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Periodic orbits around the collinear liberation points in the restricted three body problem when the smaller primary is a triaxial rigid body : Sun-Earth case

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Abstract. Periodic orbits belonging to the Strömgren families A, B and C around the collinear liberation points in the restricted three body problem have been studied when the smaller primary is a triaxial rigid body by taking different values of semiaxes of the triaxial rigid body. The Liapunov stability of each periodic solution has also been examined.

Keywords : Restricted three-body problem, triaxial rigid body, periodic orbits, Liapunov stability

1. Introduction

In the effort to understand the structure of the solutions of a non-integral dynamical system, numerical determination of its periodic solutions and their stability properties play a role of fundamental importance. The determination of the periodic solutions can of course be achieved by numerical integration of the equations of motion.

The infinitesimal periodic oscillations around the collinear Lagrangian points L_1, L_2, L_3 in the restricted three – body problem are continued to finite periodic orbits in the plane of motion of the two primaries as well as in three dimensions, Moulton (1920). In the planar case these finite orbits are grouped into the families A, B and C respectively and have been studied numerically by many investigators e.g., Strömgren

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(1935); Bartlett (1964); Henon(1965) for $\mu = 0.5$; and Markellos (1975) for $\mu = 0.00095$; Ragos and Zagouras (1991); Elipe and Lara (1997); Corbera and Llibre (2003); Henon (2003); Henrard and Navarro (2004). Concerning the three-dimensional case, Moulton (1920) has shown that there are three types of finite periodic solutions which are generated from the infinitesimal ones. Bray and Goudas (1967) have computed numerically for $\mu = 0.4$ the three families A,B and C. Ragos and Zagouras (1991) draw the periodic solutions around the collinear Lagrangian points in the photo-gravitational restricted three body problem: Sun-Jupiter case. Elipe and Lara (1997) studied the periodic orbits in the restricted three-body problem with radiation pressure. Corbera and Llibre (2003) studied the periodic orbits of a collinear restricted three-body problem. New families of periodic orbits in Hill's problem of three-bodies were found by Henon (2003). Henrard and Navarro (2004) have shown the families of periodic orbits emanating from homoclinic orbits in the restricted problem of three bodies.

In nature, the celestial bodies are not perfect spheres. They are either oblate or triaxial. So far, very little work is done by taking the primaries as triaxial bodies.

In this paper, we have studied the effect of oblateness of the smaller primary on the periodic orbits in the restricted three body problem when smaller primary is a triaxial rigid body and more massive body is a point mass with its equatorial plane coincident with the plane of motion in sun-earth–satellite system.

We have drawn the exact periodic orbits in the Strömgren families A, B and C. Here we have used the predictor-corrector method for the numerical determination of periodic solutions around the collinear liberation points.

2. Equation of motion and variation

In the usual barycentric, rotating and dimensionless coordinate system (X, Y), with the two main bodies having masses m_1 and m_2, the equations of motion of the third particle m_3 in the phase space (X_1, X_2, X_3, X_4) are

$$\dot{X}_i = f_i(X_1, \dots, X_4), \quad i = 1 \dots 4, \tag{1}$$

with

$$\begin{aligned}
f_1 &= X_3, \quad f_2 = X_4 \\
f_3 &= 2nX_4 + n^2 X_1 - \frac{(1-\mu)(X_1-\mu)}{r_1^3} - \frac{\mu(X_1+1-\mu)}{r_2^3} - \frac{3\mu(2\sigma_1-\sigma_2)(X_1+1-\mu)}{2r_2^5} \\
&\quad + \frac{15\mu(\sigma_1-\sigma_2)(X_1+1-\mu)X_2^2}{2r_2^7}, \\
f_4 &= -2nX_3 + n^2 X_2 - \frac{(1-\mu)X_2}{r_1^3} - \frac{\mu X_2}{r_2^3} - \frac{3\mu(2\sigma_1-\sigma_2)X_2}{2r_2^5} - \frac{3\mu(\sigma_1-\sigma_2)X_2}{2r_2^5}
\end{aligned}$$

$$+\frac{15\mu(\sigma_1-\sigma_2)X_2^3}{2r_2^7}, \tag{2}$$

where

$$X_1 = X, X_2 = Y, X_3 = \dot{X}, X_4 = \dot{Y},$$
$$r_1^2 = (X_1-\mu)^2 + X_2^2, r_2^2 = (X_1+1-\mu)^2 + X_2^2,$$
$$\mu = \frac{m_2}{m_1+m_2} \leq \frac{1}{2},$$

