Deconvolution in Radio Astronomy

by Subhashis Roy

$V(u_k, v_k) = \hat{V}(u_k, v_k) + \epsilon(u_k, v_k)$

$V(u,v) = W(u,v)\,[\hat{V}(u,v) + \epsilon(u,v)]$

$W(u,v) = \sum_k W_k\, \delta(u - u_k, v - v_k)$

Convolution by $F(W)$ in the Image domain.

$I^P_{p,q} = \sum_{p',q'} B_{p-p', q-q'}\, I_{p'q'} + E_{pq}$

E_{pq}: F.T. of $\epsilon(u,v)$

MEM $\quad H = -\sum_K I_K \ln \frac{I_K}{M_K}$ — I_K → Image, M_K → model

problem with point src. decon. in extended emission

What is it and requirements

No. of independent measurements:

$V(u, \quad v) \quad = \quad \sum_{p=1}^{N_l} \quad \sum_{q=1}^{N_m} \quad I'(p\Delta l, \quad q\Delta m)$ $e^{-2\pi i(p\Delta l.u+q\Delta m.v)}$

These many sampling in u, v plane should be there.

Only a small fraction measured.
~~Rest are Z~~ have measurement with zero weight.

Invisible distribution and Principle solution

Unique solution to $I_D = B * F(V')$ to determine $F(V')$?

Unmeasured frequencies in u, v plane:

Consider intensity distribution Z, such that $B * Z = 0$
F.T. of I_D and $I_D + n.Z$ both fits the V^S.
$n = 0$ is called Principal solution.

$n = 0$ gives only Dirty image with side-lobes. Non-linear deconvolution needed.

No unique solution from Mathematical side.

A priori information

F.T. of Sampling fn. is beam, having positive and negative values.

Dirty image has negatives,
real source is positive only.

Could be used during Deconvolution and is used in MEM.

Algorithms in common use

The CLEAN algorithm

Hogbom algorithm

Developed by Hogbom (1974). Assumes a combination of a few point sources in the field of view.

Steps:

1. Find the strength and position of the peak in I_D.
2. Subtract from I_D at the position of the peak, ~~the~~ B times the Peak with a 'loop gain' $(I_D - B.S.\alpha)$.
3. Record the position and strength of the point source subtracted in a model (file).
4. Go to 1, unless the remaining Peak is below user threshold.
5. Convolve the accumulated Point-source model with an idealisd Clean beam (usually elliptical Gaussian fitted to the central lobe of the Dirty beam).
6. Add the residuals of the Dirty image to the Clean image from 5.

Clark algorithm

Minor cycle:
Only a small part of Beam is used to subtract source diffraction from the Dirty map.
Major cycle:
FFT of accumulated Point source models, multiplication by weighted Sampling function and again FFT to make a Dirty image.
This Dirty image subtracted from original I_D.

The Cotton-Schwab algorithm

Variant of Clark algorithm.
In major cycles, Point source models are subtracted from the V^S (ungridded visibilities).

Implemented in IMAGR.
Can use DFT for a small no. of Point sources.
Gridding errors can be avoided.
w term can be used.

Practical issues regarding Imaging

(a) Use of Boxes.

(b) No. of iterations, loop gain and Beam patch size.

(c) The problem of Short spacings.

(d) Clean instabilities and extended sources.

Other Topics

(i) S.D.I Clean for extended sources.

(ii) The understanding of Clean is poor in presence of noise.

Schwarz (1979) has shown Hogbom Clean is ≠ least square ~~fitting of~~ minimisation ~~of models~~ betw. obs. and model visibility.

(iii) The Clean beam (restoring)

(iv) Use of a-priori models → model of disk for planets.

(vi) Non-Uniqueness → Depends on Clean control parameters.

(vii) Instability p-102 → modulation at unsampled parts.