# Statistics of cosmic Radio waves

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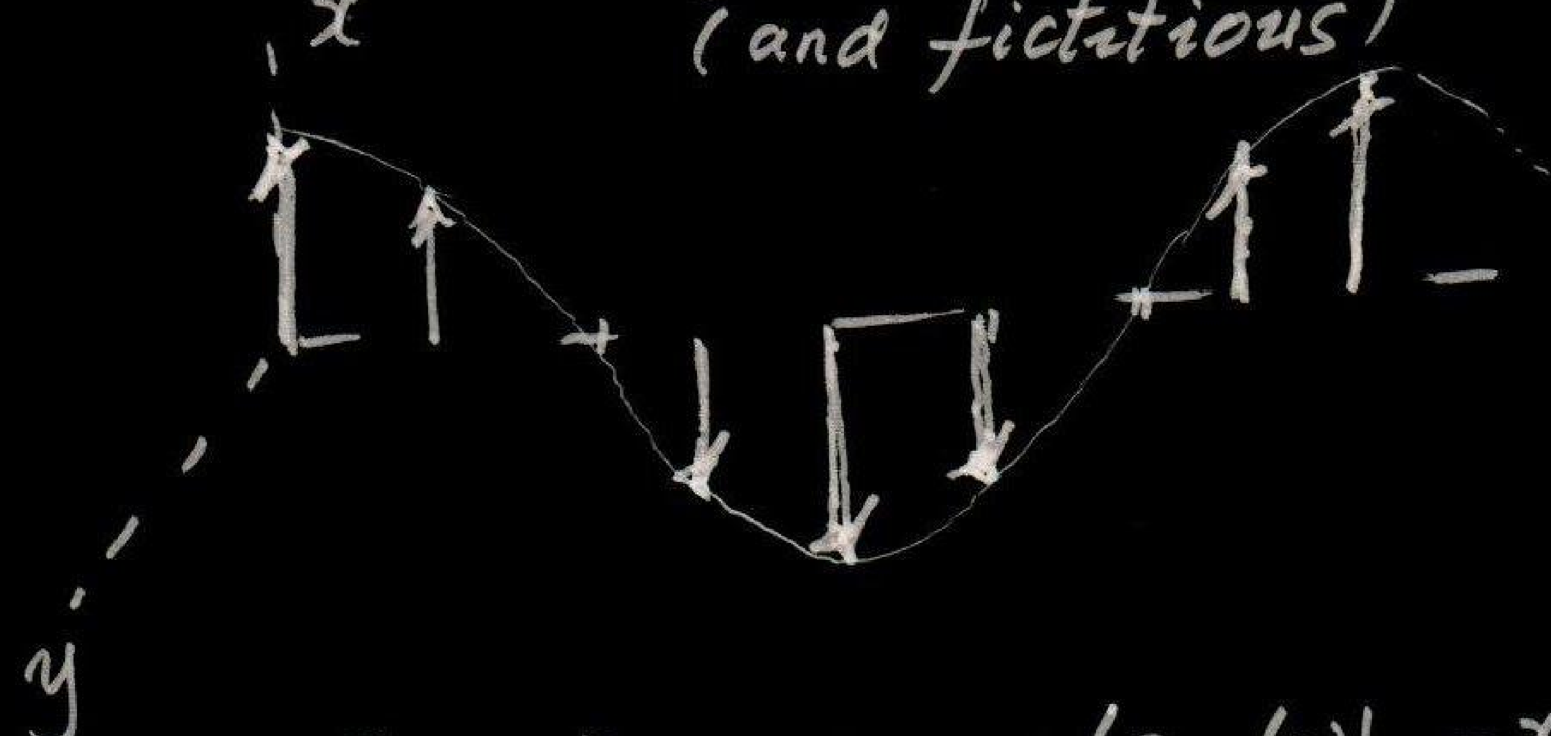
# The simplest EM wave