$m_1, m_2 (m_1 \geq m_2)$ being the masses of the primaries,

$$\sigma_1 = \frac{a_1^2-a_3^2}{5R^2}, \sigma_2 = \frac{a_2^2-a_3^2}{5R^2}, \sigma_1, \sigma_2 \ll 1,$$

where a_1, a_2 and a_3 are the semi-axes of the earth and R is the dimensional distance between the earth and the sun. Here, we have taken only the first order terms of σ_1, σ_2. The mean motion n of the primaries is given by

$$n = 1 + \frac{3}{4}(2\sigma_1 - \sigma_2).$$

The coordinates of the infinitesimal particle in phase space $X_1, \ldots, X_4$ depend uniquely, along any solution, with the initial conditions $(X_{01}, \ldots, X_{04})$ and the time t i.e.

$$X_i = X_i(X_{01}, \ldots, X_{04}, t), \quad i = 1 \ldots 4.$$

Their partial derivatives with respect to the initial conditions satisfying the equations of variation are

$$\frac{d}{dt}\left(\frac{\partial X_i}{\partial X_{oj}}\right) = \sum_{k=1}^{4} \frac{\partial f_i}{\partial X_k} \cdot \frac{\partial X_k}{\partial X_{oj}}, \quad i,j = 1\ldots 4. \tag{3}$$

If we denote the variations $\frac{\partial X_i}{\partial X_{oj}}$ by v_{ij}, we can write these equations more explicitly as follows

$$\dot{v}_{ij} = v_{(i+2)/j}, \quad i = 1,2, \quad j = 1,2,3,4,$$
$$\dot{v}_{ij} = f_{i1}v_{1j} + f_{i2}v_{2j} + f_{i3}v_{3j} + f_{i4}v_{4j}, \quad i = 3,4, \quad j = 1,2,3,4,$$

where

$$f_{ij} = \frac{\partial f_i}{\partial X_{oj}}.$$

The stability parameters a, b, c and d as used by Markellos (1975) are:

$$a = v_{11} + sv_{14}, \quad b = v_{13},$$
$$c = v_{31} - 2(1+s)v_{21} - s^2 v_{24},$$
$$d = v_{33} - (2+s)v_{23}, \quad s = s_c + s_\sigma,$$

where

$$s_c = \frac{-1}{X_{04}}\left[n^2 X_{01} - \frac{(1-\mu)}{|X_{01}-\mu|(X_{01}-\mu)} - \frac{\mu}{|X_{01}-\mu+1|(X_{01}-\mu+1)}\right],$$

$$s_\sigma = \frac{1}{X_{04}}\left[\frac{3\mu(2\sigma_1-\sigma_2)}{2|X_{01}-\mu+1|(X_{01}-\mu+1)^3}\right].$$

3. Motion around the collinear equilibrium points

We have calculated the collinear equilibrium points in different cases by assuming the different values of the semiaxes of smaller primary. (Table 1)

We know for collinear equilibrium points,

$$f_3 - 2nX_4 = 0, \quad X_2 = 0,$$

i.e.

$$n^2 X_1 - \frac{(1-\mu)(X_1-\mu)}{|X_1-\mu|^3} - \frac{\mu(X_1+1-\mu)}{|X_1+1-\mu|^3} - \frac{3\mu(2\sigma_1-\sigma_2)(X_1+1-\mu)}{2|X_1+1-\mu|^5} = 0.$$

