Radio Techniques
Single Dish Radioastronomy
Part I

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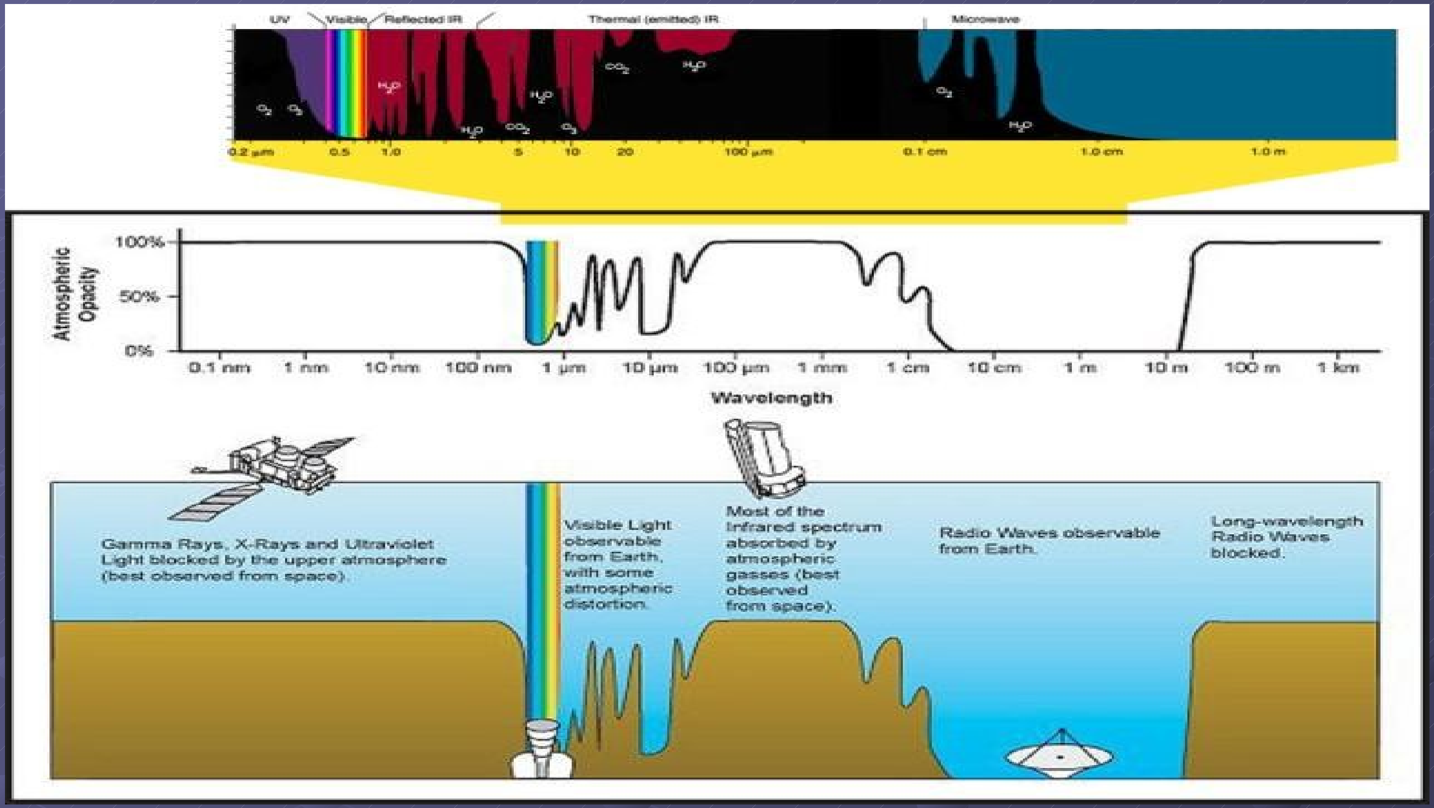
NCRA