- plane, linearly polarised, monochromatic (and fictitious)




$$E_x = B_y = a\cos\left(2\pi\left(\nu t - \frac{x}{\lambda}\right)\right)$$

$$\nu = \frac{2\pi}{T} = c/\lambda$$

# A real radio wave from a natural source (e.g. a hot body)

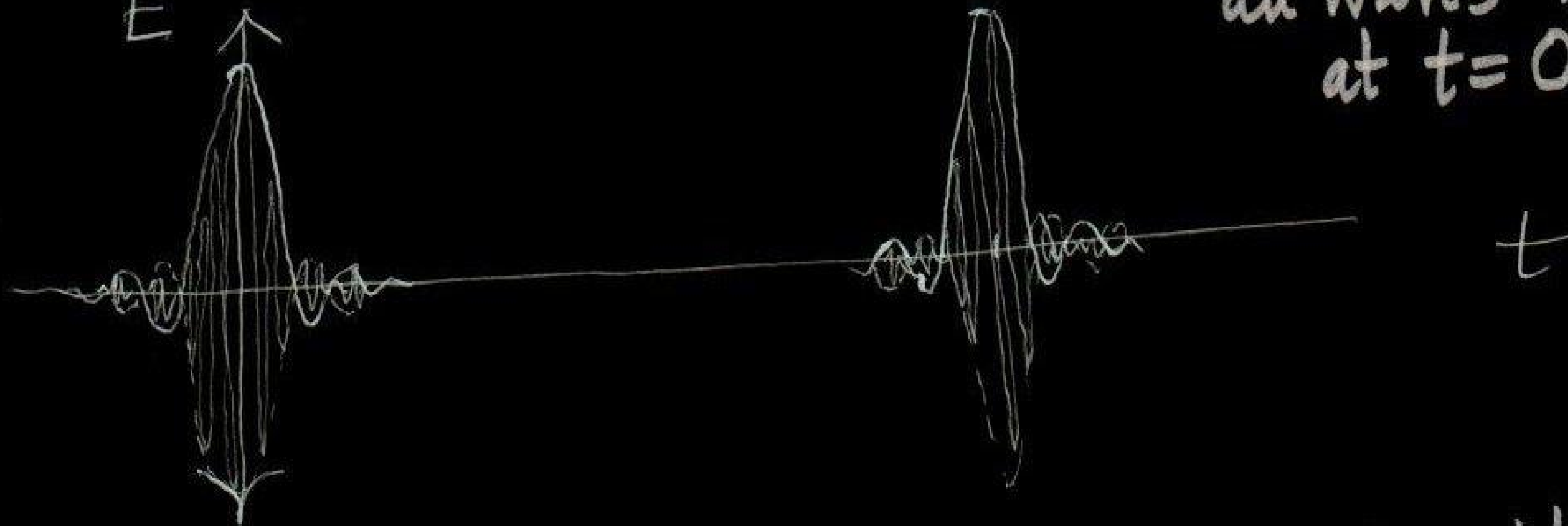
- The E (& B) fields fluctuate in amplitude, polarisation, and phase in a random way – hence statistics
- Model – superpose a large number of waves with frequencies