The characteristic roots are

$$\lambda_i = \pm\left(\frac{\lambda_{c1}}{\sqrt{2}} + \lambda_{\sigma 1}\right), \quad i = 1,2$$

$$\lambda_i = \pm\left(\frac{\lambda_{c2}}{\sqrt{2}} - \lambda_{\sigma 2}\right), \quad i = 3,4$$

$$\lambda_{c1} = [R - 2 + (9R^2 - 8R)^{1/2}]^{1/2},$$

$$\lambda_{c2} = [R - 2 - (9R^2 - 8R)^{1/2}]^{1/2},$$

$$\lambda_{\sigma 1} = \frac{q_1 - q_2 - 6(2\sigma_1-\sigma_2)}{2\sqrt{2}\lambda_{c1}} + \frac{6(2-R)(2\sigma_1-\sigma_2)}{2\sqrt{2}(9R^2-8R)^{1/2}\lambda_{c1}} + \frac{q_2(4+3R) - q_1(4-3R)}{2\sqrt{2}(9R^2-8R)^{1/2}\lambda_{c1}},$$

$$\lambda_{\sigma 2} = \frac{q_1 - q_2 - 6(2\sigma_1-\sigma_2)}{2\sqrt{2}\lambda_{c2}} + \frac{6(2-R)(2\sigma_1-\sigma_2)}{2\sqrt{2}(9R^2-8R)^{1/2}\lambda_{c2}} + \frac{q_2(4+3R) - q_1(4-3R)}{2\sqrt{2}(9R^2-8R)^{1/2}\lambda_{c2}},$$

and their corresponding angular frequencies are

$$\omega_{ci} = -\frac{\lambda_{ci}}{\sqrt{2}}, \qquad \omega_{\sigma i} = -\lambda_{\sigma i}, \quad i = 1,2$$

where

$$R = \frac{(1-\mu)}{|X_{Lj}-\mu|^3} + \frac{\mu}{|X_{Lj}+1-\mu|^3},$$

$$q_1 = \frac{3}{2}(2\sigma_1-\sigma_2) + \frac{6\mu(2\sigma_1-\sigma_2)}{|X_{Lj}+1-\mu|^5},$$

$$q_2 = \frac{-3}{2}(2\sigma_1-\sigma_2) + \frac{3\mu(2\sigma_1-\sigma_2)}{2|X_{Lj}+1-\mu|^5} + \frac{3\mu(\sigma_1-\sigma_2)}{|X_{Lj}+1-\mu|^5}.$$

X_{Lj} is the X- coordinate of the collinear libration points $L_j, j = 1, 2, 3$.

4. Second order approximation of periodic solution

Let L be any collinear equilibrium points $L_j, j = 1, 2, 3$. If a new coordinate system is defined with L as origin and L_{X1}, L_{X2}, as axes, parallel to OX and OY as defined by Szebehely (1967) respectively, the proper transformation between the two systems is given by the relations

$$X_1 = X_L + x_1, \quad X_2 = x_2. \tag{4}$$

The eqns. (2) are transformed through eqs. (4) in the (x_1, x_2) coordinate system and the equations obtained are expanded by Taylor series up to the second order terms to obtain

$$\begin{aligned} \ddot{x}_1 - 2n\dot{x}_2 &= (A_1 + A_1')x_1 + (A_2 + A_2')x_1^2 + (A_3 + A_3')x_2^2, \\ \ddot{x}_2 - 2n\dot{x}_1 &= (B_1 + B_1')x_2 + (B_2 + B_2')x_1 x_2, \end{aligned} \tag{5}$$