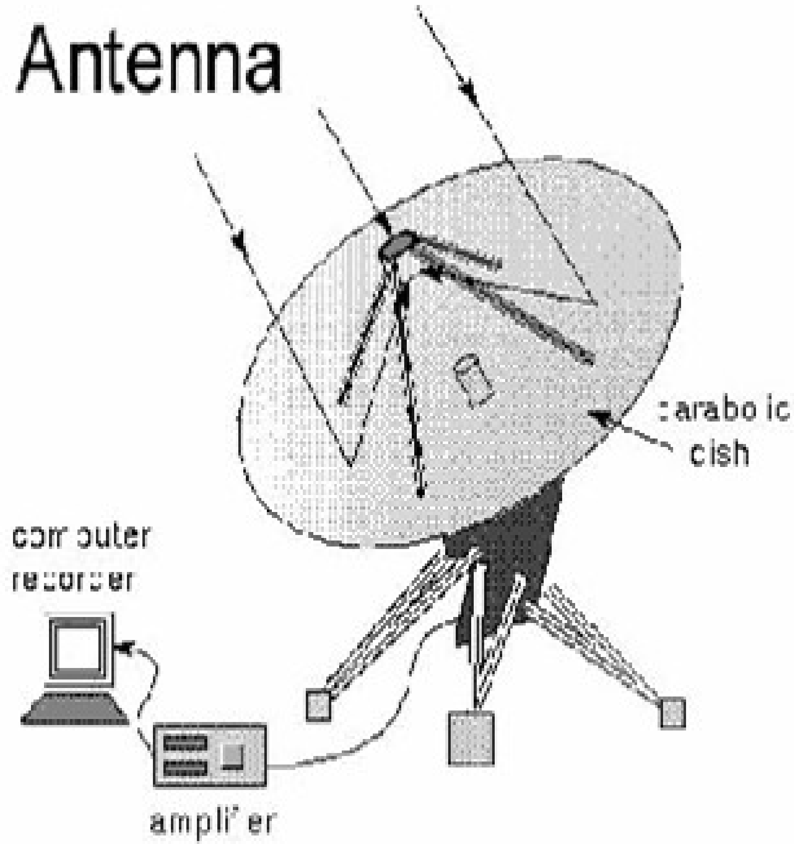
ATMOSPHERIC WINDOW



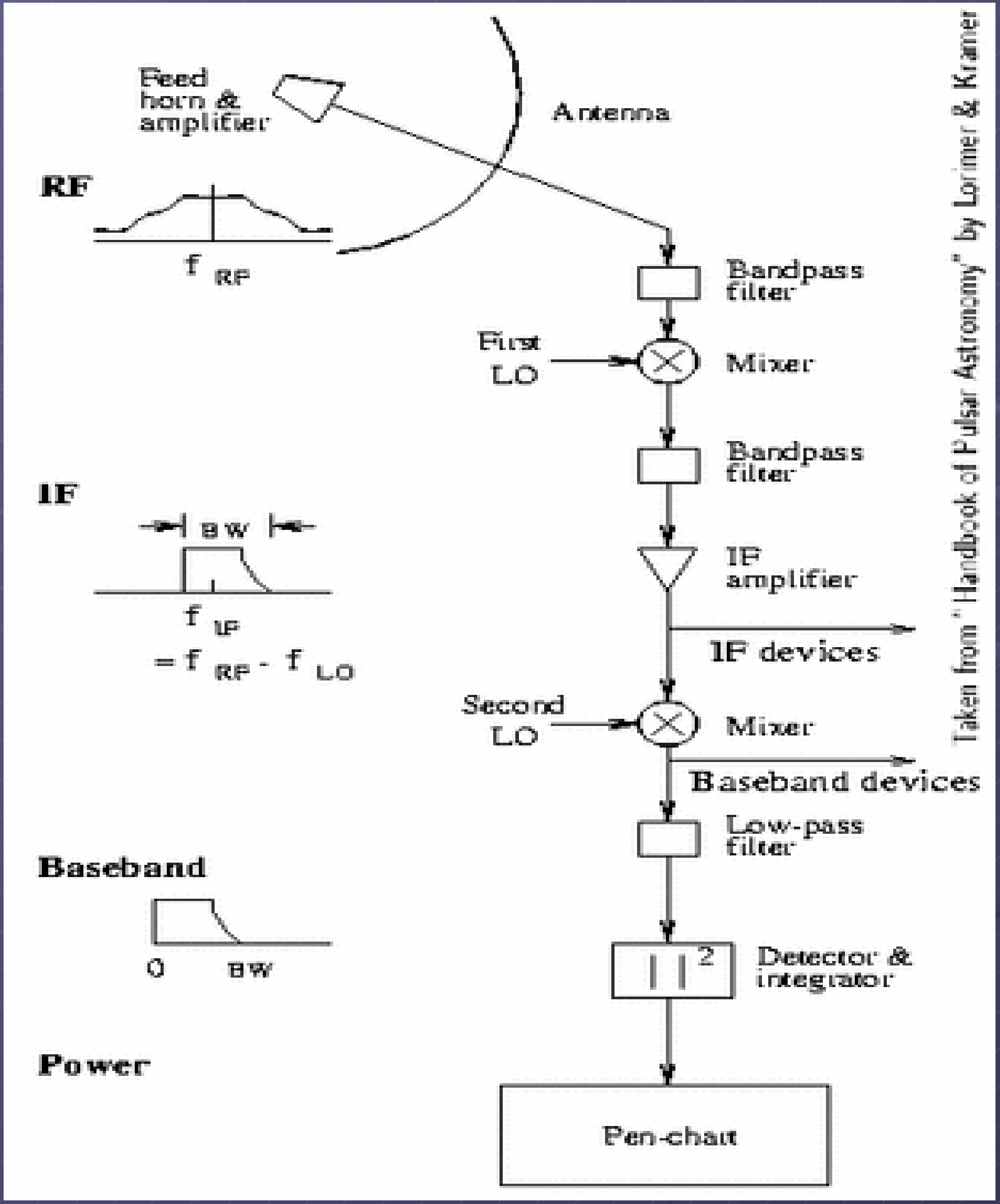
HIGH FREQUENCY CUTOFF IN RADIO IS: 600 GHz (or 0.5 mm)

LOW FREQUENCY CUTOFF IS : 15 MHz (or 20 m)

Schematic of a Single Dish Radio Telescope

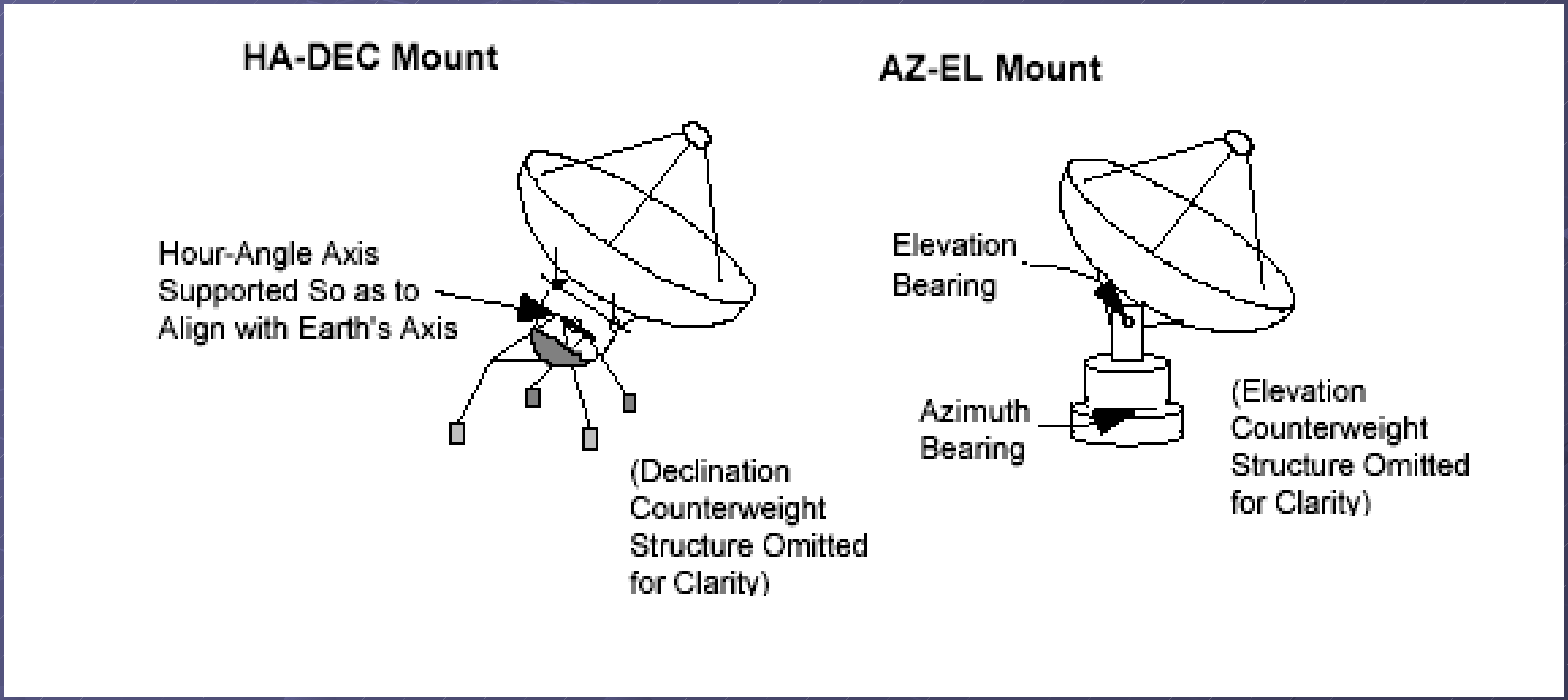


A radio telescope reflects radio waves to a focus at the antenna.