$$\nu_0 - B/2 < \nu < \nu_0 + B/2 \quad \text{and}$$

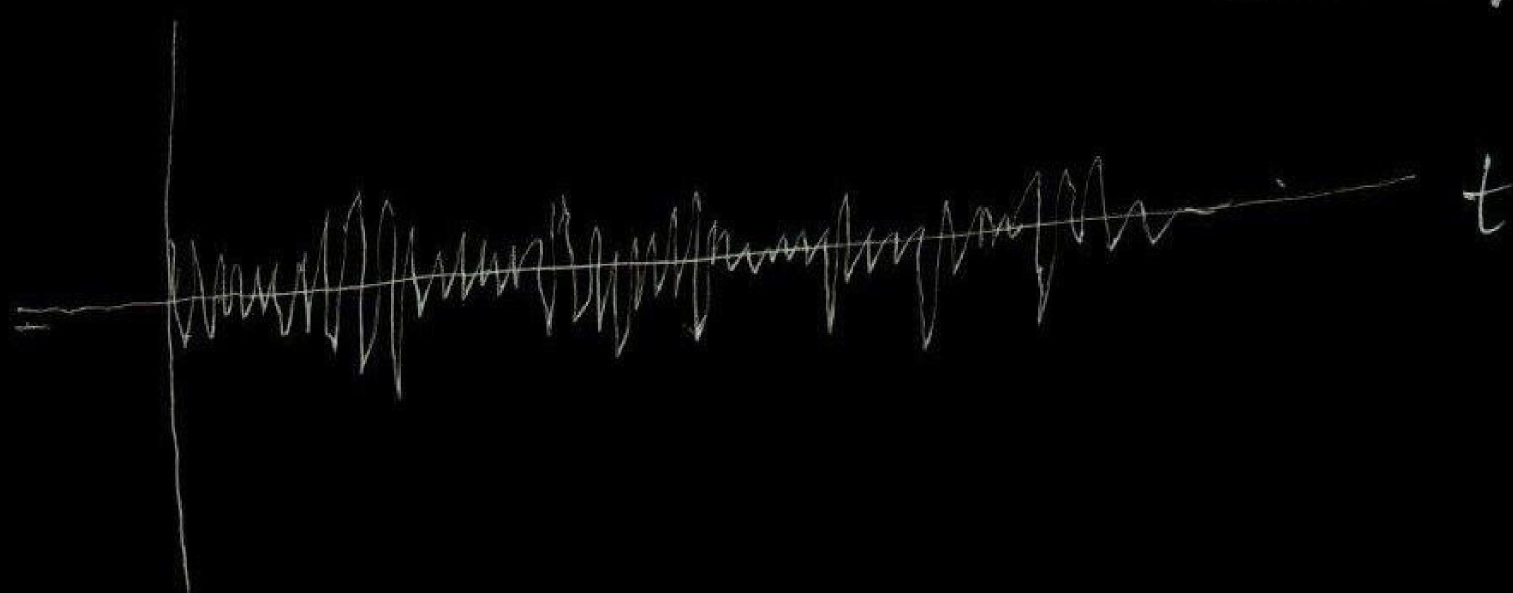
random phases

# Two kinds of Superposition

all waves in phase
at $t=0$



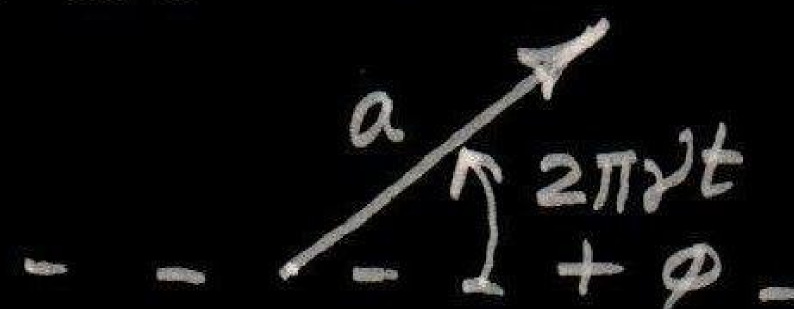

random phases



# Phasors: how to superpose waves with the right side of your brain

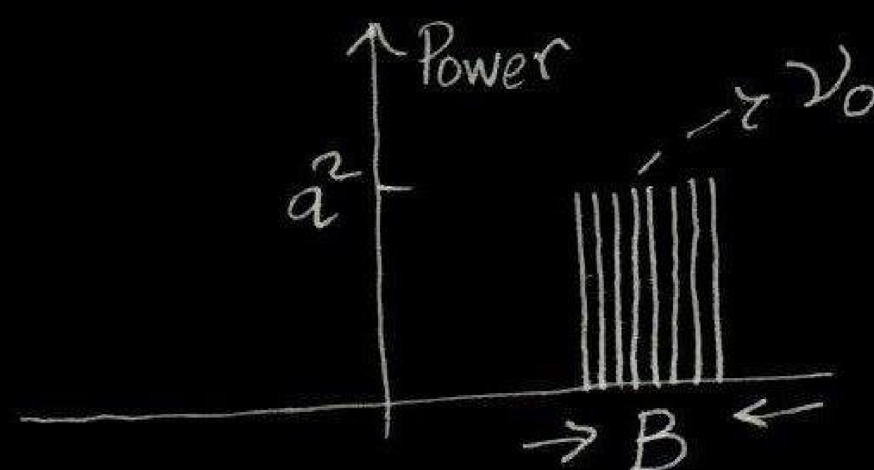
$$a\cos(2\pi\nu t+\varphi) = Re\; a e^{-i\varphi} e^{-2\pi i\nu t}$$