where

$$\begin{aligned} A_1 &= (1 + 2R), \quad A_1' = 3(2\sigma_1 - \sigma_2)\left[\frac{1}{2} + \frac{3\mu}{|X_{Lj} + 1 - \mu|^5}\right], \\ A_2 &= -3\left[\frac{(1-\mu)(X_{Lj} - \mu)}{|X_{Lj} - \mu|^5} + \frac{\mu(X_{Lj} + 1 - \mu)}{|X_{Lj} + 1 - \mu|^5}\right], \\ A_2' &= \frac{-15\mu(2\sigma_1 - \sigma_2)(X_{Lj} + 1 - \mu)}{|X_{Lj} - \mu + 1|^7}, \\ A_3 &= -\frac{A_2}{2}, \quad A_3' = \frac{A_2'}{4} - \frac{15\mu(\sigma_1 - \sigma_2)(X_{Lj} + 1 - \mu)}{2|X_{Lj} + 1 - \mu|^7}, \\ B_1 &= (1 - R), \quad B_1' = \frac{3}{2}(2\sigma_1 - \sigma_2)\left[1 - \frac{\mu}{|X_{Lj} - \mu + 1|^5}\right] - \frac{3\mu(\sigma_1 - \sigma_2)}{|X_{Lj} + 1 - \mu|^5}, \\ B_2 &= -A_2, \quad B_2' = -A_2' + \frac{15\mu(\sigma_1 - \sigma_2)(X_{Lj} + 1 - \mu)}{|X_{Lj} - \mu + 1|^7}. \end{aligned}$$

We search for periodic solutions in the form of second order expansions in powers of parameter ε

$$\begin{aligned} x_1(\tau) &= x_{11}(\tau)\varepsilon + x_{12}(\tau)\varepsilon^2, \\ x_2(\tau) &= x_{21}(\tau)\varepsilon + x_{22}(\tau)\varepsilon^2, \end{aligned} \tag{6}$$

where time is expanded by the expression

$$t = (1 + t_1\varepsilon + t_2\varepsilon^2)\tau.$$

In order to erase any secular term in further analysis, we substitute the relations (6) into (5). Retaining terms of powers in ε not greater than two and denoting by dot $(\cdot)$ the τ - derivatives and ignoring the terms $\sigma_i\varepsilon^2$, $i = 1, 2$, we get

$$\begin{aligned}(\ddot{x}_{11}\varepsilon + \ddot{x}_{12}\varepsilon^2) - 2n(\dot{x}_{21}\varepsilon + \dot{x}_{22}\varepsilon^2) &= (A_1 + A_1')(x_{11}\varepsilon + x_{12}\varepsilon^2) \\ &\quad +(A_2 + A_2')(x_{11}\varepsilon + x_{12}\varepsilon^2)^2 \\ &\quad +(A_3 + A_3')(x_{21}\varepsilon + x_{22}\varepsilon^2)^2, \\ (\ddot{x}_{21}\varepsilon + \ddot{x}_{22}\varepsilon^2) + 2n(\dot{x}_{11}\varepsilon + \dot{x}_{12}\varepsilon^2) &= (B_1 + B_1')(x_{21}\varepsilon + x_{22}\varepsilon^2) \\ &\quad +(B_2 + B_2')(x_{11}\varepsilon + x_{12}\varepsilon^2)(x_{21}\varepsilon + x_{22}\varepsilon^2). \quad (7)\end{aligned}$$

4.1 The first order system

Defining the differential operator and taking terms up to the first order in ε in eqs. (7), we have

$$F_1(D) = \begin{pmatrix} D^2 - (A_1 + A_1') & -2nD \\ 2nD & D^2 - (B_1 + B_1') \end{pmatrix}$$

and

$$F_1 D \begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (8)$$

The general solution of the eqs. (8) is

$$x_{11}(\tau) = \sum_{i=1}^{4} c_i Exp(\lambda_i \tau), \quad x_{21}(\tau) = \sum_{i=1}^{4} d_i Exp(\lambda_i \tau) \quad (9)$$

where λ_i, $i = 1, 2, 3, 4$ are the characteristic roots of the system (8). By a suitable choice of the coefficients of the exponential terms of eq. (9), we may have a special periodic solution, which contains only the frequency corresponding to a specific imaginary part. We denote this frequency by ω. The eqs. (8) admit the periodic solution.

