlinear hyperbolic partial differential equations (PDEs). The mathematical properties of hyperbolic PDEs are well studied. In particular, the eigenvalues of the Jacobian $\partial \mathbf{F} / \partial \mathbf{U}$ define the characteristic (wave) speeds in MHD.

In fact, probably the most important property of hyperbolic PDEs is that they admit wave-like solutions. Much of the dynamics of magnetized plasmas can be interpreted using the properties of linear and nonlinear wave solutions. The properties of linear waves can be studied using the dispersion relation, derived by looking for solutions for small amplitude disturbances of the form $\exp i(\omega t + \mathbf{k} \cdot \mathbf{x})$, where ω is the frequency and \mathbf{k} the wavevector, in a stationary, isotropic, homogeneous medium. Inserting this form for the solution into the equations of motion, and keeping only terms which are linear in the disturbance amplitude results in a system of linear equations, which have solutions only if the frequency and wavenumber are related through the following dispersion relation

$$\left[\omega^2 - (\mathbf{k} \cdot \mathbf{V}_A)^2 \right] \left[\omega^4 - \omega^2 k^2 (V_A^2 + C^2) + k^2 C^2 (\mathbf{k} \cdot \mathbf{V}_A)^2 \right] = 0 \quad (8)$$

where $\mathbf{V}_A = \mathbf{B} / \sqrt{4\pi\rho}$ is the Alfvén velocity, and $C^2 = \gamma P / \rho$ the adiabatic sound speed. The

dispersion relation has three pairs of solutions, which represent right- and left-going waves of three different families. (Note that MHD is immediately different from hydrodynamics, which has only one wave family: sound waves). The MHD wave families are the Alfvén wave (an incompressible transverse wave propagating at speed V_A), and the fast and slow magnetosonic waves (which are both compressible acoustic modes with phase velocity modified by the magnetic pressure). To complicate matters even more, the phase velocity for each mode depends on the angle between the wavevector and the magnetic field, as well as the strength of the magnetic field as measured by the ratio V_A/C . The angular dependence is most easily demonstrated using Friedrichs diagrams, which plot the relative phase velocity of each mode versus the angle between \mathbf{k} and \mathbf{B} in a polar diagram (see section 14.1 in Sturrock 1994 for an example). Such plots clearly demonstrate important properties of MHD waves, for example for directions parallel to the magnetic field, the Alfvén wave has the same phase velocity as either the fast or the slow magnetosonic wave (which one depends on whether the Alfvén speed is faster or slower than the sound speed). In this case, the modes are degenerate. Mathematically, this reflects the fact that the equations of MHD are not *strictly hyperbolic*, since in some circumstances the eigenvalues of the Jacobian are degenerate. This fact makes finding solutions to the equations of ideal MHD even more complicated.

Another important property of MHD waves in comparison to hydrodynamics is that, because they involve transverse motions, Alfvén waves can be polarized. The sum of two linear polarizations with different phase shifts can lead to circularly polarized Alfvén waves. This means in MHD, all three components of velocity must be kept, even in one dimensional flows, in order to represent all polarizations. Moreover, in non-ideal MHD the left- and right-circularly polarized Alfvén waves can have different phase velocities, these are the whistler waves in the Hall MHD regime (where ions and electrons can drift due to collisions with neutrals). Again, this new behavior is a direct consequence of the complexity of MHD waves, and it is fair to say that the rich dynamics of MHD results in part from this complexity.

Finding analytic solutions to the equations of MHD, beyond those representing linear waves, is very difficult. Usually, very restrictive assumptions are required, such as steady (so that $\partial/\partial t = 0$), and/or one dimensional flow. Today, the most important tools for solving the MHD equations are numerical methods. Grid based methods for MHD are now quite mature, and a variety of public codes are available to study MHD flows in fully three-dimensions, including a rich set of physics beyond ideal MHD. Most grid based methods for MHD adopt the same approach: the conserved variables are discretized on a grid, with volume averaged values stored at cell centers. In order to enforce the divergence-free constraint on the magnetic field, it is better to store area averages of each component of the magnetic field at corresponding cell faces, and evolve these components using electric fields at cell edges, using a technique called “constrained transport”. Figure 1 shows the basic discretization of the variables.