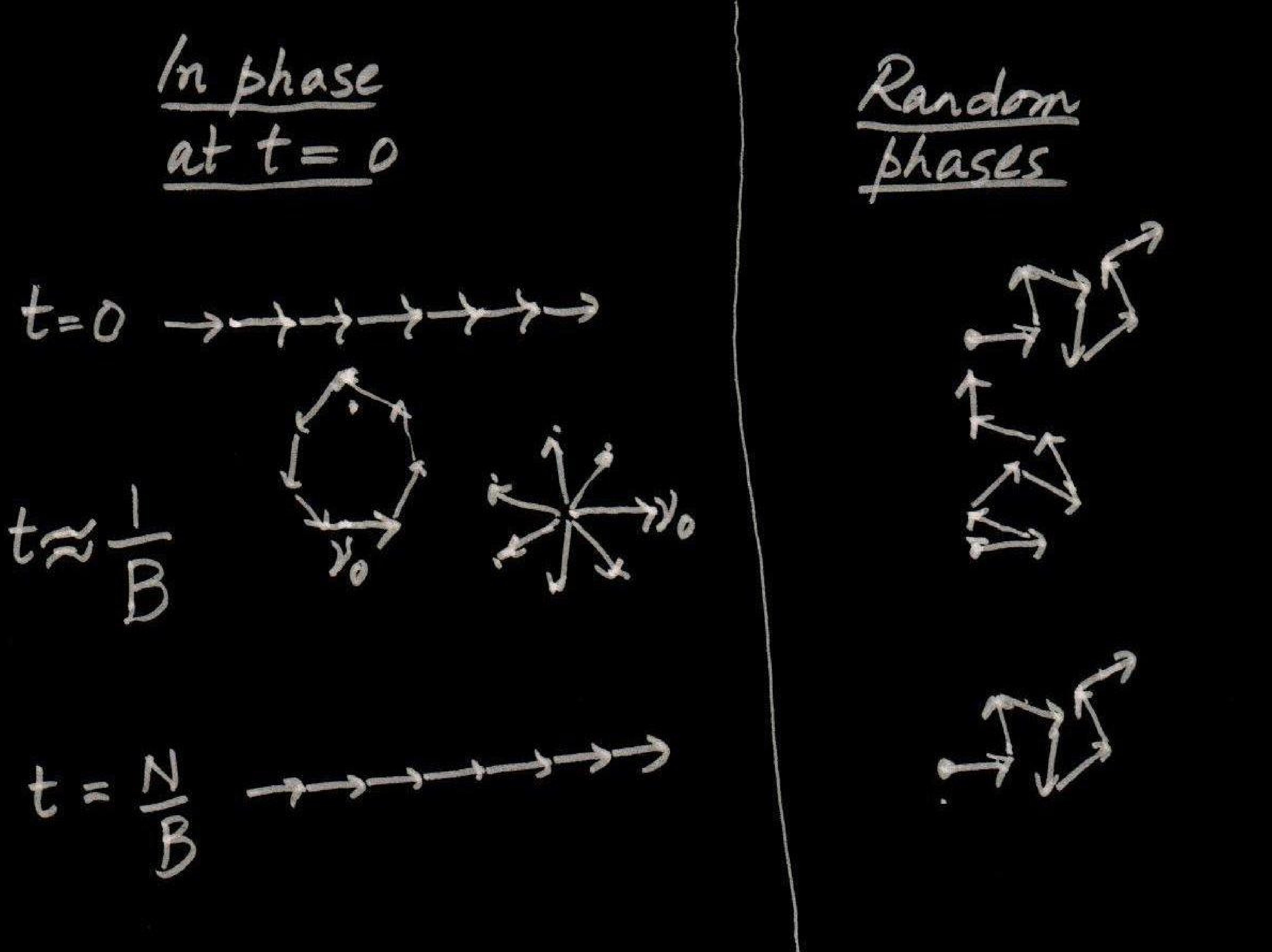
= horizontal projection of




Example: $N$ waves, all of amplitude $a$ separated by $\Delta\nu$ around $\nu_0$,

