INTRODUCTION TO INTERFEROMETRY

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Astronomy: A personal perspective

- The only source of information - the tiny amounts of radiation incident on the telescopes
- You can only ‘observe’ not ‘experiment’
- Observations + laws of physics + logic

- Over the years we have learnt an amazing amount about our grand universe from analysing a minuscule amount of light which happens reach our vantage point.
- Detective work – the art and science of logical deduction

Andromeda, our nearest galactic Neighbour

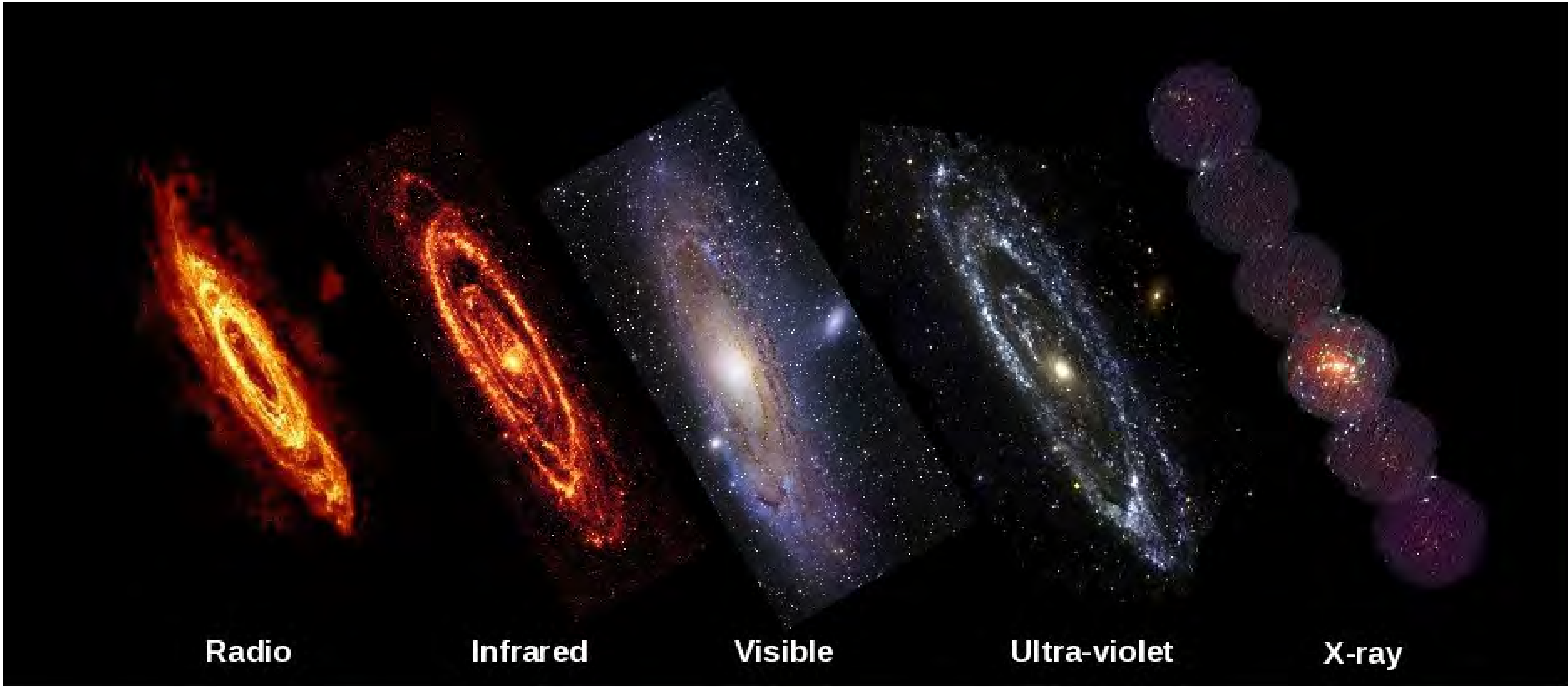
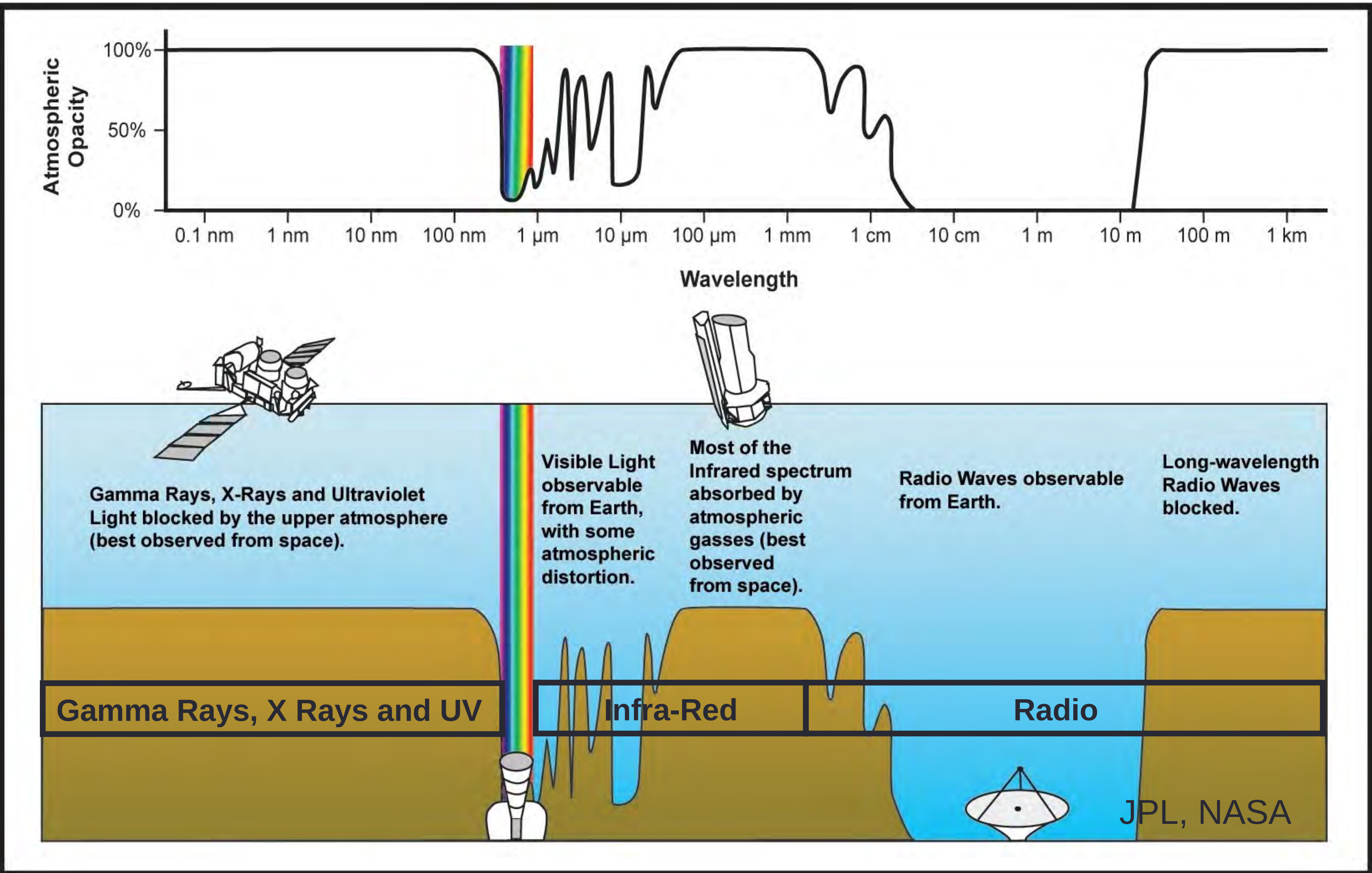
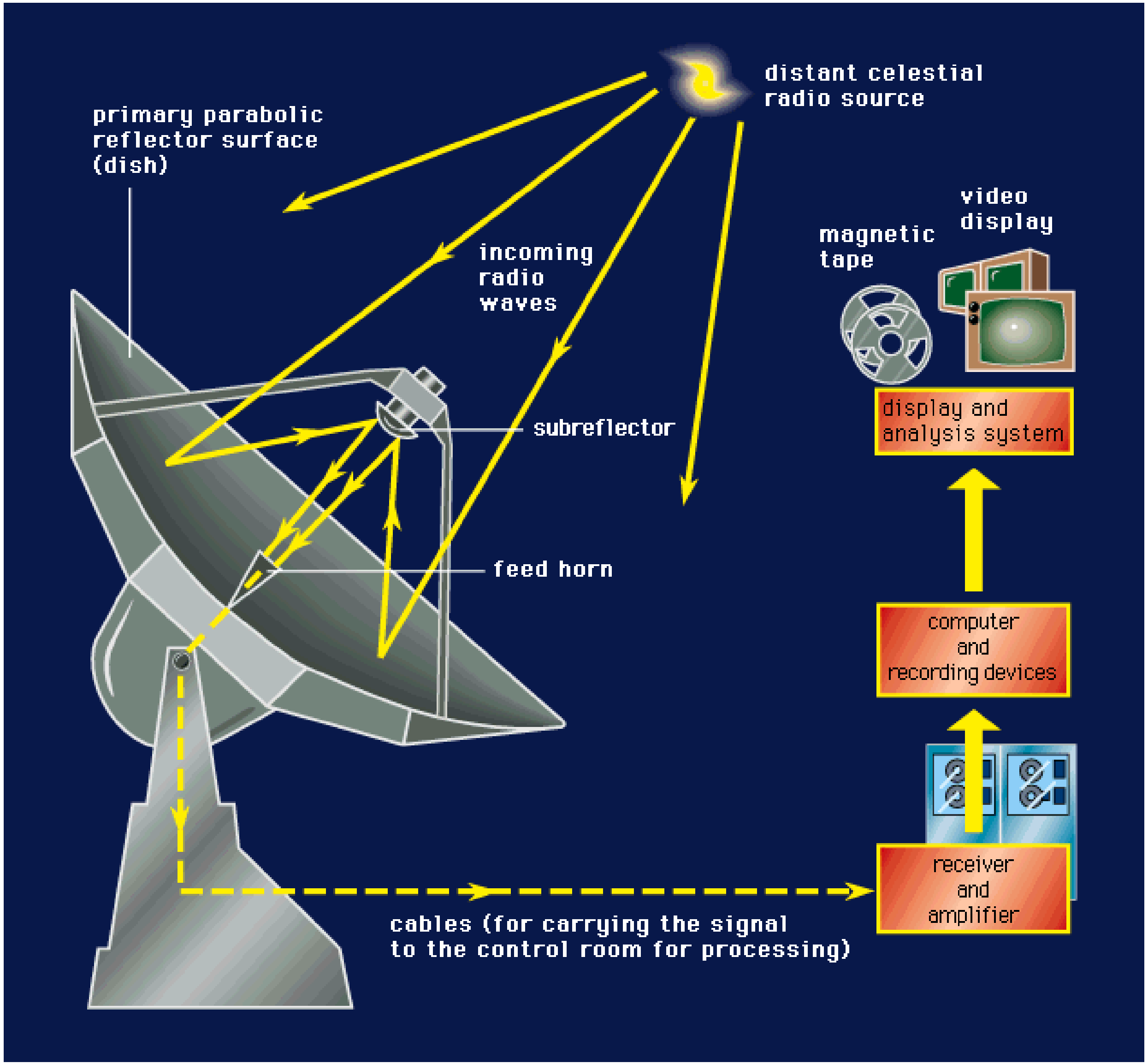