$$\begin{aligned} x_{11}(\tau) &= A\cos(\omega\tau) + B\sin(\omega\tau), \\ x_{21}(\tau) &= A^*\cos(\omega\tau) + B^*\sin(\omega\tau), \end{aligned}$$

where the coefficients A, B, A^*, B^* are connected by the relations

$$\begin{aligned} A &= A_c + A_\sigma, \\ A^* &= \frac{2n\omega B}{B_1 + B_1' + \omega^2}, \quad B^* = B_c^* + B_\sigma^*, \end{aligned}$$

where

$$\begin{aligned} B_C^* &= -\frac{2A_c}{B_1 + \omega_c^2}\omega_c, \\ B_\sigma^* &= -\frac{2A_c}{B_1 + \omega_c^2}\left[\omega_\sigma - \frac{\omega_c(B_1' + 2\omega_c\omega_\sigma)}{B_1 + \omega_c^2} + \frac{3}{4}(2\sigma_1 - \sigma_2)\omega_c\right] - \frac{2A_\sigma\omega_c}{B_1 + \omega_c^2}, \end{aligned}$$

$$A_c = \frac{1}{A_2}\left[-A_1 - \omega_c^2 + \frac{4\omega_c^2}{B_1+\omega_c^2}\right],$$
$$A_\sigma = \frac{1}{A_2}\left[-A_1' - 2\omega_c\omega_\sigma + \frac{4}{B_1+\omega_c^2}\left\{2\omega_c\omega_\sigma + \frac{3}{2}(2\sigma_1-\sigma_2)\omega_c^2 - \frac{\omega_c^2(B_1'+2\omega_c\omega_\sigma)}{B_1+\omega_c^2}\right\}\right]$$
$$+\frac{A_2'}{A_2^2}\left(A_1+\omega_c^2-\frac{4\omega_c^2}{B_1+\omega_c^2}\right).$$

Without any loss of generality, we put $x_{21}(0) = 0$. Then $A^* = 0$ and consequently $B = 0$. This means that $\dot{x}_{11}(0) = 0$. Finally, the above solution becomes

$$x_{11}(\tau) = A\cos(\omega\tau),$$
$$x_{21}(\tau) = B^*\sin(\omega\tau). \tag{10}$$

4.2 The second order system

Defining the deferential operator and taking terms of the second order in ε in eqs. (7), we have

$$F_2(D) = \begin{pmatrix} D^2 - A_1 & -2nD \\ 2nD & D^2 - B_1 \end{pmatrix}$$

and

$$F_2(D)\begin{pmatrix} x_{12} \\ x_{22} \end{pmatrix} = \begin{pmatrix} g_1(\tau) \\ g_2(\tau) \end{pmatrix} \tag{11}$$

where

$$g_1(\tau) = A_2x_{11}^2 + A_3x_{21}^2, \quad g_2(\tau) = B_2x_{11}x_{21}.$$

Substituting eqs. (10) into eqs.(11), functions g_i, $i = 1, 2$ become

$$g_1(\tau) = K_0 + K_1\cos(2\omega\tau), \quad g_2(\tau) = \Lambda_1\sin(2\omega\tau).$$

where

$$K_0 = \frac{1}{2}[A_2A_c^2 + A_3B_c^{*2}],$$
$$K_1 = \frac{1}{2}[A_2A_c^2 - A_3B_c^{*2}],$$
$$\Lambda_1 = \frac{1}{2}[B_2A_cB_c^*].$$

A periodic solution of system (11) is

$$x_{12}(\tau) = M_0 + M_1\cos(2\omega\tau),$$
$$x_{22}(\tau) = N_1\sin(2\omega\tau). \tag{12}$$

where

$$M_0 = -\frac{K_0}{A_1}, \quad M_1 = \frac{1}{\psi}[-4K_1\omega_c^2 + 4\Lambda_1\omega_c - B_1K_1],$$
$$N_1 = \frac{1}{\psi}[-4\Lambda_1\omega_c^2 + 4K_1\omega_c - A_1\Lambda_1],$$
$$\psi = 16\omega_c^4 - 4[4 - A_1 - B_1]\omega_c^2 + A_1B_1.$$