$N\Delta\nu = B$




In phase at t = 0
Random phases
t=0
t≈1/B
ν0
ν0
t = N/B

# The two dimensional random walk.

$$\vec{r}_N = a(\vec{n}_1 + \vec{n}_2 + \cdots \vec{n}_N)$$

$$\ell_N^2 = \vec{r}_N \cdot \vec{r}_N$$

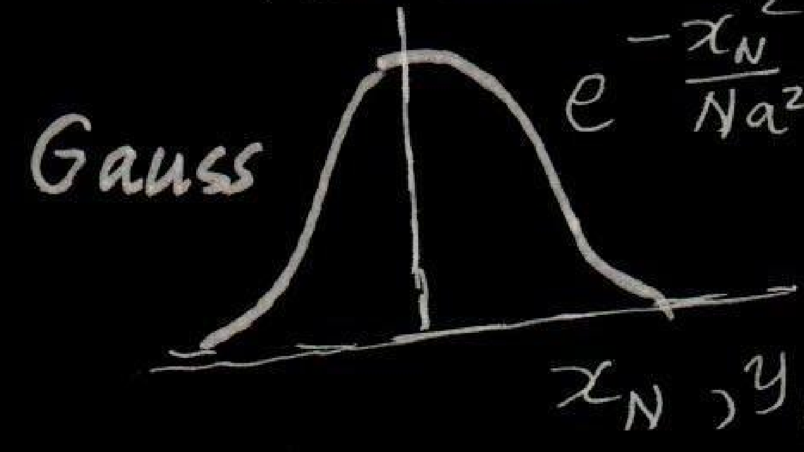
$$= a^2(\vec{n}_1 \cdot \vec{n}_1 + \cdots \vec{n}_N \cdot \vec{n}_N)$$

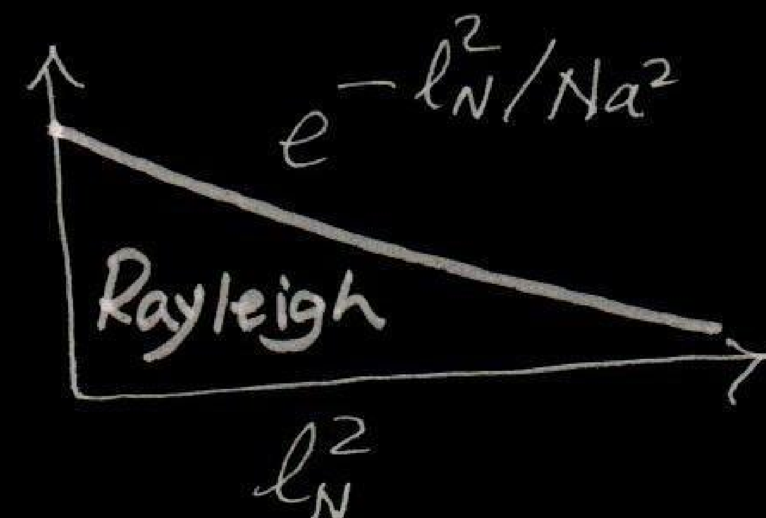
$$+ 2a^2(\vec{n}_1 \cdot \vec{n}_2 + \cdot + \vec{n}_i \cdot \vec{n}_{j>i})$$

$$\overline{\ell_N^2} = Na^2$$

i.e typical $\ell_N \uparrow N^{1/2}$

Distributions



## Coherence/Correlation: A measure of resemblance:

$$\overline{\left(\mathcal{E}(0)+\mathcal{E}(t)\right)\left(\mathcal{E}(0)+\mathcal{E}(t)\right)^*}$$

$$= \overline{\mathcal{E}(0)\,\mathcal{E}^*(0)} + \overline{\mathcal{E}(t)\,\mathcal{E}^*(t)}$$

$$+ \underbrace{\overline{\mathcal{E}(0)\,\mathcal{E}(t)^*} + \overline{\mathcal{E}(t)\,\mathcal{E}(0)^*}}_{\text{Interference term}}$$

$$= \overline{2\,|\mathcal{E}(0)|\,|\mathcal{E}(t)|\cos\left(\phi(0)-\phi(t)\right)}$$

