Coherence, Correlation, convolution and matched-filtering

--desh

Raman Research Institute, Bangalore, India

RAS-2013 (22 Aug)

What keeps radio astronomers busy… OR what are they after really…mostly.?

Sky Noise !, which brings us the cosmic news, unlike in communications, where signal and noise have explicit distinction.

Noise properties (necessarily statistical)….

related to color (spectrum), prob. density fn.

coherence, if any (spatial, temporal)

and their variation in time, and polarization state, etc.

What keeps radio astronomers busy… OR what are they after really…mostly.?

Intensity (i.e. variance of noise)(1) as a function of

- Sky co-ordinates (2,3)
- Frequency (4)
- Time (5)
- Polarization state (6)

The spans and resolutions in these six parameters, i.e. the 12 numbers, together would essentially define vital-statistics of a telescope.

But not all can be specified mutually independently !

But not all can be specified mutually independently !

Intensity : span/resolution : dynamic range

: dependence on collecting area,

spectral bandwidth, observation time (duration)

sky coverage, angular resolution:

depend on collecting area & frequency

Frequency : resolution limited to total time-span

Time : resolution limited to total spectral-span

Polarization state : antenna response in polarization

Transducer for sensing incident radiation at radio frequencies

- Individual photon energies ($h\nu$) are too small at radio frequencies, hence even a swarm of millions of radio photons can not free an electron from its atom, or initiate photo-chemical changes.

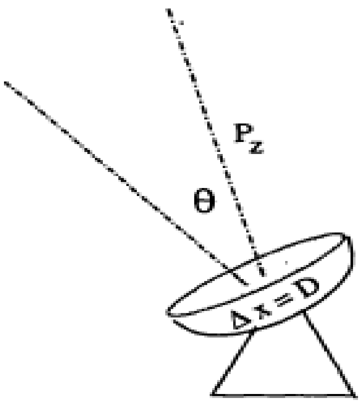
- The radio photons do however induce currents in antenna. The currents are too weak to trigger any detector, unless can be amplified in suitable device (e.g. by amplifiers).

Unavoidables and relevant in even the single photon case

A

Take a telescope of diameter D operating at any wavelength λ

Let its beamwidth be θ



Now uncertainty of photon arrival position is clearly $\Delta x = D$

Uncertainty of photon transverse momentum ΔP_x is $P_z \theta$

$$\Delta x \, \Delta P_x = D P_z \theta \sim h, \text{ or } \theta \approx \frac{h}{D P_z}$$

now $$P_z = \frac{\text{energy}}{\text{velocity}} = \frac{h\nu}{c}$$

$$\theta \approx \frac{h c}{h \nu D} = \frac{\lambda}{D}$$

YOUR DIFFRACTION FORMULA !!