Image credits: Radio: WSRT/R. Braun; Infrared:NASA/Spitzer/K. Gordon; Visible: Robert Gendler; Ultraviolet: NASA/GALEX; X-ray: ESA/XMM/W. Pietsch

What makes it to the Earth surface?

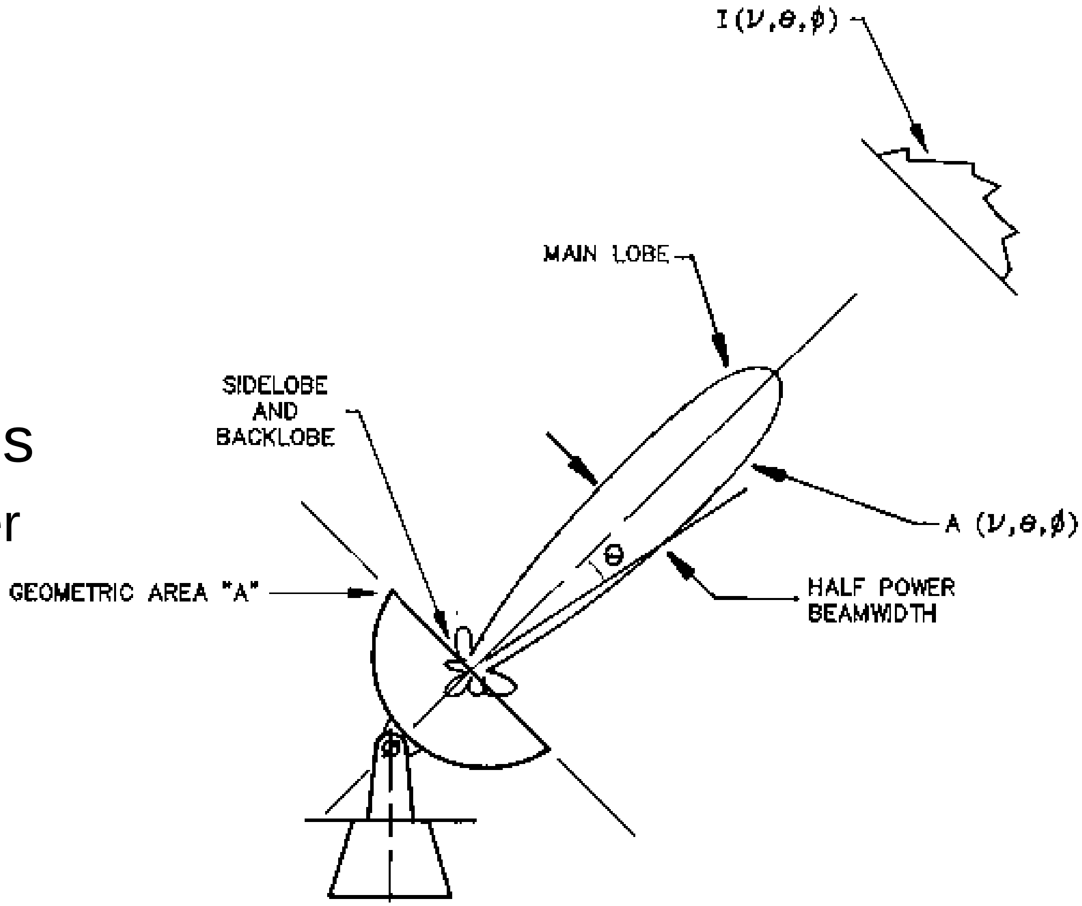


A typical radio telescope



Beam size and resolution

- Size of the main lobe in radians ~λ/D
  - λ is the wavelength
  - D is the diameter
- Better resolution requires
  - Shorter wavelength (higher frequency)
  - Bigger telescopes



Why Interferometry?

- Resolution ~ λ/D

  λ - wavelength of observation

  D - size of aperture (diameter of lens/mirror)
- A 4m optical telescope is ~5×10^6 λ (8000 Å)

  (1arc sec resolution requires D ~2×10^5 λ; ~16 cm)
- In radio λ ranges from ~0.5 mm to ~10 km

  (1 arc sec requires D ~100 m to ~2×10^3 km)
- Impossible to build apertures of required dimensions and surface accuracy
- *Interferometry - resolutions corresponding to the separation between the elements (telescopes)*

The concept behind an interferometer

The important property of a parabolic dish is that it adds parallel light rays coherently

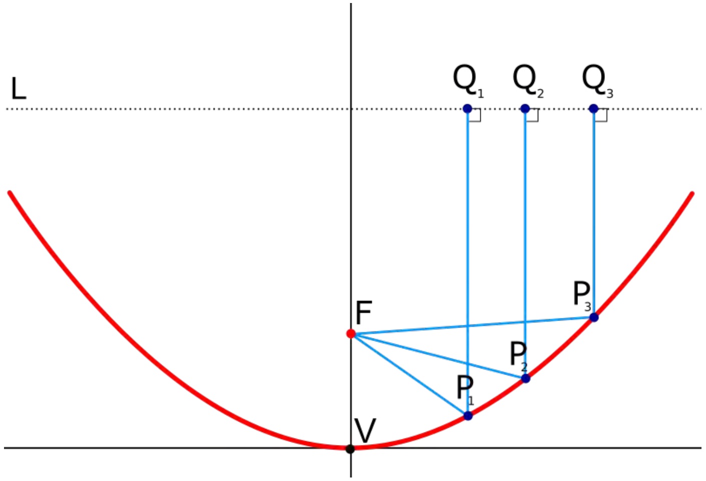
Parallel rays (from infinity) have equal path lengths to the focus, so they all arrive in phase

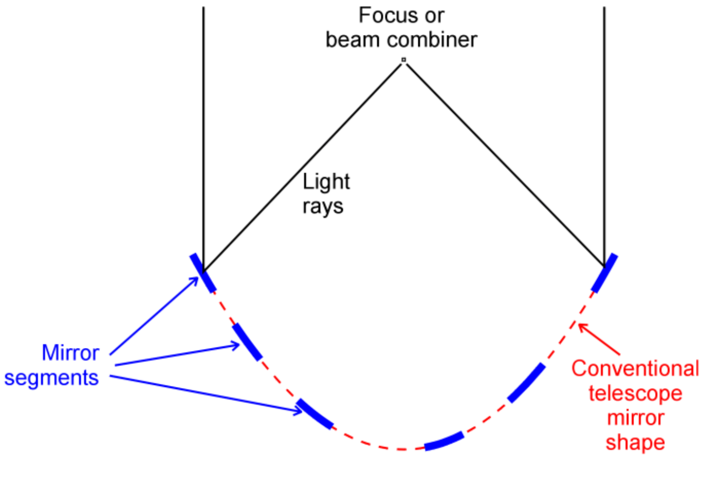
This is still true if we remove segments of the parabola – remaining rays still reach focus in phase