One example of a publicly-available grid code for MHD is Athena (Stone et al. 2008), available at <https://trac.princeton.edu/Athena>. Athena implements a higher-order Godunov scheme based on directionally unsplit integrators, piecewise-parabolic reconstruction, and constrained transport, with a variety of Riemann solvers available to compute the fluxes. With this

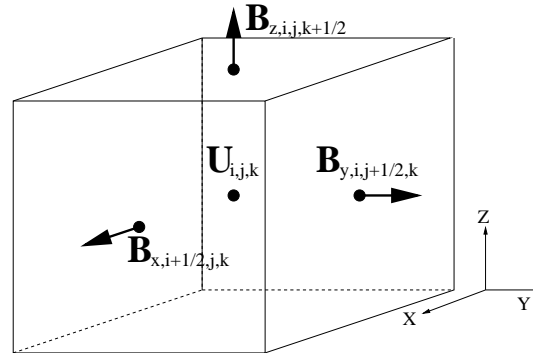


Figure 1. Basic centering of variables for a grid-based numerical method for MHD using constrained transport. Volume averages of conserved variables are stored at cell centers, while area averages of each component of the magnetic field are stored at cell faces.

approach, mass, momentum, energy, and magnetic flux are all conserved to machine precision. Of course, there are many other codes available which implement different algorithms than those used in Athena, and this is a very good thing, because by comparing solutions to the same problem generated by different algorithms, we can gauge whether those solutions are reliable. Throughout the rest of this paper, I will discuss solutions to MHD problems generated by Athena and other codes.

3. Solar magnetoconvection

The best evidence of the importance of magnetic fields to the dynamics of astrophysical plasmas comes from observations of the outer layers of the Sun. Both the presence of sunspots in the photosphere, and structures such as filaments, prominences, and flares in the solar corona, demonstrate the key role that magnetic fields play in shaping the dynamics. In fact, the very existence of the hot corona is now interpreted as due to heating by MHD effects. Beautiful images and animations that show magnetic fields in action in the solar corona have been obtained by recent spacecraft missions such as SOHO, TRACE, Yokoh, Hinode, and SDO.

It is thought that most of the magnetic activity of the Sun is driven by the combination of rotation and turbulent flows in the convection zone. In fact, the properties of MHD turbulence driven by convection was one of the problems that first interested Chandra in plasma physics (for examples, see papers in Chandrasekhar 1989a).

Understanding the origin and evolution of the Sun's magnetic field via a dynamo process has been a challenging problem for many decades. In addition to generation of the dipole field due to differential rotation, a process first proposed by Parker (1955), there are also small-scale multipole fields thought to be generated by the convective turbulence that play a role in shaping sunspots

and coronal activity. Both the processes that produce sunspots, and the large-scale magnetic field of the Sun, are very active areas of research.

In the case of sunspots, direct numerical simulations of magnetoconvection in the outer layers, including realistic radiative transfer to capture the outer radiative zone, can now reproduce details of observed sunspots, including the penumbral filaments; a beautiful example is given in Rempel et al. (2009).

In the case of the solar dynamo, the dipole field is now thought to originate in the *tachocline*, a region of strong shear between the radiative core (which is in solid body rotation, according to results from helioseismology) and the outer convective zone (which is in differential rotation). However, although the sophistication of modern global MHD simulations of magnetoconvection in spherical and rotating stars is impressive, they still fail to explain both the origin of the differential rotation in the convective zone, and the origin of the cyclic dipole field. Solving the solar dynamo problem is important, as we are unlikely to understand magnetic fields in other stars if we cannot first understand the Sun.

4. The MRI in accretion disks

Moving beyond stars, the next set of astrophysical systems where magnetic fields have been identified as being important is accretion disks. Such disks are ubiquitous, occurring in protostellar systems, close binaries undergoing mass transfer, and in active galactic nuclei.

The most basic property of an accretion disk is the angular momentum transport mechanism. This mechanism controls the rate of accretion, which in turn controls the luminosity, variability, and spectrum of the disk. Mass accretion in disks is analogous to nuclear fusion in stars: it is the mechanism that powers the entire system.

It has been known for decades that kinetic viscosity in an astrophysical plasma is too small to explain the angular momentum transport and mass accretion rate, so that some form of “anomalous” viscosity is required (Shakura & Sunyaev 1973). It has also been long suspected that the transport was associated with turbulence in the disk, but disks with Keplerian rotation profiles are linearly stable according to the Rayleigh criterion, that is, so long as the specific angular momentum increases outwards. So the question becomes: what drives turbulence in disks?