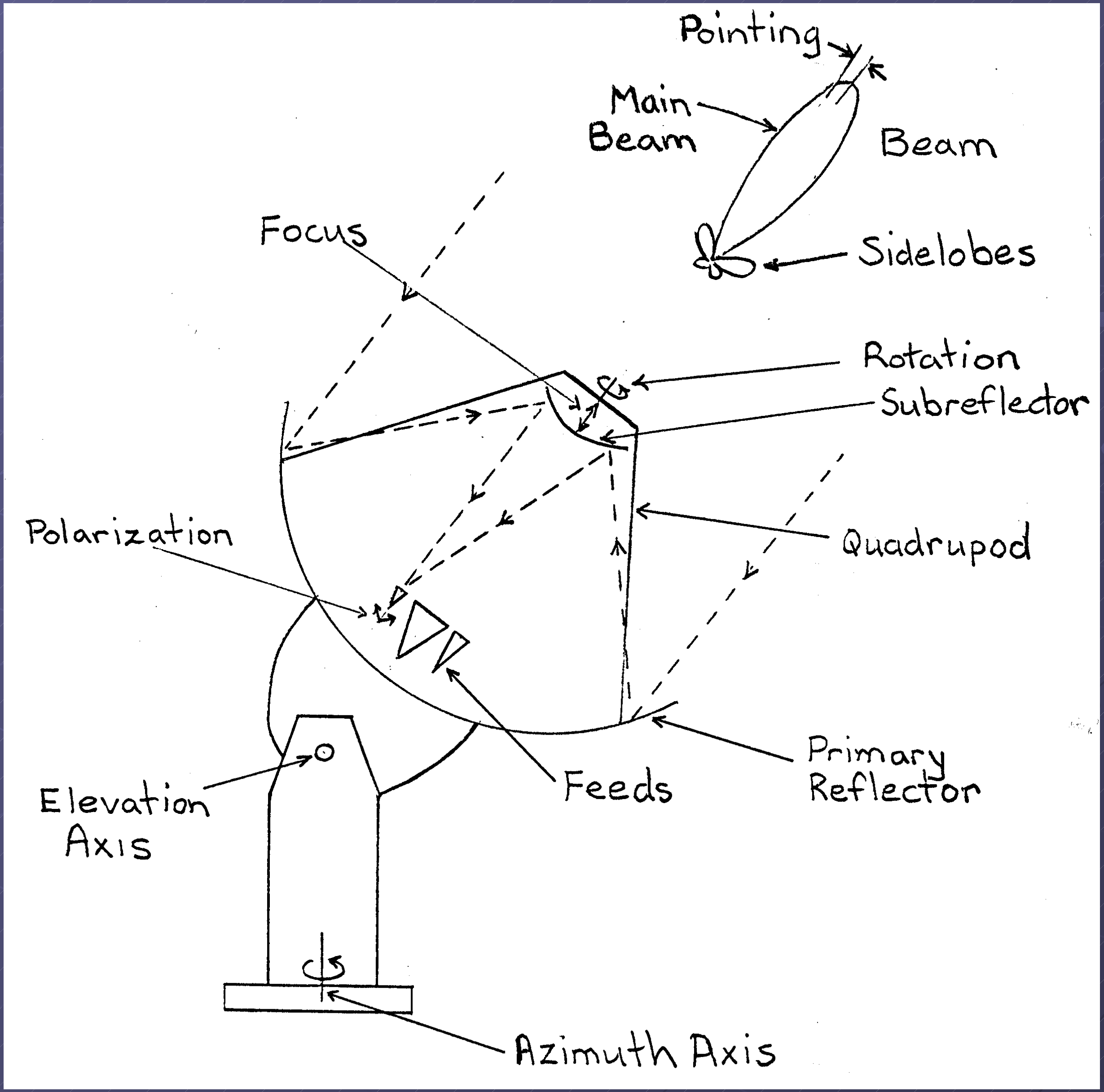


Feed horn & amplifier
Antenna
RF
f RF
Bandpass filter
First LO
Mixer
Bandpass filter
IF
BW
IF amplifier
f IF
= f RF - f LO
IF devices
Second LO
Mixer
Baseband devices
Low-pass filter
Baseband
0
BW
Detector & integrator
Power
Pen-chart
Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Mounting of Antennas



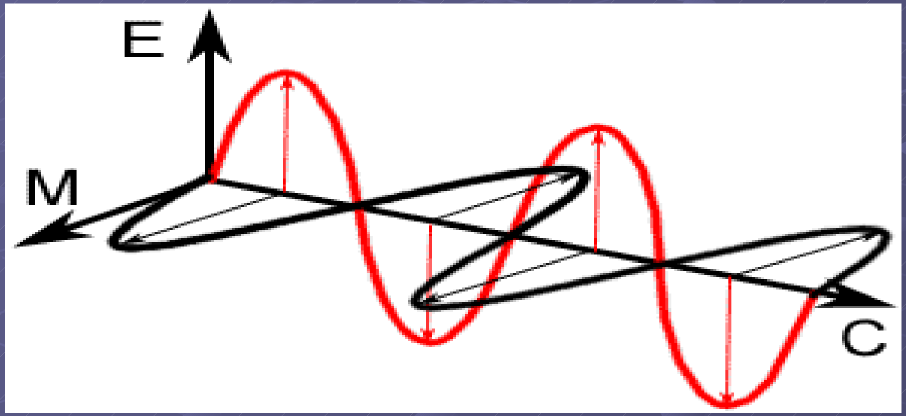
Key elements of an Antenna

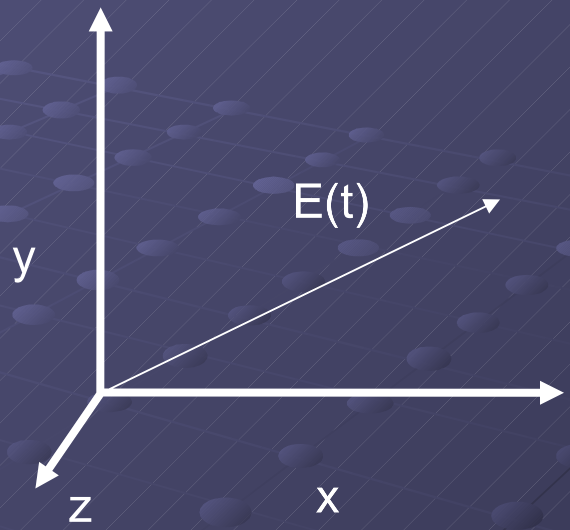


Nature of the radio emission of a Celestial source

- At radio frequencies the EM radiation can be treated as waves
- EM waves are generated by acceleration of charged particles
- Inverse square law: Flux of the source decrease as $1/R^2$
- Celestial sources are so far away that we can ignore the curvature of the wave front (plane wave)