## Autcorrelation of our $(\nu_0, B)$ signal

$$C(t) = \overline{\sum_{\nu_n} a_n e^{i\varphi_n} \sum_{\nu_m} a_m e^{-i\varphi_m} e^{-i2\pi\nu_m t}}$$

only $n=m$ survive $\because$ random phases

$$= \sum_{\nu_n} a_n^2 e^{-i2\pi\nu_n t} \equiv \int d\nu\, P(\nu) e^{-2\pi i\nu t}$$

$P(\nu)\,d\nu$ = Sum of all $\overline{a_n^2}$ inside $d\nu$

$P(\nu) \equiv$ "power spectrum"

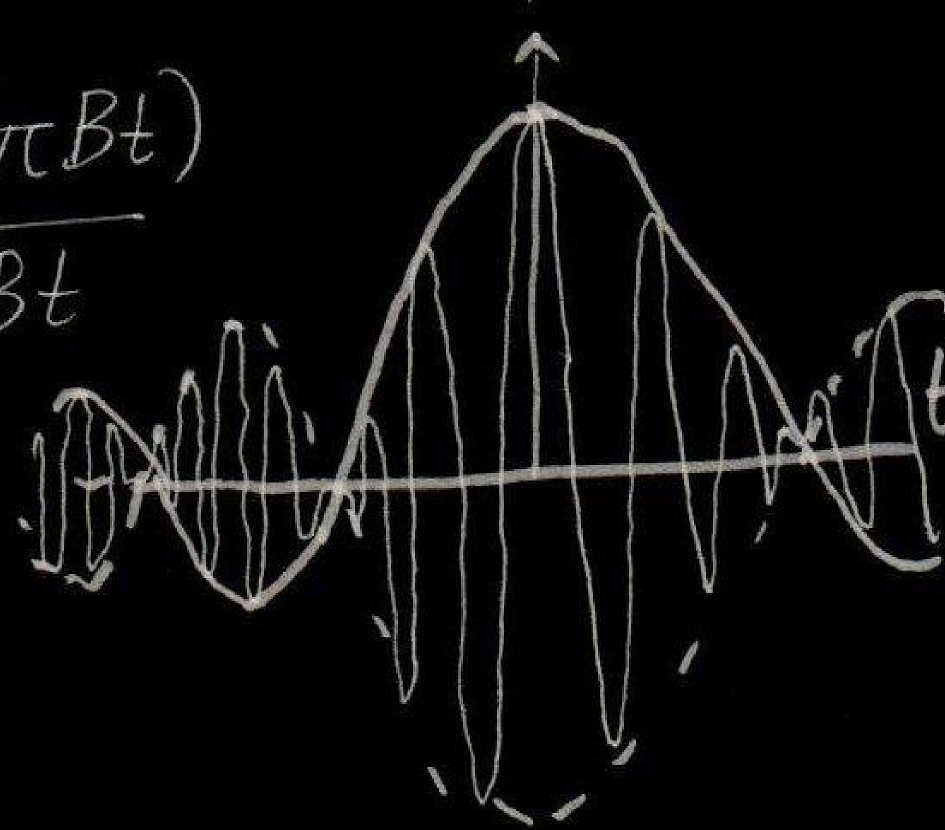
# Case of a narrow & flat band

"Quasimonochromatic"

$$\int_{\nu_0-B/2}^{\nu_0+B/2} P\, e^{-2\pi i \nu t}\, d\nu = P e^{-2\pi i \nu_0 t} \int_{x=-B/2}^{B/2} dx\, e^{-2\pi i x t}$$

$$= P e^{-2\pi i \nu_0 t}\, B\, \frac{\sin(\pi B t)}{\pi B t}$$

i.e a "sinc"
(modulated at $\nu_0$)
vanishing at $\frac{1}{2B} = \tau_N$)



# Qualitative picture: wave packets

- for $t \ll \frac{1}{B}$, we have a 'monochromatic' wave with freq. $\nu_0$ & amplitude and phase stable.