Now imagine moving the remaining segments of the dish off the surface of the paraboloid

So long as we know very precisely where the segments are located, we can delay their signals appropriately and still add them together coherently

This, in essence, is what an interferometer does

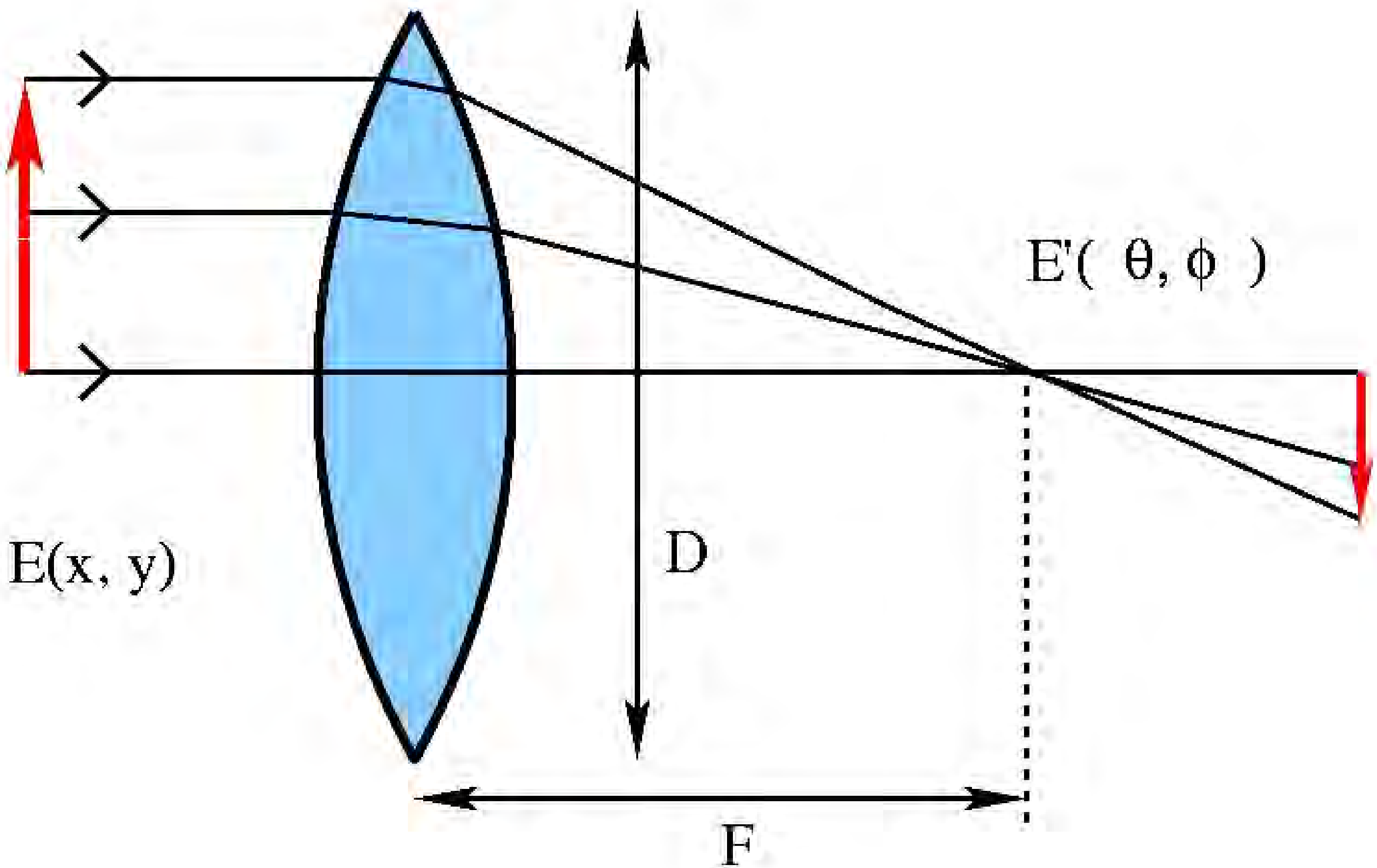




Images: wikipedia

Vincent Fish, MIT Haystack Observatory

Imaging with a lens (mirror)



It ensures that the optical path lengths from all points on a plane wavefront (perpendicular to the optical axis) to the focal point are the same.