Plane Wave Basics





$E_x = A_x \cos(2\pi\nu t + \delta_x)$

$E_y = A_y \cos(2\pi\nu t + \delta_y)$

- For A_x or $A_y = 0$, linearly polarized light.
- For $A_x = A_y$, $\delta x = 0$, $\delta y = -\pi/2$, circularly polarized light

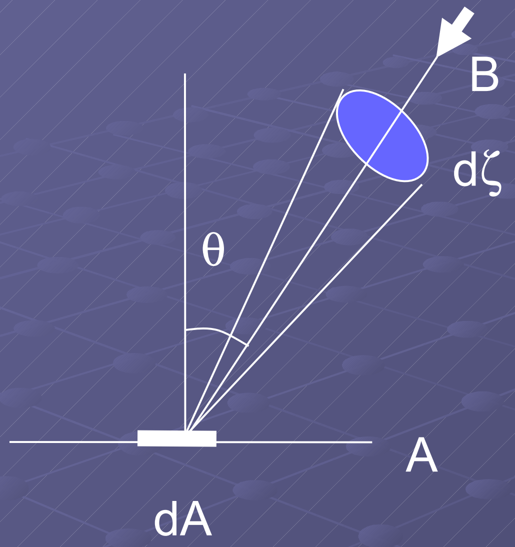
Plane wave cont…

Variation of the Plane wave in space and time can be described as

$$E(z,t) = A \cos(2\pi\nu t - k z)$$

or $$E = \text{Real}\left(A\ e^{j(2\pi\nu t - kz)}\right)$$

Some Basic Definitions in Radioastronomy



infinitesimal power from a solid angle $d\zeta$ is
$dW = B \cos(\theta)\, d\zeta\, dA\, d\nu$ watts

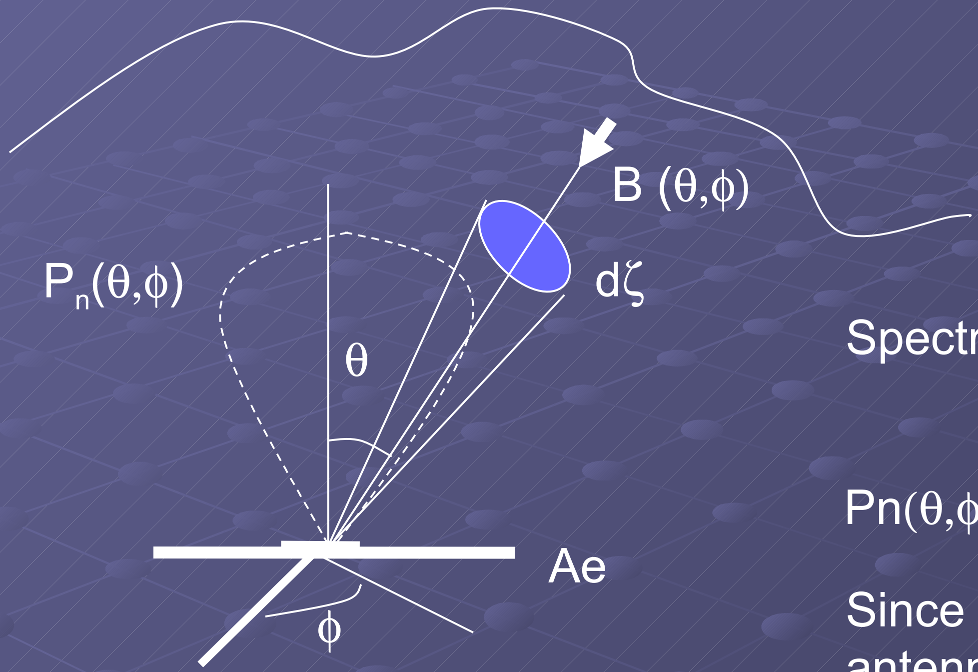
B = Brightness , watt m^{-2} Hz^{-1} rad^{-2}

$$\text{Power } W = A \int_{\nu} \iint_{\zeta} B \cos(\theta)\, d\zeta\, d\nu$$

$$\text{Spectral power : } dw = B \cos(\theta)\, d\zeta\, dA \quad \text{watt Hz}^{-1}$$

For a constant brightness the spectral power $w = \pi A B$

Definitions continued .....



Spectral Power : $w = Ae \int B(\theta,\phi)\ P_n(\theta,\phi)\ d\zeta$

$Pn(\theta,\phi)$ is the power pattern of the antenna

Since the radiation received by the antenna is of incoherent type and since any antenna is only responsive to 1 polarization the spectral power is

$$w = \tfrac{1}{2}\ Ae \int B(\theta,\phi)\ Pn(\theta,\phi)\ d\zeta$$

Flux Density of a source

$$S = \int \mathbf{B(\theta,\phi)\ P(\theta,\phi)\ d\zeta}$$

The unit of flux is jansky (Jy) which is equal to 10^{-26} watt m^{-2} Hz^{-1}

Case 1) For a point source, where the source is much smaller than the antenna beam

$$S = \int B(\theta,\phi)\, d\zeta$$

Case 2) When the source is much larger than the main lobe of the antenna and the brightness is constant over the main lobe

$$S = B(\theta,\phi) \int P(\theta,\phi)\, dz \sim B(\theta,\phi)\ \zeta_m$$

Concept of Temperature

Radiation from a blackbody is described by Planks law

$B(\nu) = (2h\nu^3 / c^2) \quad 1/(e^{h\nu/kT} - 1)$ Watt m^{-2} Hz^{-1} rad^{-2}

For a typical radio frequency like 10^9 Hz, $h\nu/k \sim 0.048$, Hence

$e^{h\nu/kT} \sim 1 + h\nu / kT$ or, $B(\nu) = (2\nu^2/c^2)\, k\, T = 2\, k\, T / \lambda^2$ or $T = (\lambda^2 / 2\, k)\, B(\nu)$

This approximation is called the Rayleigh-Jeans Approximation of the Plank spectrum.

The brightness temperature of a source is hence defined as $T_b = (\lambda^2 / 2\, k)\, B(\nu)$

In general T_b has no relation to the physical temperature of the source (except some sources in the sky which are emitting as a black body)

Why do we think that pulsar radio emission is coherent?