- Every stretch of $\frac{1}{B}$ can be regarded as having a different amplitude & phases drawn from the $x_N, y_N$ distribution.

## Accuracy of power measurement

From Rayleigh, $\Delta P_{rms,\text{one sample}} = P\sqrt{2}$

$$P_{estimate} = \frac{\bar{P} + \Delta P_1 + \bar{P} + \Delta P_2 + \ldots \bar{P} + \Delta P_N}{N}$$

$$= \bar{P} + \frac{(\Delta P_1 \ldots + \Delta P_N)}{N} = \bar{P} \pm \frac{\Delta P_{rms,1}}{\sqrt{N}}$$

random sign errors!

# N stands for Nyquist

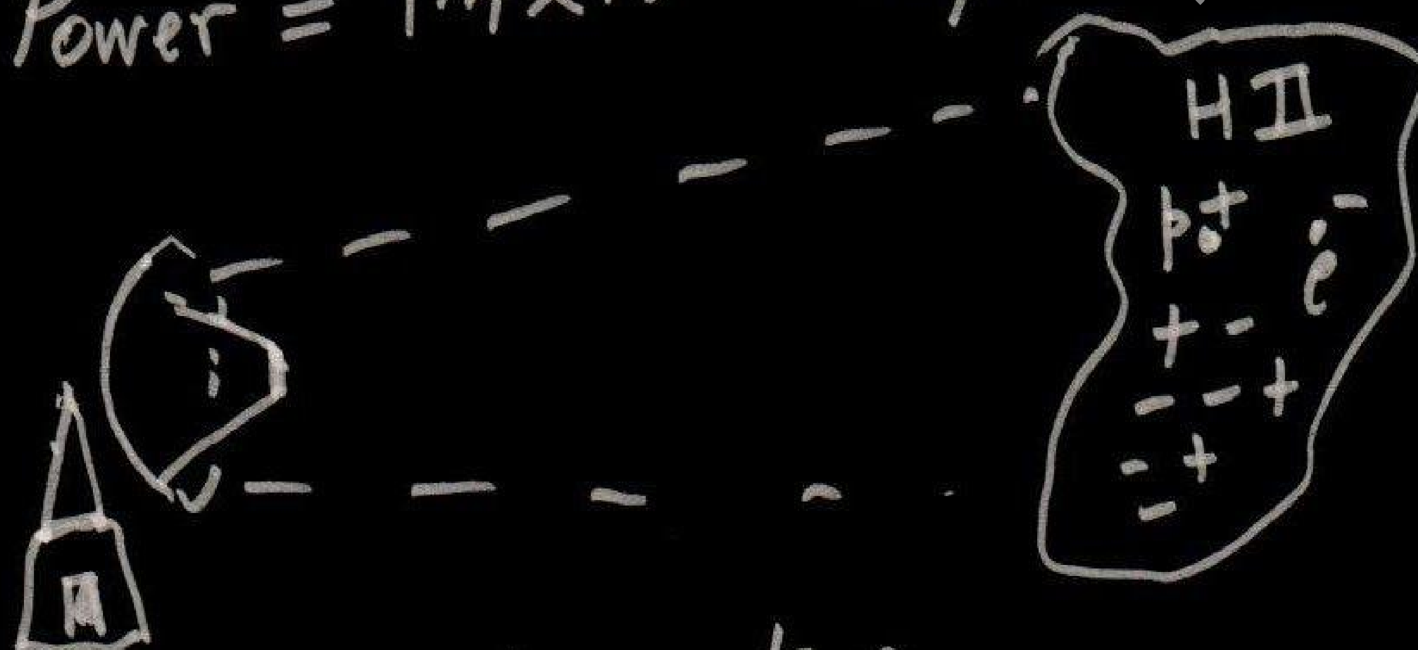
- For a flat spectrum, samples spaced by $\frac{1}{2B}$ are independent.
- # measurements = $2Bt$

$\therefore \Delta P_{rms,t} = \overline{P}/\sqrt{B \cdot t}$

$\overline{P}$ = energy/time/frequency interval $\equiv k_B T$.
as if one is looking at a hot body!