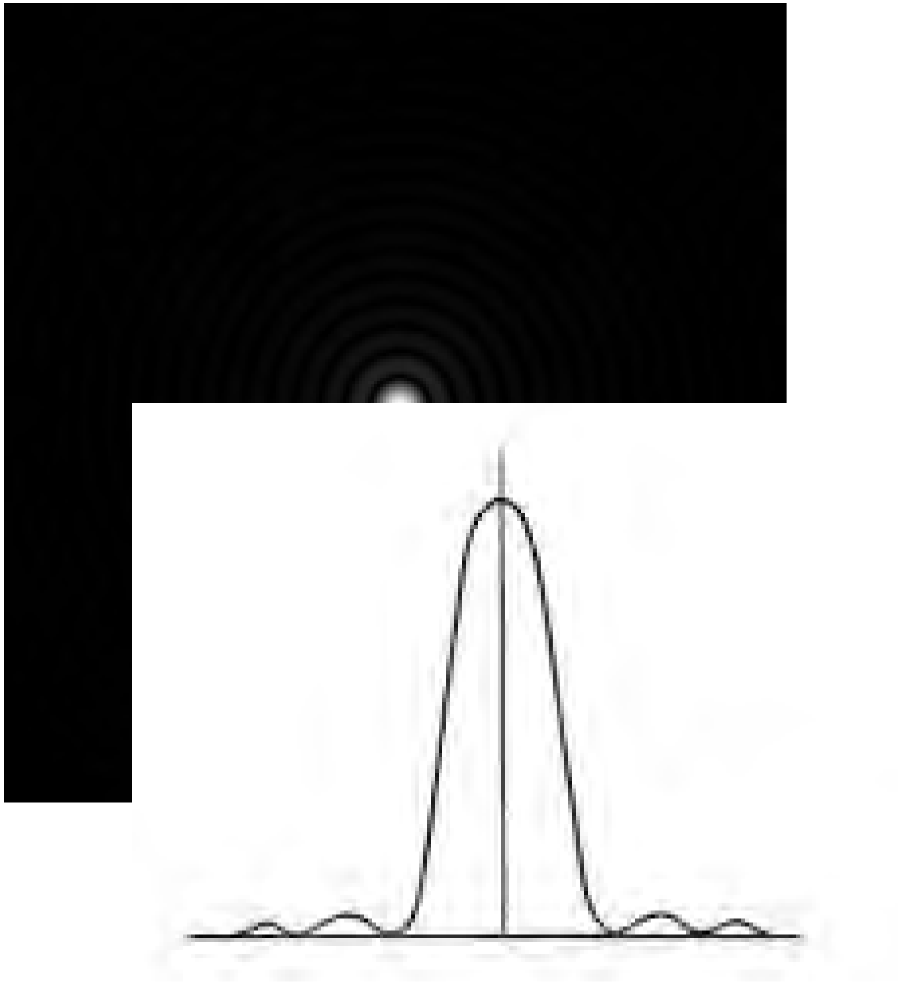
A more sophisticated perspective

Mathematically, a lens performs a Fourier Transform of the incident wavefront

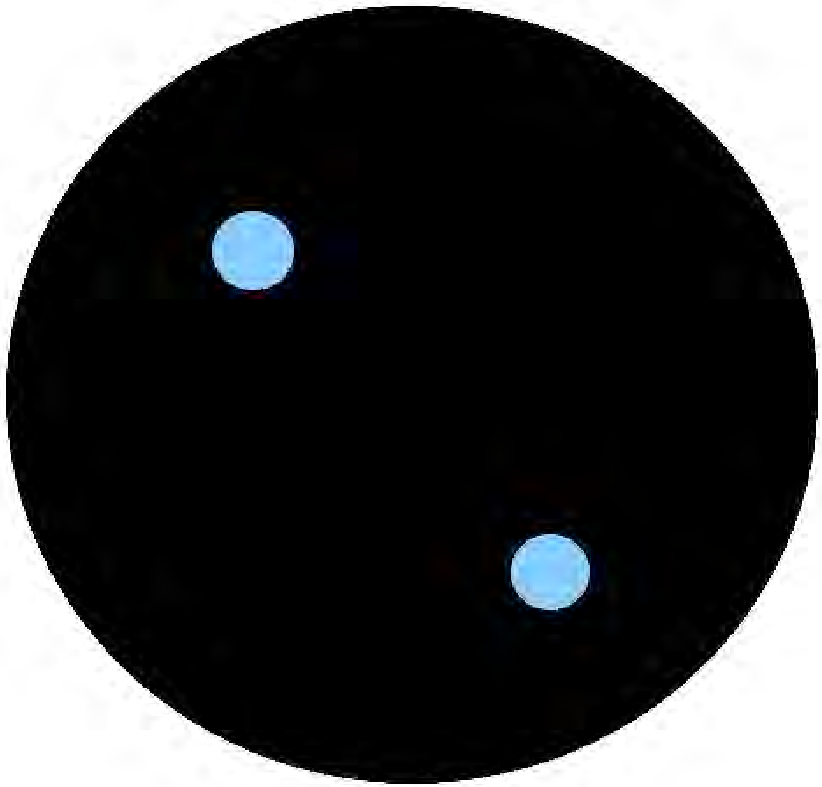
$E(x,y) \leftrightarrow E'(\theta,\varphi)$