Pulsars have high brightness temperature

$$T_b = (c^2 / 2\nu^2 k) I_\nu$$

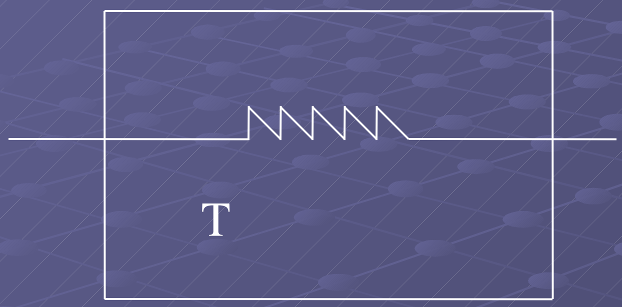
$$F_\nu = I_\nu \, \Omega, \quad \Omega \sim l^2/d^2$$

$$T_b \sim (3.1 \times 10^{23}) \, \nu_9^{-2} \, F_\nu \, (\mathrm{mJy}) \, d^2 \, (\mathrm{kpc}) \, l_6^{-2}$$

$$T_b \sim 10^{25} \text{ --- } 10^{30} \text{ k}$$

this is extremely high !!!

Nyquist Theorem and Noise Temperature



A resistor put in a thermal bath of temperature T will have an output power per unit frequency given by

$P = k\,T$, called the Nyquist formula

When radiation falls in an antenna, the power absorbed is expressed in temperature units. The power available in the antenna due to a source the sky is termed as the antenna temperature Ta

$$T_a = P_a / k$$

The power introduced in a radio telescope due to several noise contributions (e.g from sky, ground, receiver etc) is called the system temperature Tsys

$$T_{sys} = P_{sys} / k$$

Connecting Flux and Temperature

Recall that the spectral power is

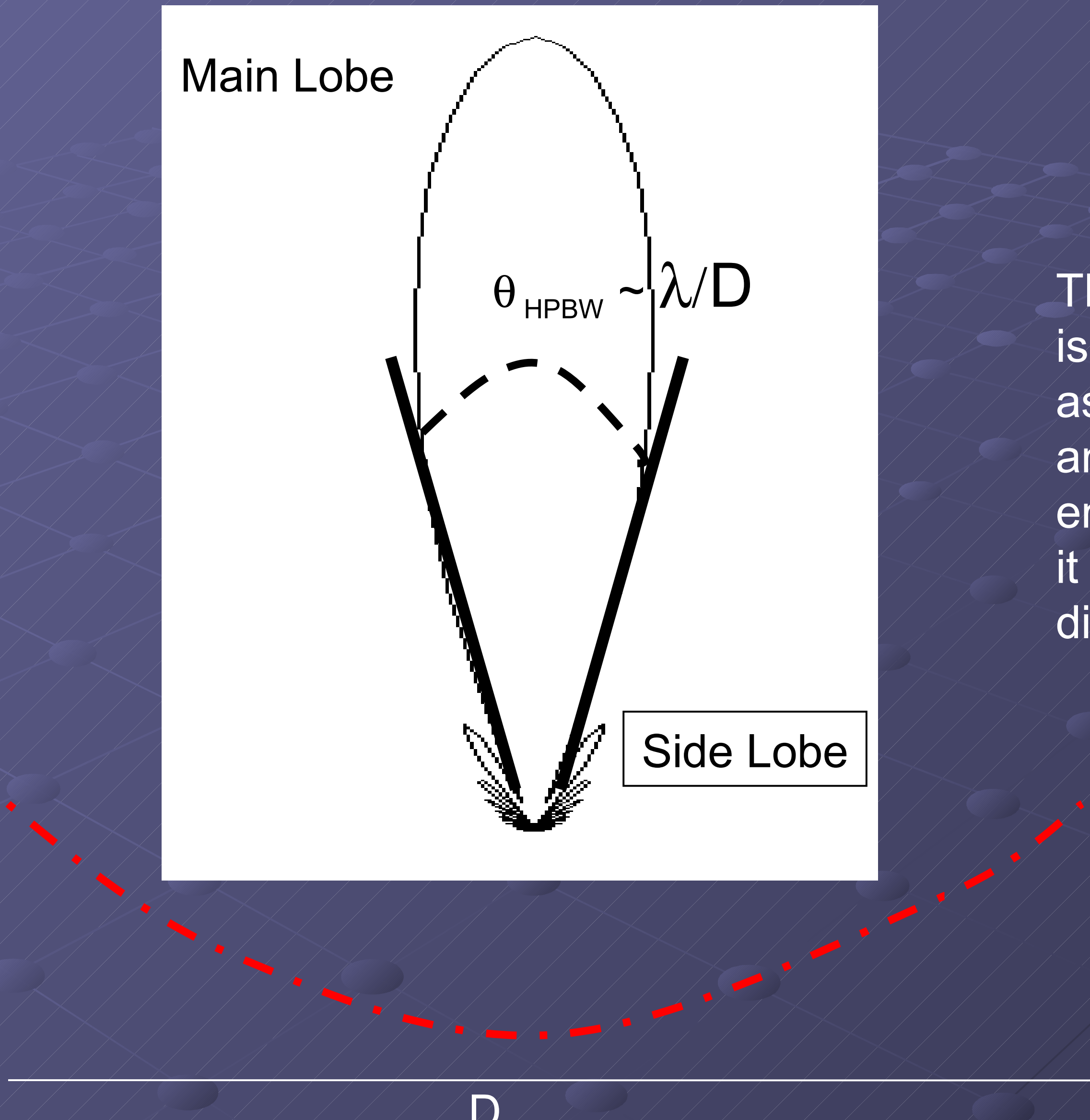
$$w = \tfrac{1}{2}\ A_e \int B(\theta,\phi)\ P_n(\theta,\phi)\ d\zeta$$

again $w = k\,T$ as per Nyquist theorem

Hence we get a relation between temperature and flux as

$$kT = \tfrac{1}{2}\, A_e\, S$$

Beam of an Antenna



Reciprocity

The pattern of an antenna is same whether it is used as a transmitting or receiving antenna, i.e if the antenna emits efficiently in one direction it will receive efficiently in that direction.

Antenna Patterns

The effective aperture of the antenna

$$Ae(\theta,\phi) = \frac{\text{Power density available at the antenna}}{\text{flux density of the wave incident on the antenna}}$$

The power pattern of the antenna

$$P(\theta,\phi) = \frac{Ae(\theta,\phi)}{Ae^{max}}$$

$$\text{Directivity of the antenna, } D(\theta,\phi) = \frac{4\pi\, P(\theta,\phi)}{\int P(\theta,\phi)\, d\zeta} = \frac{\text{Power emitted in } (\theta,\phi)}{\text{Total power emit./ } 4\pi}$$

Gain= Power emitted in (θ,φ) / Total power input/4π

cont…….

…….