B

Similarly, $\Delta z \, \Delta P_z \sim h$

Δz = Uncertainty in longitudinal position
$= c \, \Delta t$

And ΔP_z = Uncertainty in longitudinal momentum

But longitudinal momentum $= \dfrac{h \nu}{c}$

so $\Delta P_z = \Delta\nu \dfrac{h}{c}$

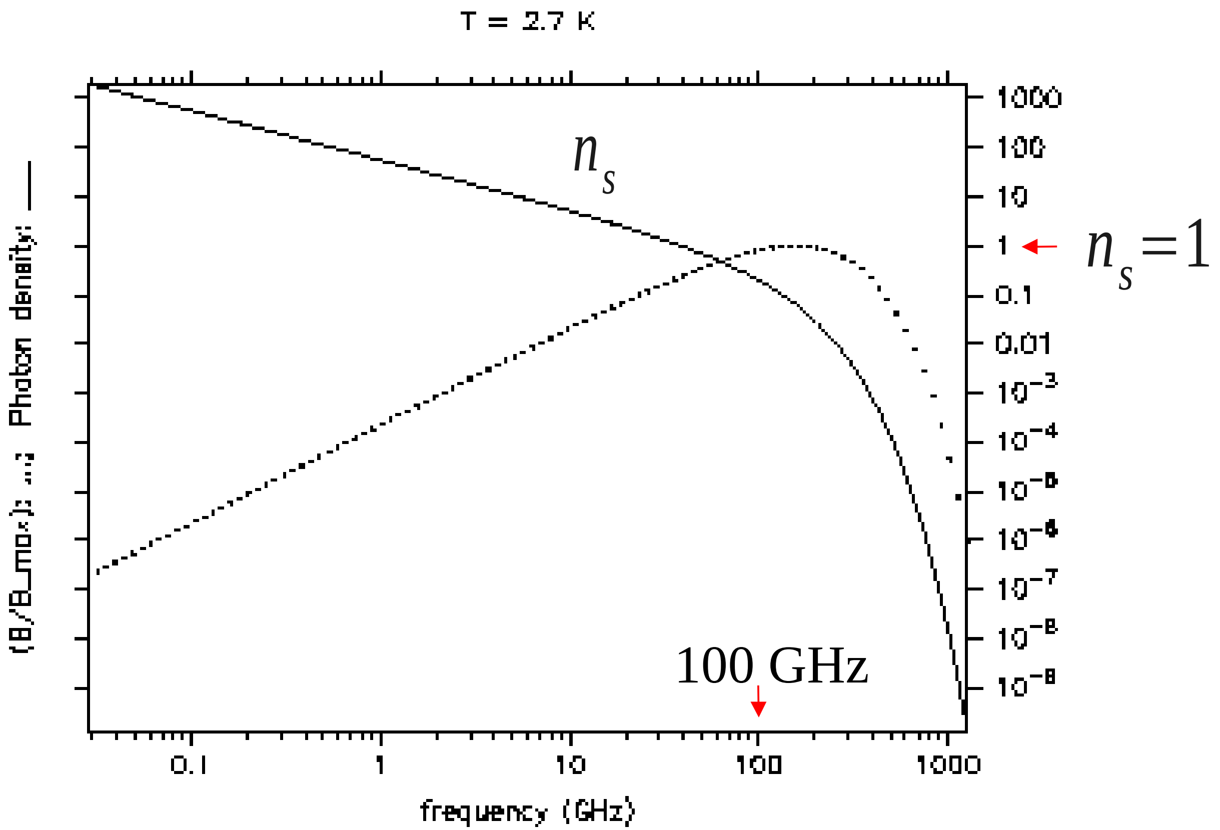
so, $\Delta z \, \Delta P_z \sim h \longrightarrow c \Delta t \, \Delta\nu \dfrac{h}{c} \sim h$

or $$\Delta\nu \approx \frac{1}{\Delta t}$$

i.e. RESPONSE TIME OF A FILTER !!