The answer seems to be: magnetic fields. Remarkably, disks with Keplerian rotation profiles which contain weak magnetic fields (weak in the sense that the gas pressure is larger than the magnetic pressure) are linearly *unstable* to the magnetorotational instability (MRI), as first recognized by Balbus & Hawley (1991). The MRI can be identified by calculating the linear dispersion relation for MHD waves in a Keplerian shear flow. The simplest analysis which captures the MRI assumes incompressible axisymmetric perturbations, a purely vertical magnetic field, and ideal MHD (all of these assumptions have been relaxed in later analyses, e.g. see Balbus &

Hawley 1999 for a review). The resulting dispersion relation is

$$\omega^4 - \omega^2 \left[\kappa^2 + 2 (\mathbf{k} \cdot \mathbf{V}_A)^2 \right] + (\mathbf{k} \cdot \mathbf{V}_A)^2 \left([\mathbf{k} \cdot \mathbf{V}_A]^2 + \frac{d\Omega^2}{d \ln r} \right) = 0 \quad (9)$$

where V_A is the Alfvén speed, and

$$\kappa^2 = \frac{1}{R^3} \frac{d(R^4 \Omega^2)}{dR} \quad (10)$$

is the epicyclic frequency (R is the cylindrical radius). Note that the coefficient of the first and second terms in equation (9) are positive and negative respectively, therefore solutions with $\omega^2 < 0$ (that is, instability) are possible if the third term is negative. This occurs when

$$(\mathbf{k} \cdot \mathbf{V}_A)^2 < -\frac{d\Omega^2}{d \ln r} \quad (11)$$

Physically, this states that if the rotation frequency in the disk is decreasing outwards (as is true in Keplerian flows), then there are always sufficiently small wavenumbers that will be unstable. Note that this is in direct contradiction to the Rayleigh criterion, which requires the angular *momentum* (not frequency) decrease outward for instability. How small is “sufficiently small” for instability depends on the magnetic field strength (V_A). In practice, if the field is weak ($V_A < C$), there always are unstable modes with wavenumbers large enough that the corresponding wavelength is less than the vertical scale height (thickness) of the disk.

In fact, studies of the MRI have a long and interesting history. The MRI was first identified by Velikhov (1959) in a study motivated by a rotating plasma experiment. Chandrasekhar (1960) made important contributions, showing the instability was present in a global analysis of magnetized Couette flow. Fricke (1969) found the instability in differentially rotating stars. However, the importance of the MRI to accretion disks was not recognized by any of these authors, in fact Safronov (1972) argued that the inclusion of finite resistivity and viscosity effects would make the MRI unimportant in disks. A key element of confusion seems to be over the lack of recovery of the Rayleigh criterion as the magnetic field strength is decreased to zero. The stability properties of hydrodynamic flows (based on angular momentum gradients) and MHD flows (based on angular velocity gradients) are incompatible, a point discussed in detail by Balbus & Hawley (1991). It was not until their paper that the important role that the MRI plays in disks was identified.

Over the past 20 years, there has been considerable effort to understand the nonlinear regime and saturation of the MRI, mostly using computational methods. Figure 2 shows images from typical simulations of the MRI in both *global* domains, in which the entire disk is evolved over a wide range of radii, and *local* shearing box simulations, in which only a small radial extent of the disk is evolved. The advantage of the shearing box is that by focusing all of the computational resources on a small patch, much higher numerical resolution is possible.

Perhaps the most important result from local shearing box simulations is that in the nonlinear regime, the MRI produces MHD turbulence which has both significant Maxwell and Reynolds stresses that transport angular momentum outward. It is remarkable that the inclusion of a weak

