Bull. Astr. Soc. India (2007) **35**, 481–486

Study of MHD surface waves along the interfaces of the magneto sheath, boundary layer and magnetosphere with the effect of solar wind

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Abstract. The magnetospheric boundary layer, which is the interface between the magneto sheath and the magnetosphere, has been considered to be the plasma slab surrounded by moving and static plasma media on either side. The compressibility effect on surface waves propagating along the slab has been discussed. The effect of variation of the boundary layer thickness is studied as a special case, since the observed variation of the boundary layer thickness leads to unstable modes due to Kelvin-Helmholtz instability. The solar wind driven magneto sheath plasma is considered as the moving plasma medium and the effect of the velocity of the solar wind is also taken into account.

Keywords : Sun : MHD surface waves – magneto sheath – boundary layer – solar wind

1. Introduction

The magnetospheric boundary layer is a layer of plasma lying immediately earthward of the magneto pause, which is the magnetic discontinuity separating the interplanetary magnetic field from the planetary magnetic field. The observations of the magneto pause-boundary layer region by Isee satellite indicated the boundary layer thickness to be highly variable. Paschmann (1979), Sckopke et al. (1979) and Sonnerup (1980) suggested the

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variation in the boundary layer thickness to be caused by the Kelvin-Helmholtz (K-H) instability in the magneto pause-boundary layer region. In the study of the K-H instability in the magneto pause boundary layer region, Lee et al. (1981) have considered the three uniform plasma regions with the incompressible flow. In continuation of this work, Uberoi (1986) has studied the effect of the angle and the finite thickness to arrive at the marginal instability condition. Since the plasma parameters in the regions one and three (i.e., the layers above and below the region under consideration) have been considered to be identical, this assumption is more unrealistic for the dayside magnetosphere (Uberoi, 1986). Spitzer et al. (1966) have studied the behaviour of magneto sheath in terms of properties of waves that propagate in the liquid and illustrated the point by calculating the sonic line and characteristics of sound waves. Though they dealt with compressible hydrodynamic flow, the magnetospheric plasma medium was considered to be field free ($B = 0$). Siscoe et al. (2002) have taken the magnetic field into account, however, much attention has been given to the night side magnetosphere only and not to the day side magnetosphere. In all the above mentioned works, the cases of high beta plasma have alone been taken into account. One of the most successful theories of the origin of geomagnetic pulsations is based on the steady state resonant interaction of the surface waves in the magneto pause with local field line oscillations in the magnetospheric boundary. So in this work, we have considered the low beta compressible plasma to study the MHD wave characteristics of moving magneto sheath plasma over the magnetospheric boundary layer due to the solar wind and the parameters of the plasma in each region is assigned different values to validate the realistic nature of the day side magnetosphere.

2. Geometry and formulation

The geometrical features of the configuration to be studied are shown in Fig. 1. We consider a compressible MHD model consisting of the solar wind driven flowing magneto sheath plasma (region 2) separated from the magnetosphere (region 3) by the boundary layer (region 1). Further these three layers are separated from each other by two idealized interfaces, namely, magneto pause (regions 1 and 2)and inner boundary (regions 1 and 3) of the boundary layer. The densities and the uniform background magnetic fields of the boundary layer, magneto sheath and magnetospheric plasmas have been considered to be ρ_{01}, ρ_{02}, ρ_{03}, $\mathbf{B_{01}}$, $\mathbf{B_{02}}$, $\mathbf{B_{03}}$, respectively. The magneto sheath plasma is flowing parallel to the boundary layer with the uniform velocity along z-direction. This model is justified to be a plasma slab (boundary layer) surrounded by the moving plasma layer (magneto sheath) in one side and the static plasma layer (magnetosphere) on the other side, since the magnetic field changes discontinuously at two locations (Roberts 1981), namely, at magnetopause and inner boundary of the boundary layer.

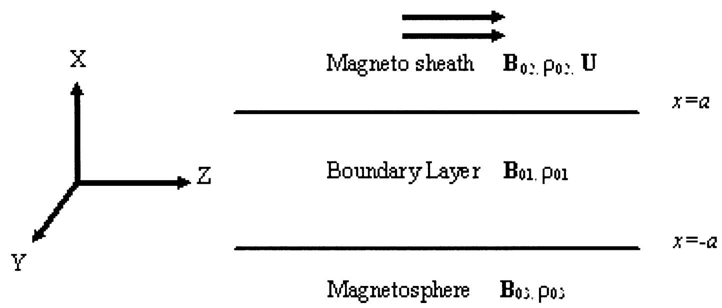

Figure 1. The geometry.