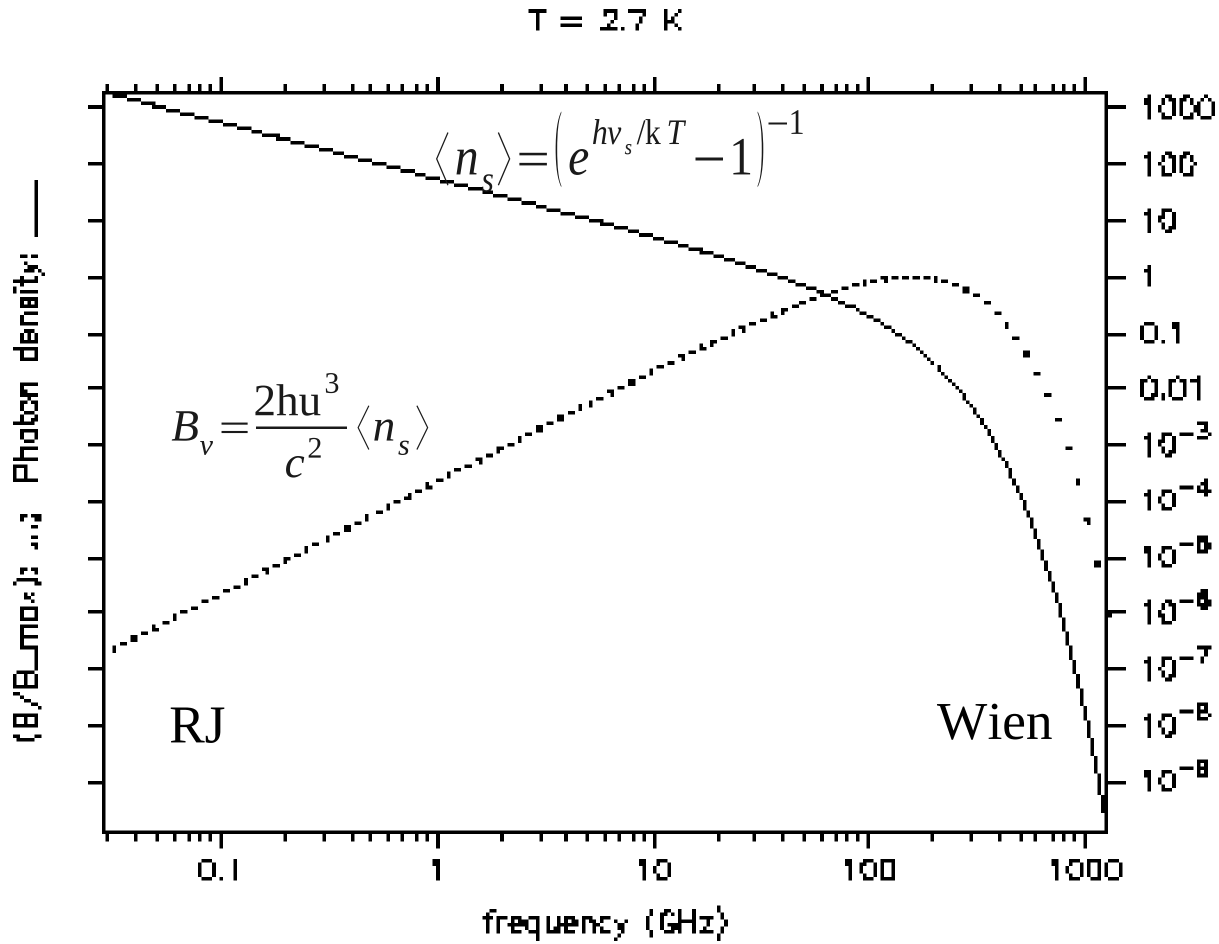
Figure 33–3. Heisenberg's uncertainty principle connects naturally, A) the aperture size and resolving power of telescopes, or B) the bandwidths and response times of filters, even when dealing with single photons.

The sky is not dark in the radio!



"Even the feeble microwave background ensures that the occupation number at most radio frequencies is already high. In other words, even though the particular contribution to the signal that we seek is very very weak, it is already in a classical sea of noise and if there are benefits to be derived from retaining the associated aspects, we would be foolish to pass them up."   Radhakrishnan 1998

When is wave noise important? Photon occupation number at 2.7K

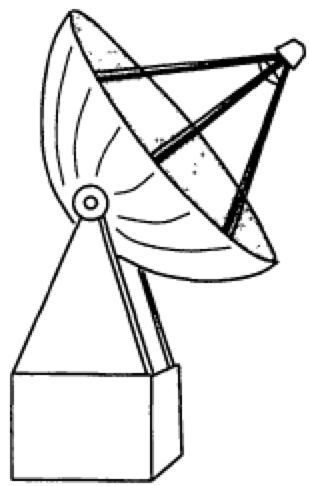


$$\langle Dn_s^2 \rangle ^o \left\langle \left(n_s - \langle n_s \rangle \right)^2 \right\rangle = \langle n_s \rangle + \langle n_s \rangle^2$$

$$\text{Wien}: n_s < 1 \Rightarrow \text{rms}\ \mu \sqrt{n_s}\ (\text{countingstats})$$

$$\text{RJ}: n_s > 1 \Rightarrow \text{rms}\ \mu n_s\ (\text{wavenoise})$$

A RADIO



IF source is strong, $T_A >> T_{REC}$

$$T_A \propto D^2$$

$$\Delta T_A = \frac{T_A}{\sqrt{t\Delta\nu}}$$

$$\therefore \frac{\Delta T_A}{T_A} = \frac{1}{\sqrt{t\Delta\nu}}$$

INDEPENDENT of DIAMETER.