MHD simulations of the MRI

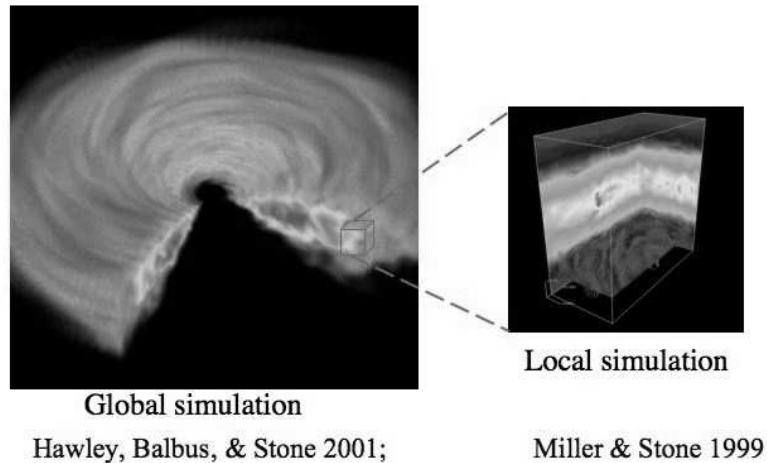


Figure 2. Images of the density from a global simulation of a MRI unstable disk (*left*), and of the density and magnetic field vectors from a local shearing box simulation (*right*).

field *qualitatively* changes the stability properties of the flow, and results in outward transport at a level required by observations. Numerical simulations of the MRI have also established that turbulence amplifies the magnetic field, and drives an MHD dynamo, and that the power spectrum of the turbulence is anisotropic, with most of the energy on the largest scales (Balbus 2003).

Still, many important questions remain. At the moment, it is not understood how the energy liberated by accretion is dissipated by the turbulence: does most of the energy go into the ions or electrons? It is not understood how MRI unstable disks drive powerful winds and outflows as are observed in many astrophysical systems, and what are the relative contributions of the MRI and winds to angular momentum transport. Finally, calculations which include radiation have only begun to be explored; it is likely many important phenomena may be related to the interaction of the radiation field with the flow field generated by the MRI. All of these questions will undoubtedly be addressed by future efforts.

5. MHD turbulence in the ISM of galaxies

Moving to ever larger scales, the next system in which magnetic fields have been observed to be important is the interstellar medium (ISM) of galaxies. The observation of polarized synchrotron

emission from the ISM of the Milky Way and other galaxies, produced by relativistic electrons spiraling around magnetic field lines, is direct proof of the presence of such fields. Moreover, the observations allow the strength and even the direction of the field to be inferred. In most cases, it is found the fields are in equipartition, with the magnetic energy density being about equal to the thermal energy of the gas, and kinetic energy of relativistic particles. Moreover, observations of the kinematics of the ISM in galaxies reveal it is highly turbulent. Thus, interpretation of the dynamics of the ISM requires an understanding of highly compressible MHD turbulence.

In fact, the statistical properties of turbulence were of considerable interest to Chandra. It is revealing to read what he wrote in his Henry Norris Russell Lecture:

We cannot construct a rational physical theory without an adequate base of physical knowledge. It would therefore seem to me that we cannot expect to incorporate the concept of turbulence in astrophysical theories in any essential manner without a basic physical theory of the phenomenon of turbulence itself, (Chandrasekhar 1949).

Fortunately, the theory of energy cascades in strong MHD turbulence has progressed enormously in the last few decades (e.g. Goldreich & Sridhar 1995), so that there now are theories of the power spectrum and statistical properties of MHD turbulence that can be tested and compared to observation. One method to investigate the properties of MHD turbulence is through direct numerical simulation.

Figure 3 shows images from high resolution (1024^3) numerical simulations of highly compressible MHD turbulence with both strong and weak magnetic fields, taken from Lemaster & Stone (2009). The turbulence is driven with a forcing function whose spatial power spectrum is highly peaked at a wavenumber corresponding to about $1/8$ the size of the computational domain. The energy input rate of the driving is held constant, and the turbulence is driven so that the Mach number of RMS velocity fluctuations $M = \sigma_v/C$ (where C is the sound speed) is about 7. The magnetic field strength corresponds to a ratio of gas to magnetic pressure $\beta = 8\pi P/B^2$ of 0.01 in the strong field case, and one in the weak field case. This means the Alfvénic Mach number of the turbulence is about one in the strong field case, and 7 in the weak field case.

It is quite clear from the images that in the weak field case, the density fluctuations are isotropic, and the magnetic field is highly tangled. In contrast, in the strong field case the density fluctuations are elongated along the field lines, and the field is more or less ordered. This suggests that the power spectrum of the turbulence will be anisotropic. In fact, this is one of the most basic predictions of the theory (Goldreich & Sridhar 1995).