A characteristic of optical imaging systems

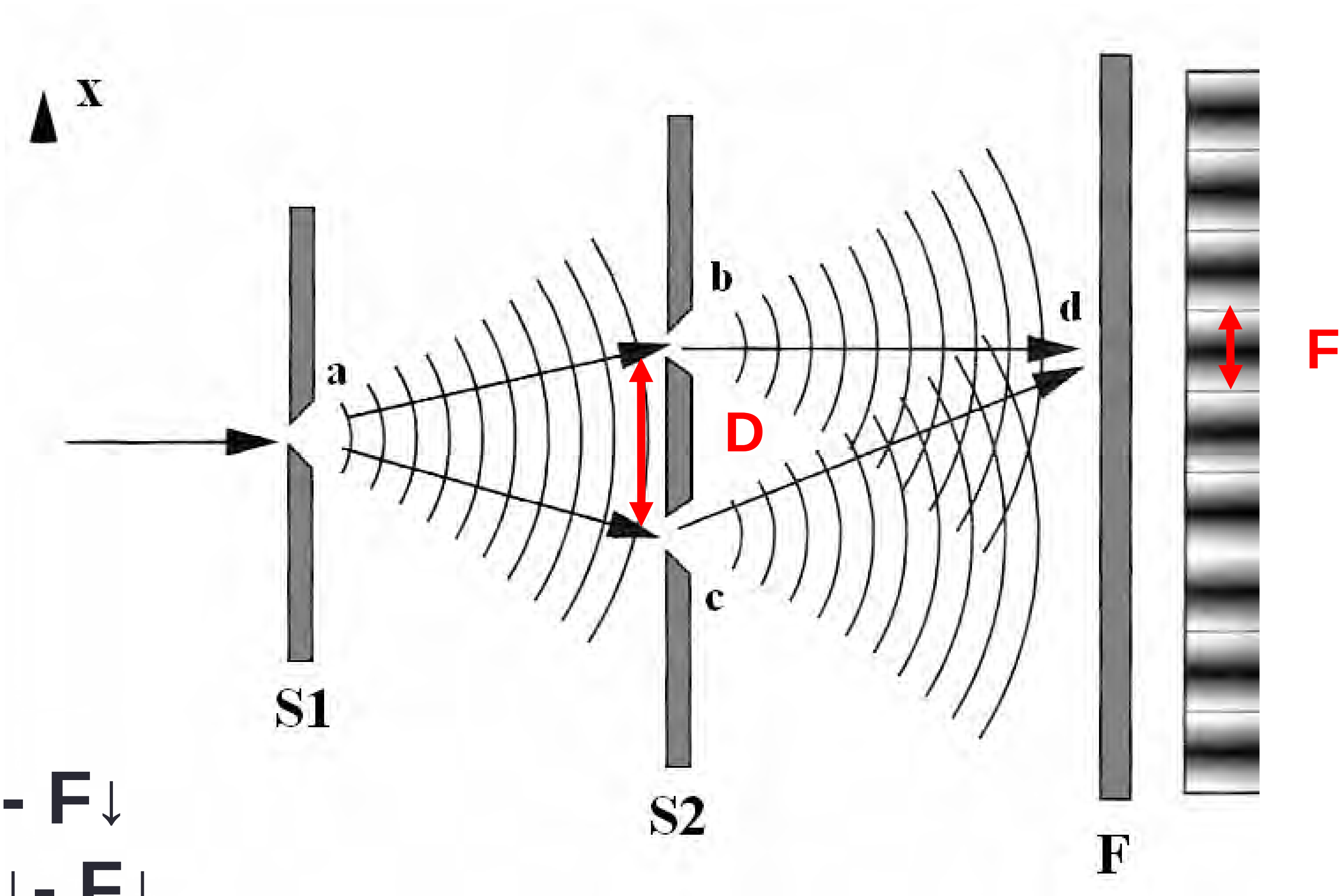
- Transfer function / Point source response / Point spread function (PSF) - Airy pattern
- Resolution = 1.22 λ/D