B OPTICAL

N (no. of photons) $\propto D^2$

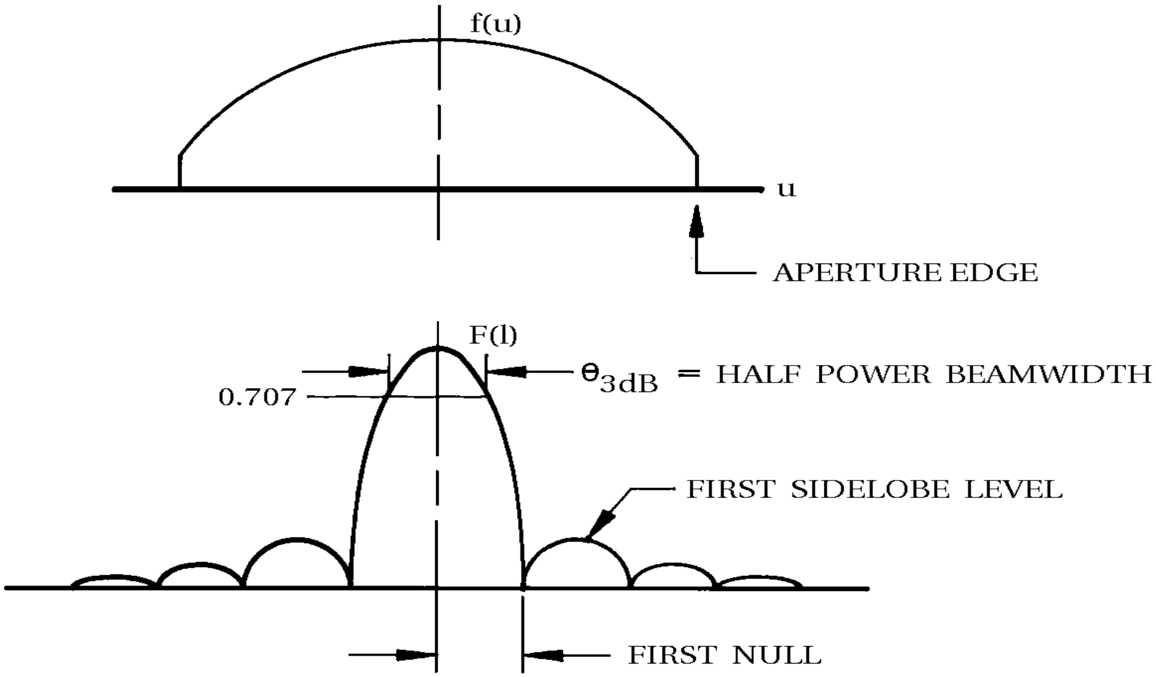
$$\frac{\Delta N}{N} = \frac{1}{\sqrt{N}}$$

$$\therefore \frac{\Delta N}{N} \propto \frac{1}{D}$$

THE BIGGER THE BETTER, ALWAYS.

Fourier pairs in astronomy:

aperture illumination ←→ far-field diffraction pattern



Another Fourier pairs in astronomy:

brightness distribution ←→ spatial-coherence function

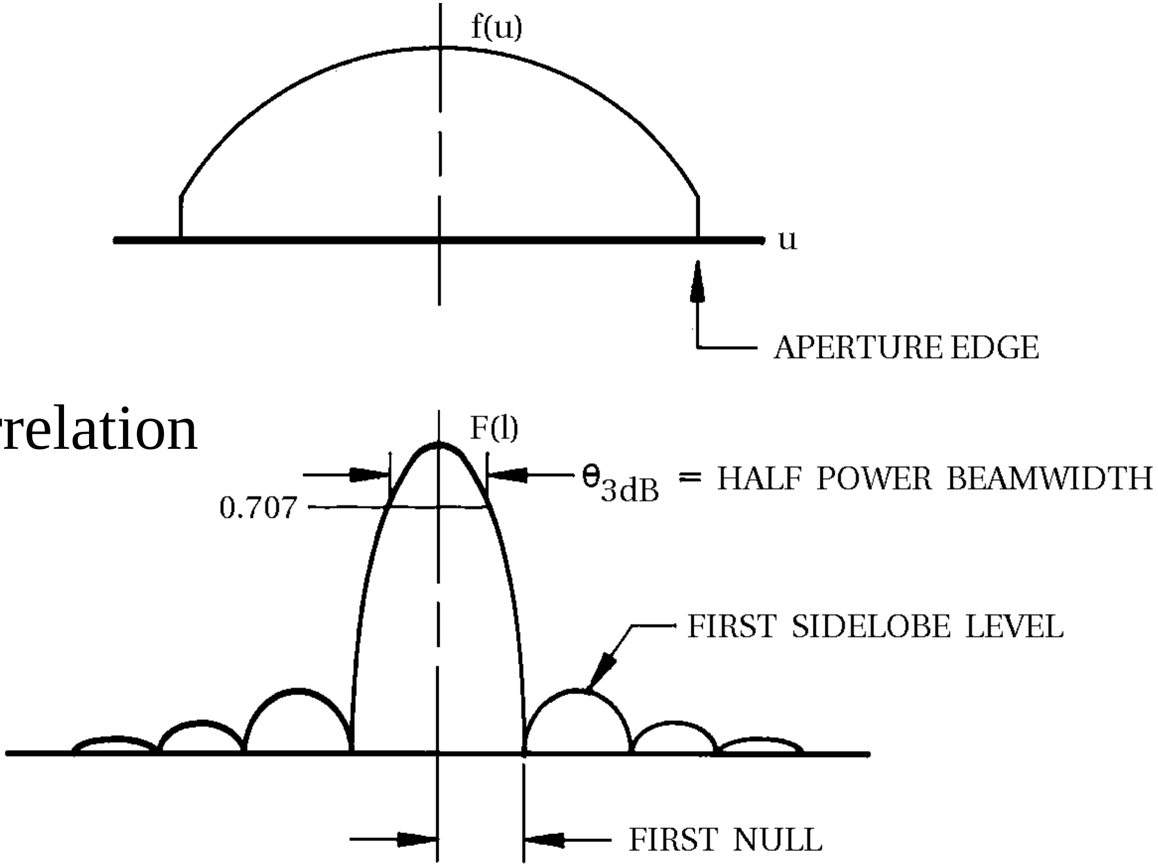
Fourier pairs in astronomy:

aperture illumination ←→ far-field diffraction pattern

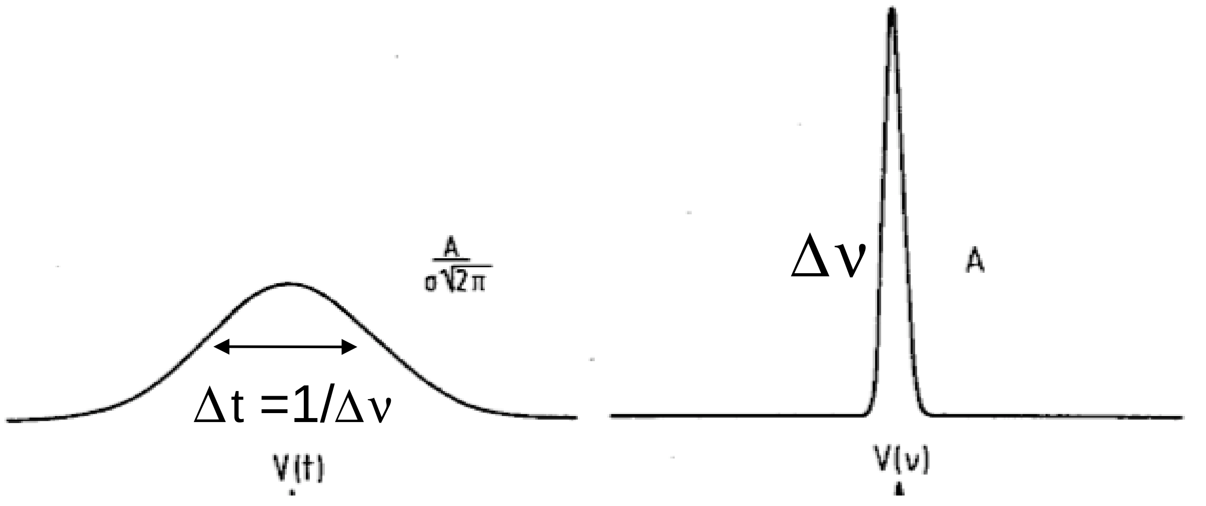
Recall FT stuff

Shift ←→ phase gradient
Product ←→ convolution
Mod square ←→ auto-correlation
Also, linear combinations
 preserved across the
two domains