In addition to investigating the spectrum of fluctuations, such simulations can be used to measure properties such as the decay rate of the turbulence, and how it depends on the magnetic field strength. Early predictions suggested the decay rate of strongly magnetized turbulence would be very low, since it would be dominated by incompressible Alfvén waves. In fact, the simulations (Stone, Ostriker & Gammie 1998; MacLow 1999) found the decay rate of *supersonic* MHD turbulence was very fast, with the decay time about equal to an eddy turn over time on

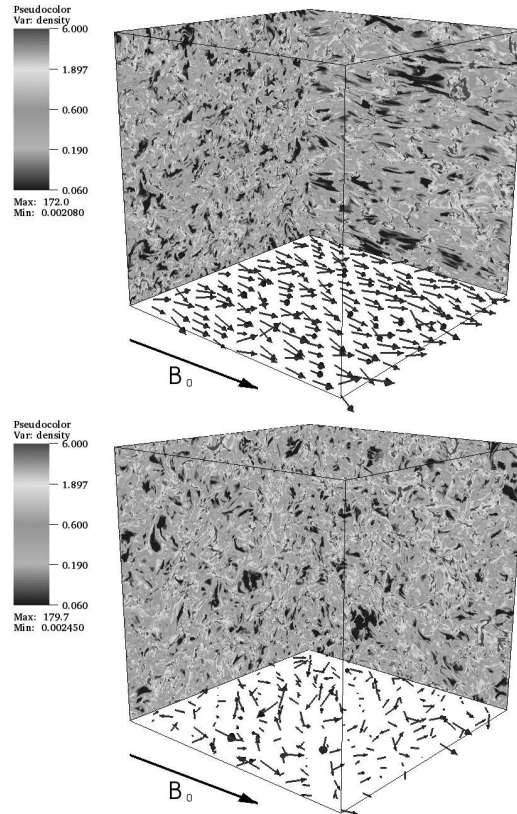


Figure 3. Structure of the density (grayscale) and magnetic field (arrows) in driven supersonic MHD turbulence for strong (top) and weak (bottom) fields.

the largest scales, regardless of the field strength. Most of the dissipation was found to occur in shocks. Thus, while Alfvén waves are important to the energetics, the coupling of large amplitude nonlinear Alfvén waves to compressible modes, in particular slow magnetosonic waves, cannot be ignored. This coupling pumps energy into the compressible modes, which then decay in shocks. The result has important implications for the decay of supersonic turbulence in the ISM of galaxies.

Finally, more direct comparison between the simulations and observations is possible using properties such as the polarization angle of background star light. In many regions of the ISM, spinning dust grains become aligned with their long axis perpendicular to the magnetic field. When background stars are viewed through these aligned grains, their light is polarized, with the strength and direction of the polarization vector related to the column density of gas, and the magnetic field strength in the plane of the sky. Using numerical simulations of MHD turbulence, it is possible to compute theoretical maps of the polarization vectors along different viewing angles