Finally, a second order approximation of the periodic solution around the collinear equilibrium points, as a function of parameter ε, is obtained from eqs. (10) and (12) as

$$x_1(\tau,\varepsilon) = [A\cos(\omega\tau)]\varepsilon + [M_0 + M_1\cos(2\omega\tau)]\varepsilon^2,$$
$$x_2(\tau,\varepsilon) = [B^*\sin(\omega\tau)]\varepsilon + [N_1\sin(2\omega\tau)]\varepsilon^2. \quad (13)$$

The period of this solution is

$$T = \frac{2\pi}{\omega}, \quad \omega \in [\omega_c, \omega_\sigma].$$

5. Numerical results $\mu = 0.000003, \varepsilon = 10^{-3}$

For comparing the effect of triaxial rigid body on the periodic orbits around the collinear liberation points, we take five different cases of different sets of semi axes in km. (a_1, a_2, a_3) of the smaller primary i.e. (6400, 6400, 6400), (6400, 6390, 6380), (6400, 6380, 6360), (6400, 6370, 6340). Now we calculate the three collinear liberation points in all the above cases by using the method given by Szebehely (1997). We computed the initial conditions $(X_o, \dot{Y}_o, T)$ at $\tau = 0$ by using the eqn. (13) and (4) in all the above cases. Then by using the linear predictor-corrector algorithm based on numerical integration of the equations of motion (1) and first order variations (3), we correct the initial conditions $(X_o, \dot{Y}_o, T)$ in all the above cases. We repeat the process up to the exact periodic orbit. At that point we note down $(X_o, \dot{Y}_o, T)$ and draw the periodic orbits in all the five cases. At that point we further calculate the stability parameters a, b, c and d for each periodic orbit. For liapanov stability, the periodic orbit is stable if $|a| < 1$ Markellos (1975).

The results for each family are represented in tabular and graphical form. The results of the families A, B, and C are given in Tables 2, 3 and 4 respectively. The results are graphically shown in Figs 1, 2 and 3.

In Table 2, the first column represents parameters $(X_o, \dot{Y}_o)$ initial condition for the periodic orbit around L_2, T is the period and a, b, c, and d represent stability parameters. The second, third, fourth, fifth and sixth columns give corresponding values of the parameters mentioned above. Columns in Tables 3 and 4 are defined in a similar way as Table 2.

With the initial conditions mentioned in Tables 2, 3, and 4, we have drawn periodic orbits around L_2 (Fig. 1), L_3 (Fig.2) and L_1 (Fig.3).

Table 1. Collinear liberation points (Sun-Earth case $\mu = 0.000003$, $R = 149597870.61$ km).

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5
σ_1	0	2.284×10^{-12}	4.561×10^{-12}	6.831×10^{-12}	9.094×10^{-12}
σ_2	0	1.141×10^{-12}	2.277×10^{-12}	3.408×10^{-12}	4.533×10^{-12}
L_1	−1.01003022840462	−101003022857538	−1.01003022874569	−1.01003022891555	−1.01003022908501
L_2	−0.990030437288914	−0.99003043711701	−0.99003043694558	−0.990030436774557	−0.990030436603957
L_3	1.00000125	1.000001249999829	1.00000124999658	1.00000124999487	1.00000124999317

Table 2. Periodic orbits around L_2: Family A ($\mu = 0.000003$, $R = 149597870.61$ km).

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5
X_o	−0.990030436143932	−0.9900304359313852	−0.9900304357260665	−0.9900304355241857	−0.9900304352373262
$\dot{Y}_o$	$-7.71407700632177\times10^{-9}$	$-7.987903348492486\times10^{-9}$	$-8.216069856687169\times10^{-9}$	$-8.424114962601054\times10^{-9}$	$-9.207379513247345\times10^{-9}$
T	3.011162684715132	3.01151513217258	3.011440321882915	3.011105900143039	3.011332657134634
a	$-5.03286297220881\times10^{10}$	$-4.864680894172237\times10^{10}$	$-4.728688772769945\times10^{10}$	$-4.607999752786654\times10^{10}$	$-4.2184254812411367\times10^{10}$
b	$3.5625997760525\times10^{2}$	$3.565782158795839\times10^{2}$	$3.565107143172163\times10^{2}$	$3.562089171678068\times10^{2}$	$3.564136253229897\times10^{2}$
c	$7.12380685325843\times10^{18}$	$6.649672871403511\times10^{18}$	$6.284283920487373\times10^{18}$	$5.972683242994785\times10^{18}$	$5.002581541182586\times10^{18}$
d	$-5.03286297338582\times10^{10}$	$-4.864680891925751\times10^{10}$	$-4.728688773006627\times10^{10}$	$-4.607999754157626\times10^{10}$	$-4.218425481844216\times10^{10}$