Same is relevant for the
other Fourier pair
Time-sequence & spectrum



General Fourier conjugate variable relationships

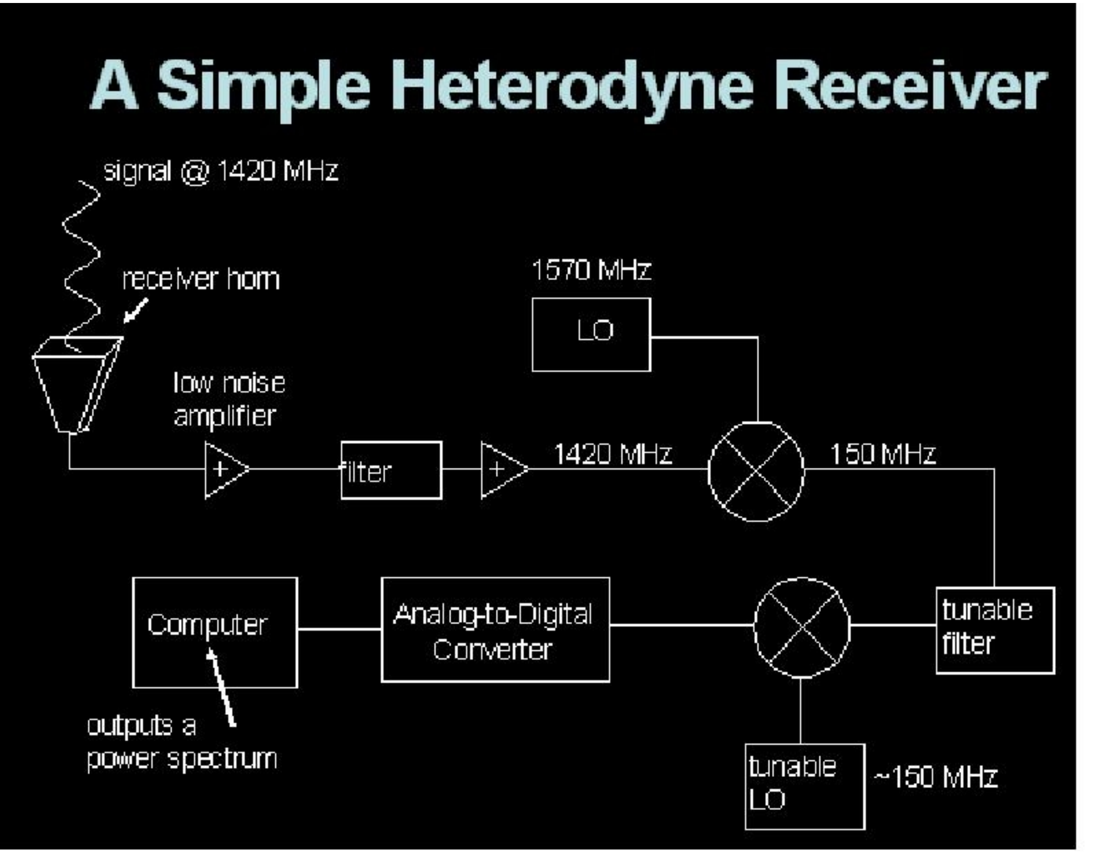


- Fourier conjugate variables, frequency -- time (or power spectrum in freq, autocorrelation in lag, eg. Weiner-Khinchin theorem)
- If V(ν) is Gaussian of width Δν, then V(t) is also Gaussian of width = Δt = 1/Δν
- **Measurements of V(t) on timescales Δt < 1/Δν are correlated, ie. *not independent***
- **Restatement of Nyquist sampling theorem: maximum information is gained by sampling at ~ 1/ 2Δν. Nothing changes on shorter timescales.**

To be revisited later, but mention key aspects: noise from receiver (Trec) and "coherent" amplification

Desired : amplification
Price paid: additional
noise from
the amplifier
and other
receiver components

even though one tries to keep
these costs to minimum
e.g. low-noise amplifiers, etc.