Recall the expression for spectral power: $w = \frac{1}{2}\ Ae \int B(\theta,\phi)\ Pn(\theta,\phi)\ d\zeta$

$$Ta(\theta',\phi') = Ae^{max} / \lambda^2 \int Tb(\theta,\phi)\ P(\theta - \theta', \phi - \phi')\ \sin(\theta)\ d\theta\ d\phi$$

Consider an antenna terminated in a resistor:

In thermal equilibrium
The power $P\_(R \rightarrow A) = k\,T$

$$P\_(A \rightarrow R) = (Ae^{max}\, kT / \lambda^2) \int P(\theta,\phi)\ d\zeta \quad \text{or} \quad Ae^{max} = \lambda^2 / \int P(\theta,\phi)\ d\zeta$$

$$Ae = Ae^{max}\ P(\theta,\phi) = \frac{\lambda^2\, P(\theta,\phi)}{\int P(\theta,\phi)\ d\zeta} \quad \text{or} \quad D(\theta,\phi) = (4\pi / \lambda^2)\ Ae(\theta,\phi)$$

Gain of the antenna is connected to the directivity through a constant factor.

$$\varepsilon = Ae^{max} / Ag$$

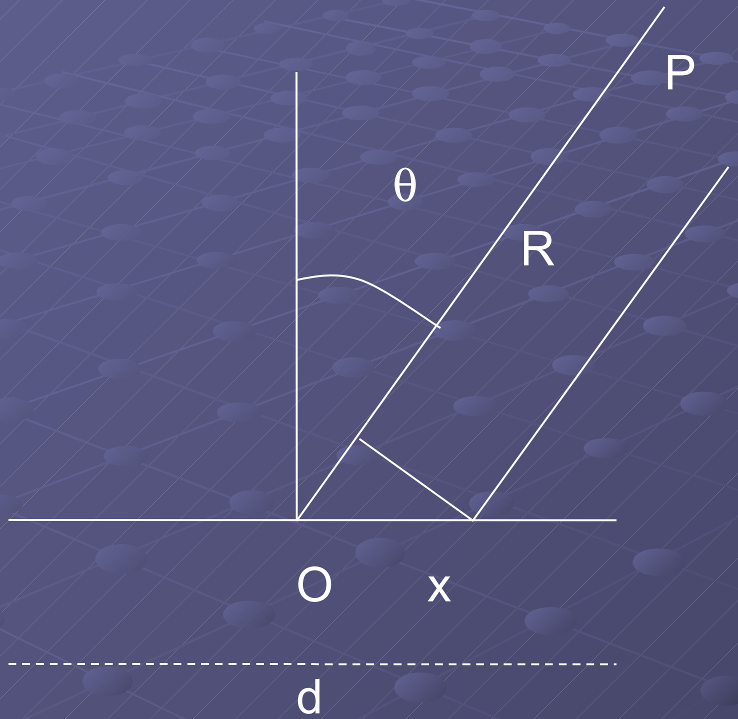
F/D ratio for an antenna

In a antenna feed system the feed should ideally illuminate the antenna with a uniform beam that only illuminates the reflecting surface

The angle subtended by the feed as seen by the reflector is given by the Focus / Diameter , F/D ratio.

When F/D ~ 0.38, the edge of the dish is about 64 deg as seen by the feed and the efficiency is close to 100 %

Computing Antenna Patterns



The total electric field at P due to a electric field distribution e(x) is

$$E(R,\theta) = \int_{-d/2}^{d/2} \frac{e(x)}{R^2}\, e^{-jk\mu x}\, dx$$

The un-normalised power has the form of a fourier transform

$$F(\mu) = \int_{-\inf}^{+\inf} e1(x)\; e^{-jk\mu x}\; dx$$

The beam is the fourier transform the of the aperture

Rules of Fourier Transform

Linearity, if $G1(\mu) = F[g1(x)]$ and $G2(\mu)=F[g2(x)]$

then: $G1(\mu)+G2(\mu)=F[g1(x)+g2(x)]$

Inverse: if $G(\mu) = \int_{-inf}^{inf} g(x)\, e^{-jk\mu x}\, dx$,

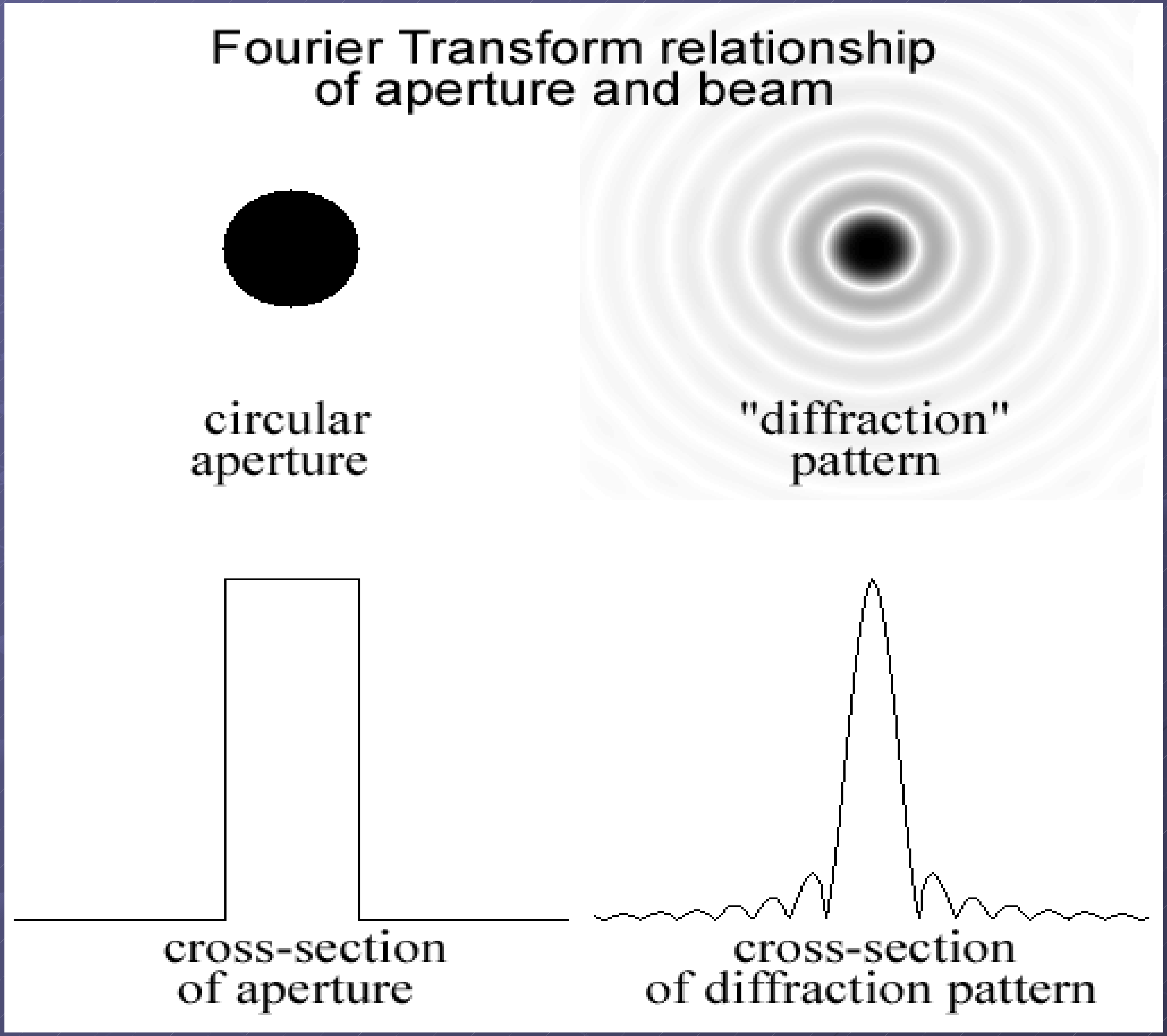
then $g(x) = \int_{-inf}^{inf} G(\mu)\, e^{jk\mu x}\, dx$