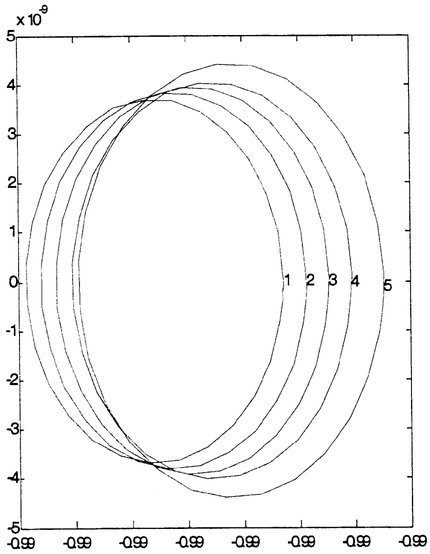

Figure 1. Periodic orbits around L_2: Family A.

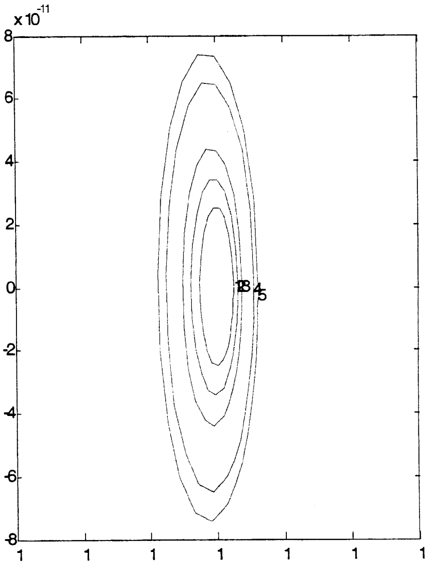

Figure 2. Periodic orbits around L_3: Family B.

Table 3. Periodic orbits around L_3: Family B ($\mu = 0.000003$, $R = 149597870.61$ km).

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5
X_o	1.00000125001256	1.000001250015503	1.000001250018664	1.000001250027542	1.00000125003045
$\dot{Y}_o$	$-2.51293384483123\times10^{-11}$	$-3.443481701619719\times10^{-11}$	$-4.417462752689427\times10^{-11}$	$-6.53399161213094\times10^{-11}$	$-7.455444338872290\times10^{-11}$
T	6.283168813947	6.28316881391477	6.28316881388254	6.28316881385031	6.28316881381827
a	1.00142000151228	1.001420114169112	1.001420114767805	1.001420125741591	1.001420132647188
b	$1.00723438364974\times10^{-4}$	$1.007234221249090\times10^{-4}$	$1.007234060345967\times10^{-4}$	$1.00723389769064\times10^{-4}$	$1.007233737281213\times10^{-4}$
c	$2.30871272679265\times10^{2}$	$2.3090653377543446\times10^{2}$	$2.309067211232234\times10^{2}$	$2.309101547215127\times10^{2}$	$2.309123166742261\times10^{2}$
d	1.00142000150503	1.001420114162944	1.001420114760661	1.001420125724428	1.001420132631970

Table 4. Periodic orbits around L_1: Family C ($\mu = 0.000003$, $R = 149597870.61$ km).