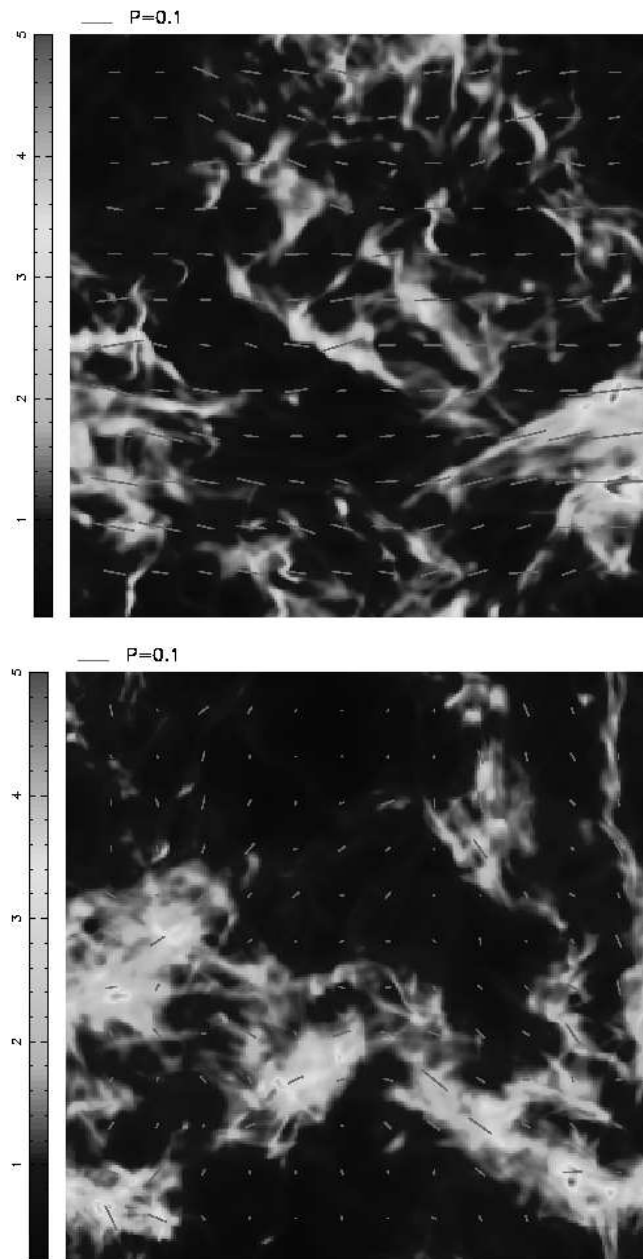


Figure 4. Scatter in polarization angle in supersonic turbulence with a strong field (*top*) and weak field (*bottom*). The grayscale shows the column density, and the line segments show the direction and amplitude of the polarization vector.

for background sources viewed through the simulation domain. Figure 4 shows an example for two simulations, both using Mach 10 turbulence with strong ($\beta = 0.01$) and weak ($\beta = 1$) magnetic fields.

It is clear from inspection that in the case of strong fields, the scatter in polarization angle is small, while in the case of weak fields the scatter is large. In fact, this effect was predicted by Chandrasekhar & Fermi (1953), who showed that the scatter in the polarization angle $\delta\phi$ should be related to the plane-of-sky magnetic field strength B_p , gas density ρ , and line-of-sight velocity dispersion δv through

$$B_p = 0.5 \frac{(4\pi\rho)^{1/2} \delta v}{\delta\phi} \quad (12)$$

Equation 12 is now known as the ‘‘Chandrasekhar-Fermi’’ formula, and is now routinely used as a technique to measure magnetic field strengths in the ISM.

6. Kinetic MHD effects in clusters of galaxies

Finally, we consider the effect of magnetic fields on the largest structures in the universe, clusters of galaxies. Radio observations of Faraday rotation in background sources indicate that the x-ray emitting plasma trapped in the gravitational potential of clusters is magnetized. Using the x-ray spectra to determine the temperature and density of the plasma shows that the mean free path of charged particles in the plasma is much smaller than the system size, but much larger than the gyroradius, that is the plasma is in the *kinetic MHD* regime.

The most important property of weakly collisional plasmas in the kinetic MHD regime, in comparison to highly collisional plasmas, is that the microscopic transport coefficients become anisotropic. For example, if the electron mean free path is much larger than the electron gyroradius, thermal conduction is primarily along magnetic field lines. Similarly, when the ion mean free path is much larger than the ion gyroradius, kinematic viscosity is primarily along magnetic field lines. The simplest description of the dynamics is therefore given by the equations of MHD supplemented by anisotropic thermal conduction and viscous transport terms (Braginskii 1965).

Remarkably, the addition of anisotropic transport *qualitatively* changes the dynamics of the plasma. For example, with anisotropic thermal conduction, the convective stability criterion no longer depends on entropy, but only on the temperature gradient (if $dT/dz < 0$, the plasma is *unstable* to convection; Balbus 2000). Convective instability in this regime has been termed the magnetothermal instability (MTI). In fact, other instabilities have also been found in the kinetic MHD regime that might be important in clusters (Quataert 2008) or in diffuse accretion flows (Balbus 2000).