Phase Shift: $G(\mu - \mu 0) = F [\, g(x)\, e^{-j2\pi\mu o\, x}\,]$, This is the principle used to steer the antenna beam

Parsevals Theorem

$$\int_{-\infty}^{\infty} |\, G(\mu)|^2 \, d\mu = \int_{-\infty}^{\infty} |\, g(x)\,|^2 \, dx$$

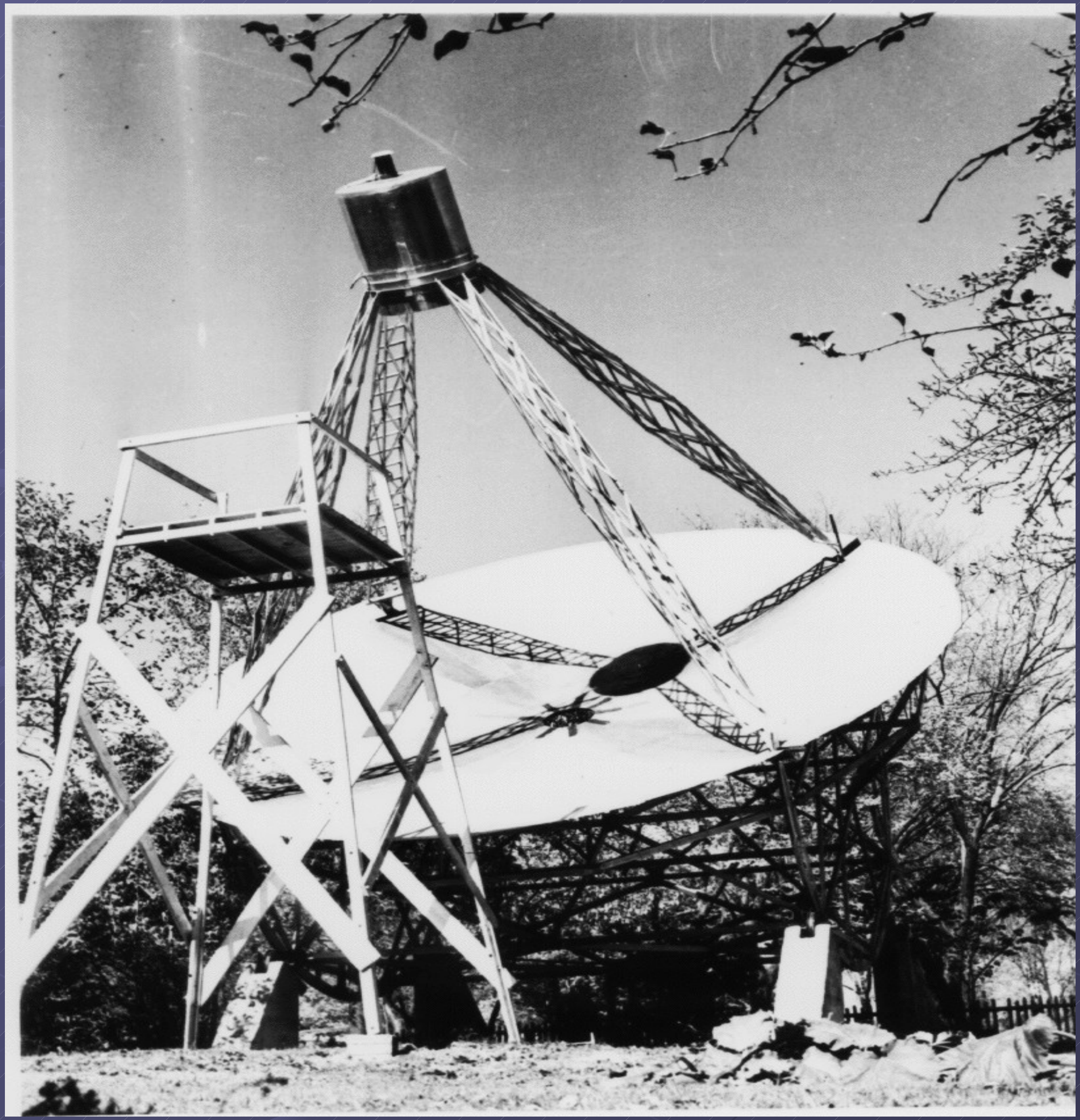
Area: $$G(0) = \int_{-\infty}^{\infty} g(x)\, dx$$


Fourier Transform relationship
of aperture and beam
circular
aperture
"diffraction"
pattern
cross-section
of aperture
cross-section
of diffraction pattern

Factors affecting Antenna Performance

- Reflector surface efficiency
- Blockage by the feed
- Spillover
- Surface accuracy (Ruze loss)
- Feed illumination efficiency (F/D ratio)
- Low sidelobe pattern
- Pointing

