Analytical studies of standing shocks in accretion flows around compact objects

Santabrata Das*

SNBNCBS, Block-JD, Sector-III, Salt Lake City, Kolkata 700098, West Bengal, India

Abstract. Shocks in advective flows around black holes have become essential ingredients in explaining wide variety of observed features. We first analytically study the properties of shocks in the special case when viscosity and other dissipative processes are negligible. We compare our analytical results with those obtained numerically. The agreement is generally good. In the course of development of the accretion disk models we study the effects of viscosity and different types of cooling mechanisms, such as bremsstrahlung and synchrotron coolings, in succession. We identify the region of parameter space which produces multiple sonic points and shocks separately in presence of heating and cooling effects. We quantify two critical viscosity parameters and a critical cooling factor which separate the flow topologies.

Keywords: accretion, accretion disk–black hole physics–hydrodynamics–shock-waves

1. Introduction

Numerical results of transonic properties of the accretion flow around a compact object in the case of adiabatic and non-adiabatic flows already exist in the astrophysical literature (Chakrabarti 1989, 1996). An analytical approach is presented recently to study the properties of standing shock waves for non-dissipative flows (Das et al. 2001). However, in a realistic flow, one has to take into account all the dissipative effects, such as, viscosity, bremsstrahlung and synchrotron coolings etc. In the present study, we work out solution topologies and obtain the parameter space for shock formation considering non-dissipative as well as dissipative system (Das & Chakrabarti 2004, Chakrabarti & Das 2004, Das
In the next Section, we present the governing equations to study the properties of the transonic flows and in Section 3, we draw our conclusions.

Figure 1. Meridional cross section of an accretion disk. CENBOL refers to the centrifugal pressure supported boundary layer that arises because of the competition between the attractive gravity force and the repulsive centrifugal force in the accretion flow. BH represents a Black hole. (Adapted from Chakrabarti & Das 2001).

2. Basic equation

We start with a thin, axisymmetric, rotating, viscous, accretion flow around a Schwarzschild black hole. The space-time geometry can be satisfactorily described by the pseudo-Newtonian potential (Paczyński and Wiita 1980) and is given by \( g(x) = -\frac{1}{2(x-1)} \), where, \( x \) is the radial distance in dimensionless unit. In the steady state, the dimensionless hydrodynamic equations that govern the infalling matter are the followings (Chakrabarti 1996).

1. Radial momentum equation: \( \frac{d}{dx} \left( \rho \frac{d\phi}{dx} \right) + \frac{1}{2} \frac{dP}{dx} - \frac{\lambda(x)^2}{x^2} + \frac{1}{2(x-1)^2} = 0 \).
2. Baryon number conservation equation: \( \dot{M} = \Sigma \vartheta x \).
3. Angular momentum conservation equation: \( \frac{\vartheta}{x^2} \frac{d}{dx} \left( x^2 W_{z\phi} \right) = 0 \).
4. The entropy generation equation: \( \Sigma \vartheta T \frac{ds}{dx} = Q^+ - Q^- \), where, \( Q^+ \) and \( Q^- \) are the heat gained and lost by the flow and rest of the quantities denote their usual meanings. The distances, velocities and masses are made dimensionless by using \( r_g = 2GM_{BH}/c^2 \), the Schwarzschild radius, \( c \), the velocity of light and \( M_{BH} \), the mass of the black hole, respectively.

The transonic flow properties are obtained by solving the above set of equations (Eq. 1-4) following standard shock finding method. For suitable set of input parameters flow may make shock transition. If Rankine-Hugoniot relations (Landau & Lifshitz 1959) are satisfied, this shock is called a standing shock. When shock oscillates, it causes quasi-periodic oscillation of observed hard X-rays which is known as QPO (Chakrabarti & Manickam 2000).
Analytical studies of standing shocks

In the present discussion, we showed that shocks, standing or oscillating, are found to be indispensable for non-dissipative as well as dissipative accretion flow. In this paper, we

Figure 2. Upper-left: Phase space diagram of the accretion flow. Upper-right: Comparison of the parameter space spanned by the specific energy $E$ and the specific angular momentum ($\lambda$) for non-dissipative flow in three different models in which shocks form. Solid boundaries and shaded regions are obtained using analytical and numerical method. Figure shows that the agreement is generally good (Chakrabarti & Das 2001). Lower-left: Division of parameter space for viscous flow on the basis of solution topologies shown in boxes (Chakrabarti & Das 2004). Lower-right: Division of parameter space for dissipative flow (including heating and cooling).

3. Conclusions

In the present discussion, we showed that shocks, standing or oscillating, are found to be indispensable for non-dissipative as well as dissipative accretion flow. In this paper, we
Figure 3. Left: Variation of critical viscosities \( \alpha_\Pi \) as a function of specific angular momentum when cooling effects are ignored. \( \alpha_\Pi \) separates standing from oscillating shocks and closed topologies from open topologies (Chakrabarti & Das 2004). Right: Critical cooling factor separating closed to open topologies. The dashed curve is obtained for higher \( \alpha_\Pi \) than the solid curve (Das & Chakrabarti 2004).

reported that shock could be studied completely analytically at least for non-dissipative flow. We also obtained the parameter space for standing as well as oscillating shock for a dissipative polytropic accretion flow. We identified the critical viscosity parameters and critical cooling factor which are strongly dependent on the inflow parameters.

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References