3. Boundary conditions and dispersion relation

The governing linearized magnetohydrodynamic (MHD) equations for the compressible plasma are the conservation of mass, momentum, equation of state and the equation connecting the magnetic field and the velocity field (Priest 1982). A perturbation of the form $f(x, y, z, t) = f(x)e^{i(ly+kz-\omega t)}$ is applied to the plasma parameters in all the regions at the interfaces. The boundary conditions (Chandrasekhar 1961; Joarder & Satya Narayanan 2000) are:

$$\frac{v_{x1}}{\omega} = \frac{v_{x2}}{\omega - Uk} \tag{1}$$

$$p_1 + \frac{B_{01}b_{z1}}{\mu} = p_2 + \frac{B_{02}b_{z2}}{\mu} \tag{2}$$

These conditions are applied at $x = a$. Another set of boundary conditions at $x = -a$ are also applied. The dispersion relation for the propagation of MHD surface waves along the magnetic slab (boundary layer) is derived as

$$\{[\frac{\rho_{01}}{K_1}(k^2V_{A1}^2 - \omega^2)]^2 + [\frac{\rho_{02}}{K_2}(k^2V_{A2}^2 - (\omega - Uk)^2) \times \frac{\rho_{03}}{K_3}(k^2V_{A3}^2 - \omega^2)]\}Tanh(2K_1a)$$

$$+\frac{\rho_{01}}{k_1}(k^2V_{A1}^2 - \omega^2)\{\frac{\rho_{02}}{K_2}(k^2V_{A2}^2 - (\omega - UK)^2) + \frac{\rho_{03}}{K_3}(k^2V_{A3}^2 - \omega^2)\} = 0 \tag{3}$$

where $V_{A1,2,3}^2 = \frac{B_{01,2,3}^2}{\mu\rho_{01,2,3}}$ are the Alfven velocities in the regions 1, 2, 3, respectively.

The wave numbers in each region are,

$$K_1 = \sqrt{n_1^2 + l^2}, K_2 = \sqrt{n_2^2 + l^2}, K_3 = \sqrt{n_3^2 + l^2}$$

and

$$n_{1,3}^2 = \frac{(k^2V_{A1,3}^2 - \omega^2)(k^2C_{1,3}^2 - \omega^2)}{(k^2C_{T1,3}^2 - \omega^2)(C_{1,3}^2 - V_{A1,3}^2)} \tag{4}$$

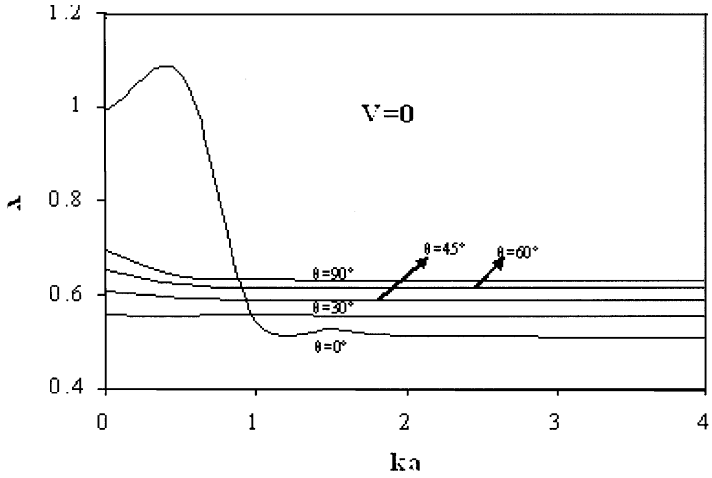

Figure 2. The dispersion curve in the absence of flow ($V = 0$).

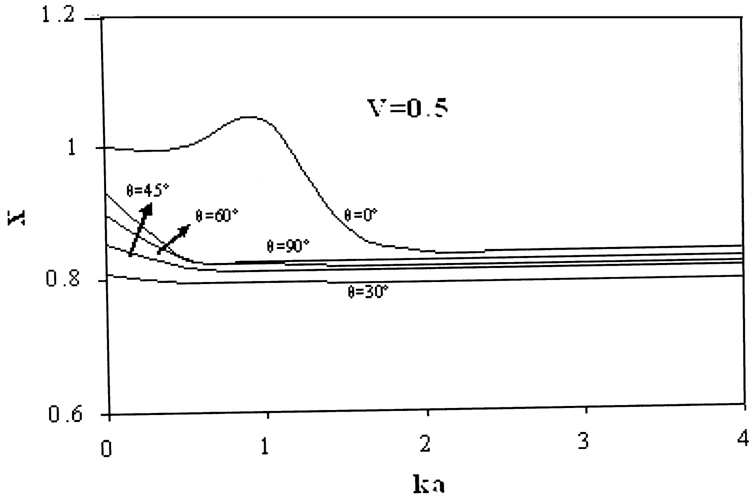

Figure 3. The dispersion curve with flow ($V = 0.5$).