# Examples of T.


Power $= 1.4 \times 10^{-17}$ W/MHz/K




Direct temperature measurement of an H II region.

## Unpolarised radiation:

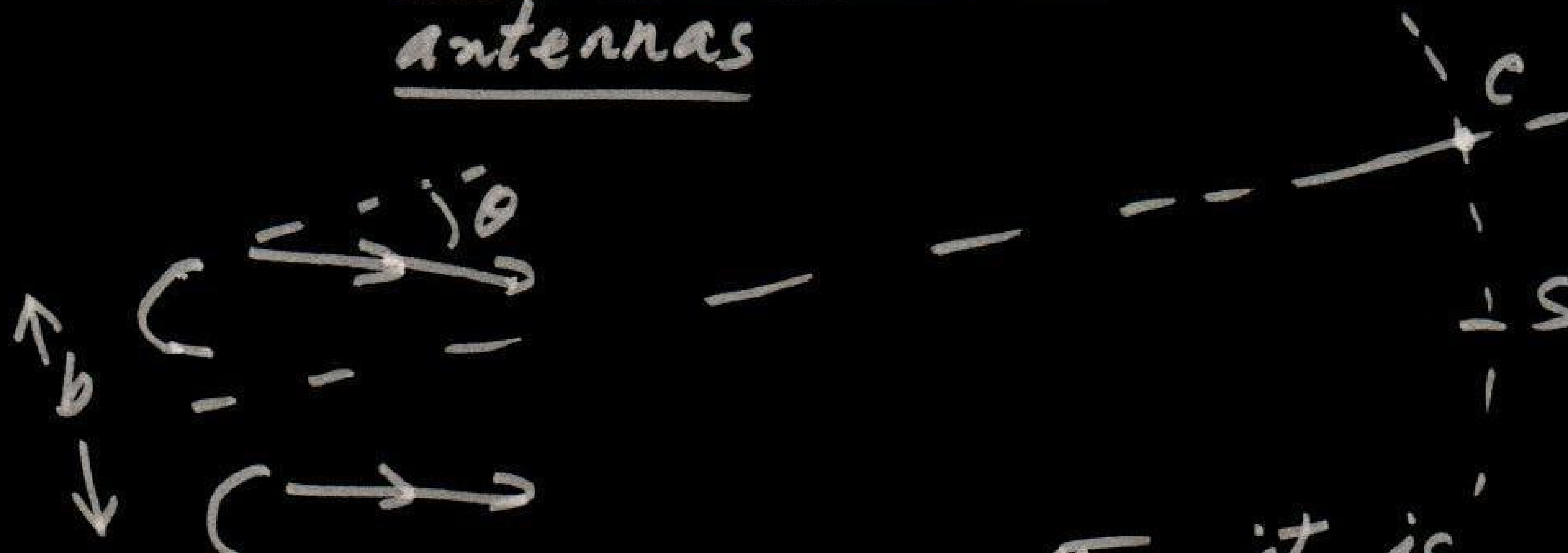
- $E_x$ & $E_y$ are both statistically similar ie same $P(\nu)$

- But they are independent ie the random phases in each are different

- For $t \ll \frac{1}{B}$, one will have the appearance of some polarised state e.g linear, elliptic ~~etc~~.

- For $t \gg 1/B$ averages over all.

# Correlation between two antennas




- If delay = 0 at C, then it is $b \cdot \theta$ for a source at S.
- Signals in a very narrow band will have a correlation $e^{-2\pi i \frac{b\theta}{c} \cdot \nu_0}$
- Different sources are independent.