Noise limit: quantum noise and coherent amplifiers

Uncertainty principle for photons:

$$D\mathrm{E}\ \ Dt=h$$

$$DE=hvDn_s$$

$$Dt=\frac{Dj}{v2p}$$

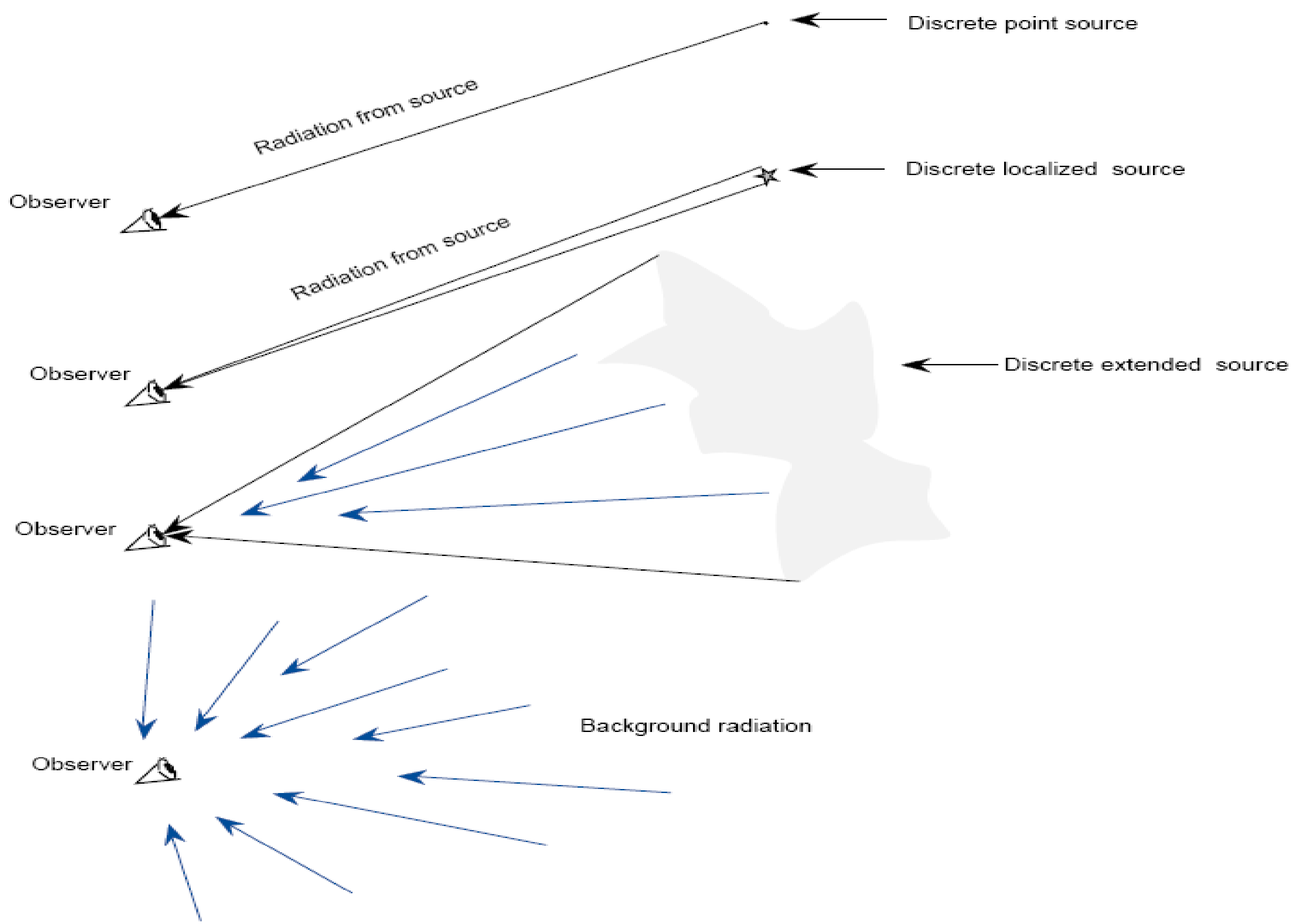
$$\Rightarrow DjDn_s=1\ \mathrm{rad\ Hz}^{-1}\mathrm{sec}^{-1}$$

CoherentAmplifier: $Dj<1\mathrm{rad}\Rightarrow Dn_s=1$ photon $\mathrm{Hz}^{-1}\mathrm{sec}^{-1}$

Phase conherent amplifier has minimum noise of n_s = 1 photon $\mathrm{Hz}^{-1}\mathrm{sec}^{-1}$

Phase coherent amplifier automatically puts signal into RJ regime => wave noise dominated

Note: phase coherent amplifier is not a detector


Discrete point source
Radiation from source
Observer
Discrete localized source
Radiation from source
Observer
Discrete extended source
Observer
Background radiation
Observer