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5
X_o	−1.01003022638504	−1.010030226509163	−1.010030226710821	−1.010030227139350	−1.010030227258375
$\dot{Y}_o$	$-1.32919839300359\times10^{-8}$	$-1.359776541803719\times10^{-8}$	$-1.339225229592130\times10^{-8}$	$-1.169630081133642\times10^{-8}$	$-1.202702486666462\times10^{-8}$
T	3.054077457438488	3.053792081858969	3.05419049640754	3.053853511038311	3.054163661767323
a	$-2.87994504507463\times10^{10}$	$-2.813185085861384\times10^{10}$	$-2.859187258874128\times10^{10}$	$-3.275651430883289\times10^{10}$	$-3.183531348077306\times10^{10}$
b	$3.47906034230882\times10^{2}$	$3.476594782098364\times10^{2}$	$3.480038787702223\times10^{2}$	$3.482041155552931\times10^{2}$	$3.479807924735031\times10^{2}$
c	$2.38884742207125\times10^{18}$	$2.281006917213038\times10^{18}$	$2.353868832256234\times10^{18}$	$3.087741334071492\times10^{18}$	$2.918396714352459\times10^{18}$
d	$-2.87994504395083\times10^{10}$	$-2.813185083788247\times10^{10}$	$-2.859187258127585\times10^{10}$	$-3.275651431451012\times10^{10}$	$-3.183531347241557\times10^{10}$

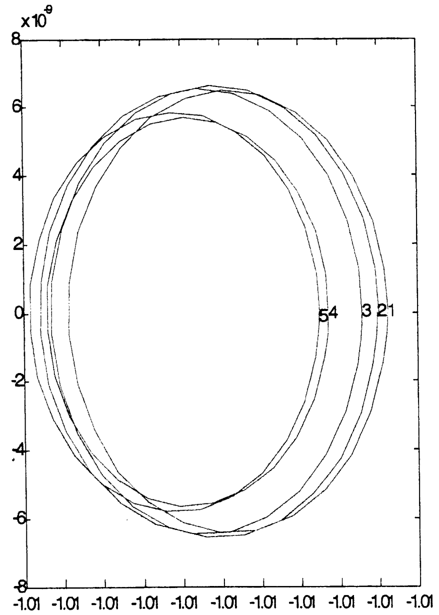

Figure 3. Periodic orbits around L_2: Family C.

6. Conclusion

We study the effect of triaxial rigid body on the periodic orbits around collinear liberation points, we also discussed the stability of these periodic orbits.

In Fig. 1, we draw the periodic orbits around L_2 in all five cases. As we increase the oblateness of the smaller primary, periodic orbits are shifting towards the origin and expanding. In Fig. 2, we draw the periodic orbits around L_3 in all five cases. In this, as we increase the oblateness of the smaller primary, we observe again that the periodic orbits are shifting towards the origin and expanding. In Fig. 3, we draw the periodic orbits around L_1 in same manner. In this, we observe that the periodic orbits go on shrinking and shifting away from the origin. In all the cases, the periodic orbits are symmetrical to x-axis. Again we have proved that the periodic orbits are unstable since in all the cases, stability parameter $|a| > 1$.

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References

Bralett, J. H., 1964, *Publ. Copenhagen obs.*, No. 179.
Bray, T. A., and Goudas, C. L., 1967, *Adv. Astr. Astrophys.*, **5**, 71.
Corbera, Montse and Llibre, Jaume, 2003, *Celest. Mech. and Dyn. Astr.*, **86**, 163.
Elipe, A. and Lara, M., 1997, *Celest. Mech. and Dyn. Astr.*, **68**, p.1.
Grau, Miquel and Noguera, Miquel, 1999, *Celest. Mech. and Dyn. Astr.*, **72**, 201.
Henon, M., 1965, *Ann. Astrophys.*, **28**, 499.
Henon, Michel, 2003, *Celest. Mech. and Dyn. Astr.*, **85**, 223.
Henrard, Jacques and Navarro, Juan F., 2004, *Celest. Mech. and Dyn. Astr.*, **89**, 285.
Markellos, V.V., 1975, *Astrophys. and Space Sci.*, **36**, 254.
Moulton, F. R., 1920, *An Introduction to Celestial Mechanics.*, Macmillan, New York.
Stromgren, E., 1935, *Publ. Copenhagen obs.*, No. 100.
Szebehely, V., 1967, *Theory of Orbits*, Academic Press, New york, p.459.