Reber’s 31.4 ft parabolic reflector



Greenbank 100-m telescope

300-m telescope in Arecibo, Puerto Rico

GMRT 45 metre dish

Note the light , see-through structure reducing wind forces, weight, and cost

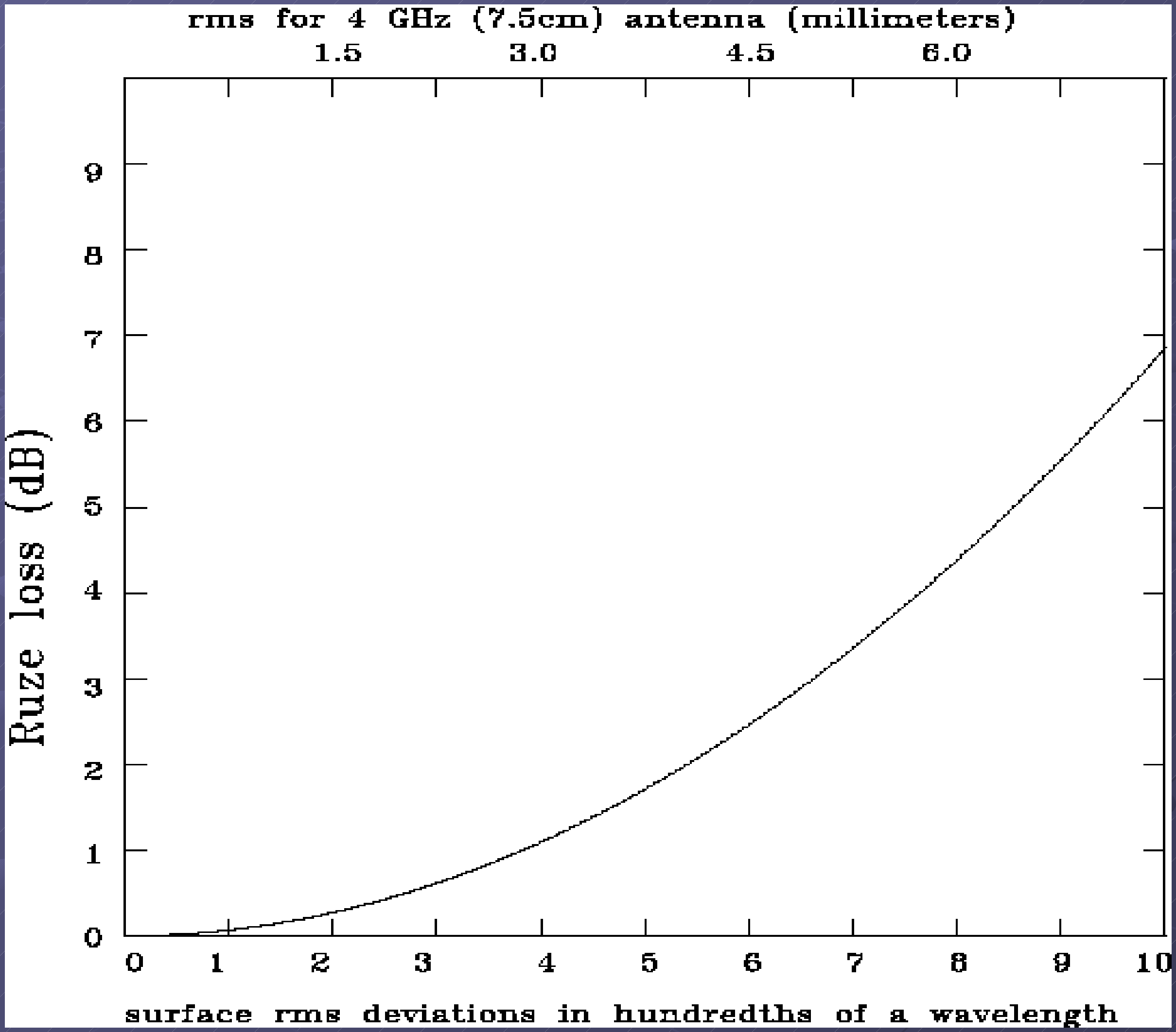
Ooty Radio Telescope 530mx30m parabolic cylinder operating at 325MHz

Gauribidnur Array at 30 MHz

$$L = \exp(-(4\pi\sigma/\lambda)^2)$$

$\sigma = \lambda/16$

causes a loss of 0.5



Antenna (Collector)

- Area ~ 10^4 m^2 , P = ½ A S $\Delta\nu$ ~ 10^{-15} watts

- Radio Telescopes are diffraction limited,

Resolution ~ λ / D ~ 0.5 deg (for wave 1 m and D=100)

Human eye ~ 20 " , Ground based optical tel: 1"

Directivity ~ Gain (Θ) / ε

- Surface accuracy ~ $\lambda/10$, such that signal can add coherently at the focus.

Mesh surface possible at low freq, like in GMRT, and is cost effective

Feed System

- At the feed the EM wave is converted into electrical signal in the cable
- Feed are resonant devices like dipoles or horns with

$\delta\nu/\nu = 10 - 20\ \%$

- Feed is small and hence have large angular beam. So to avoid picking up ground radiation feed is designed to have 1/10th gain at the edge of the dish

This leads to effective loss of collecting area

- In GMRT gain = 0.3 K / Jy, Ground radiation is 300 K,

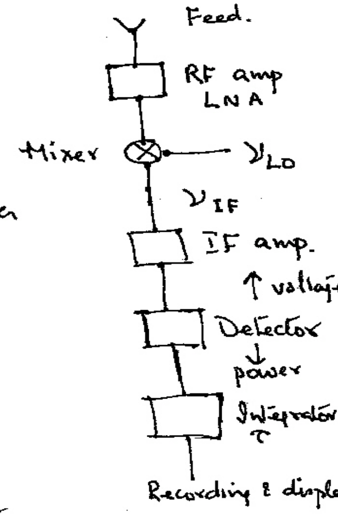
efficiency is around 70 %

Electronics

- Super heterodyne receivers – similar to household radio
- Radio source signals are very weak – amplification of 10^{6-8}
- Amplifier characterised by gain G.
- Due to collision of electrons in the amplifier, it ~~corrupts~~ adds noise to the input signal before amplification

Output power of amplifier = $G \times k\,(T_{ant} + T_{Receiver})$

- $T_{Receiver} \sim 20\,K$ to $100s\,K$
- Low Noise RF amplifiers made ~~of~~ using High Electron Mobility Transistors and others special devices
- Often (at cm & mm λ) cooled to Liquid Nitrogen or lower temperatures
- At low frequencies (< 300 MHz) LNAs not critical – galactic background emission (T_{sky} ~~Back~~ $\sim 40\,K$ @ 325 MHz $\propto \nu^{-2.5}$) is the dominant source of noise



- RF amplifier determines S/N – other amplifier in chain dont matter

Heterodyning – Mixer

For QM signal $A(t)\,e^{i[\omega t + \phi(t)]}$ all the information is in $A(t), \phi(t)$ – nothing in ω

If we multiply the QM signal by a pure sinusoid at ν_{LO}

Output = $A(t)\,e^{i[(\omega - \omega_{LO})t + \phi(t)]} = A(t)\,e^{i[\omega_{IF}t + \phi(t)]}$

Mixer is a non ~~linear~~ linear device that achieves this multiplication — [Avoid cable loss – standard components]

Distribution of Input voltage

$$P(V) = \frac{1}{(2\pi)^{1/2}\sigma} \exp\left(\frac{-V^2}{2\sigma^2}\right) ,$$

After detection $V_0 = V^2$, assuming $\sigma = 1$

$$P_\circ(V_\circ)dV_\circ = 2P(V)dV$$

for $V \geq 0$. Since $dV_\circ = 2VdV$, $dV/dV_\circ = V_\circ^{-1/2}/2$ and

$$P_\circ(V_\circ) = \frac{1}{(2\pi)^{1/2}} V_\circ^{-1/2} \exp(-V_\circ/2)$$

$\langle V_\circ \rangle = \sigma^2.$

RMS = $2^{1/2}$ times the mean output power