Figure 5, taken directly from a nonlinear simulation (Parrish & Stone 2007), demonstrates the physics of the MTI. Consider a stratified atmosphere in a constant gravitational field. Arrange the vertical profiles of the pressure and density so that the atmosphere is hotter at the bottom than the top, and so that the entropy is constant or increasing upwards. In this case, the atmosphere

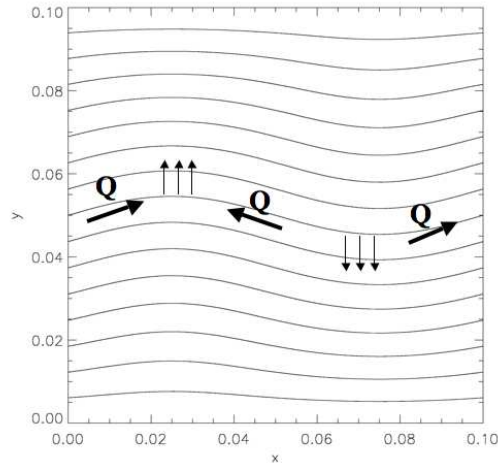


Figure 5. Basic mechanism of the MTI. The structure of the perturbed field lines in a stratified atmosphere are shown (which is hotter on the bottom than top), along with the direction of the heat flux \mathbf{Q} induced along field lines which results in amplification of the perturbations.

should be *stable* to convection by the Schwarzschild criterion. Now consider a weak, horizontal magnetic field with anisotropic thermal conduction along field lines. Initially the field lines are parallel to the isotherms, so there is no heat flux in the equilibrium state. Now consider the evolution of vertical perturbations, as shown in the figure. The peaks of the perturbations are at a slightly lower pressure than their equilibrium position, so they expand and cool. The valleys are at a slightly higher pressure, and so contract and heat up. These lead to a temperature gradient, and therefore a heat flux \mathbf{Q} , along the field lines. The net result is to *increase* the entropy at the peaks (making them more buoyant), and to *decrease* the entropy at the valleys (making them sink). This increases the perturbation, tilts the field line more to the vertical, increases the temperature gradient along the field line and therefore increases the heat flux; and this process runs away as an instability.

The nonlinear regime of the MTI has now been quite well studied using numerical simulations. With non-conducting boundaries at the top and bottom of the domain, the MTI saturates when the temperature profile becomes isothermal. If the top and bottom boundaries are held at fixed temperatures, then vigorous and sustained convection can be driven.

How does the MTI relate to galaxy clusters? Recent work shows that it can play an important role in the temperature profiles of the x-ray emitting gas. When clusters form from gravitational collapse of large-scale structure, the initial temperature profile is centrally peaked. This profile is unstable to the MTI, and simulations of hydrostatic cores with weak magnetic fields show that the MTI causes significant redistribution of the temperature profile of the cluster, along with significant amplification of the magnetic field, in a Hubble time. More recently, the role that externally driven turbulence plays in the plasma dynamics, along with the MTI and other instabilities in the

kinetic regime, has been an area of active inquiry (for example, see Parrish, Quataert & Sharma 2009).

7. Summary

I have discussed a very wide range of astrophysical systems where magnetic fields modify or even control the dynamics in order to demonstrate that MHD is now understood to be fundamental to many basic problems in astrophysics. Perhaps the best example is provided by the problem of angular momentum transport in accretion disks. For over thirty years, it was a struggle to understand why such transport occurs. With the identification of the MRI, it became clear that MHD is the key: weakly magnetized Keplerian shear flows are linearly unstable, and subsequent computational studies have shown this instability saturates as MHD turbulence with a significant Maxwell stress. In fact, both Velikhov (1959) and Chandrasekhar (1960) recognized the presence of the instability, although neither realized its importance in accretion disks, perhaps because such disks were not well recognized observationally at the time.

Many frontiers exist in astrophysical MHD, as Section 6 demonstrates. Motivated by the properties of weakly collisional plasmas in the x-ray emitting gas in clusters of galaxies, anisotropic thermal conduction was shown to qualitatively change the dynamics. In particular, it has been found that the stability condition for convection is fundamentally altered when anisotropic conduction is important: stability depends only on the temperature gradient, while the entropy profile is irrelevant. Undoubtedly, many more remarkable results remain to be discovered as ever more realistic descriptions of astrophysical plasmas are adopted.

It is impossible to describe studies of astrophysical MHD without mentioning the important role that numerical methods now play. In fact, computational methods are now the primary tool for the investigation of nonlinear, time-dependent, and multidimensional solutions to the equations of MHD. I wonder what Chandra would think of modern computational methods, and their application to problems in astrophysics?

Finally, I hope this paper has demonstrated that Chandra's contributions to plasma physics and MHD endure. In particular, his work on the MRI was before its time.

Acknowledgements

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