$$n_2^2 = \frac{(k^2 V_{A2}^2 - \Omega^2)(k^2 C_2^2 - \Omega^2)}{(k^2 C_{T2}^2 - \Omega^2)(C_2^2 - V_{A2}^2)} \tag{5}$$

Here $\Omega = \omega - Uk$ is the Doppler shifted frequency in the moving medium and $c_T = \frac{cV_A}{c+V_A}$ is the cusp speed.

We consider the low beta plasma, which is astrophysically the most interesting case, where the Alfven wave speed greatly exceeds the sound speed in all the three regions (Uberoi 1982). Defining $x = \omega/kV_A$, the dispersion relation can be normalized with the suitable plasma parameters (interface) as :

$$\left\{\Big(\frac{(1-x^2)}{\sqrt{1-x^2\cos^2\theta}}\Big)^2+\frac{(1-(R_1x-V)^2)}{\sqrt{1-(R_1x-V)^2\cos^2\theta}}\times\frac{(1-R_2^2x^2)}{\sqrt{1-R_2^2x^2\cos^2\theta}}\right\}\tanh[2ka\sqrt{1-x^2Cos^2\theta}]$$

$$+\frac{(1-x^2)}{\sqrt{1-x^2Cos^2\theta}}\left\{\frac{(1-(R_1x-V)^2)}{\sqrt{1-(R_1x-V)^2\cos^2\theta}}+\frac{(1-R_2^2x^2)}{\sqrt{1-R_2^2x^2\cos^2\theta}}\right\}=0.$$

4. Discussion

The dispersion relation in the absence of boundary layer (a = 0) and the flow (U = 0) reduces to the well known dispersion relation as

$$\rho_{02}(k^2V_{A2}^2-\omega^2)(n_3^2+l^2)^{1/2}+\rho_{03}(k^2V_{A3}^2-\omega^2)(n_2^2+l^2)^{1/2}=0 \tag{6}$$

obtained by Wentzel (1979) and Roberts (1981) for a single interface and this equation is discussed at length by Uberoi (1982). In the figures (2) - (3), the real part of the phase velocity of the surface wave (x_r) is plotted against the wavelength (ka) for different angles of propagation of surface waves ($\theta = 0°, 30°, 45°, 60°, 90°$) with respect to the magnetic field. The ratios of the Alfven velocities have been taken from Lee et al (1976). The wavenumber is kept constant to study the effect of boundary layer thickness (a) on surface waves. In the absence of the magneto sheath flow (V=0), from Fig. (2), there is a significant change in the phase velocity for surface waves when $\theta = 0°$ along the magnetic field direction. Two windows of generation of waves are found, in which one lies approximately between ka = 0 and 0.7 in the higher phase velocity region and the other between ka = 1.0 and 1.4 in the lower phase velocity region. This behavior is observed only in higher wavelength or lower thickness side. When the flow velocity is introduced (Fig. 3) (with U less than the Alfven velocity of the magneto sheath plasma), the phase velocity is increased for all angles compared to Fig. 2. For $\theta = 0°$, the wave generation in the lower phase speed range vanishes and the existing window shifts towards the lower wavelength or higher thickness side.

The observations on the fluctuation of magnetospheric boundary layer thickness have been studied by many authors (Paschmann 1979; Sckopke et al. 1979) and it is suggested that this may be due to the Kelvin - Helmholtz instability at the magneto pause (Sonnerup 1980). Thus it becomes imperative to study the thickness variations of boundary layer on surface waves.

5. Conclusion

The flow of magneto sheath plasma enhances the phase speed of surface waves for all angles of propagation. The mode reversal of surface waves has been observed, when the flow speed dominates the Alfven speed. The wave generating windows of wavelengths are found in the propagation of waves along the magnetic field ($\theta = 0°$), which move towards the lower wavelength region when the flow speed increases. More computations needs to be done, taking realistic values of the observed parameters into consideration. This is being done and will be communicated shortly.

Acknowledgement

GDR warmly acknowledges the hospitality and facilities provided by IIA, Bangalore, during his visit, where part of this work has been completed.

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