Imaging with an *unfilled* aperture



Young's double slit experiment



D↑- F↓

λ↓- F↓

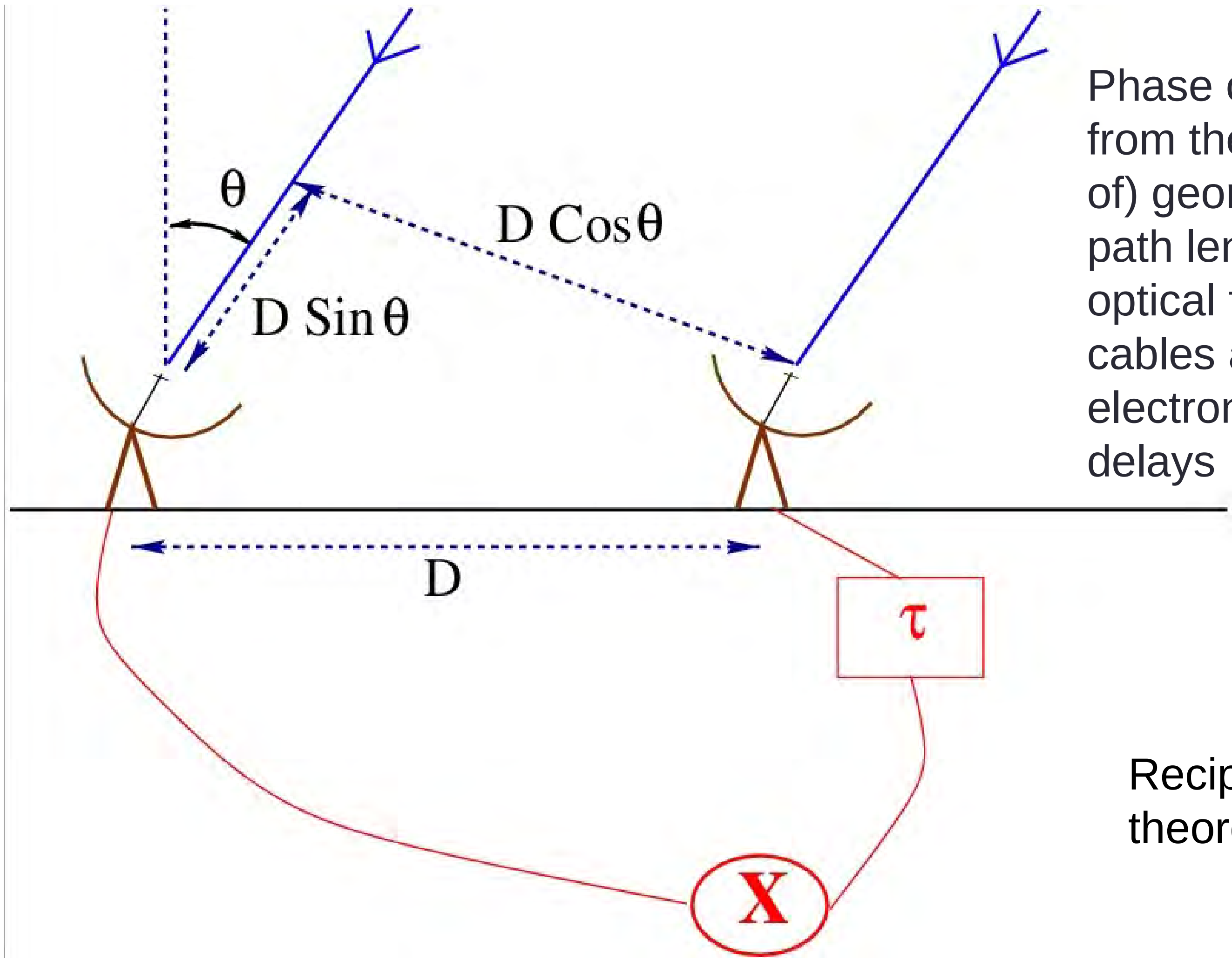
2 element interferometer

- An *antenna* is a device for converting electrical currents in conductors into electromagnetic radiation in space or vice-versa
- Radiating characteristics are identical to receiving characteristics.

- Imagine a radio double slit experiment, except that the slits are now ‘receiving’ rather than ‘emitting’ radio waves – a 2 element interferometer

A two element interferometer

Slits → Telescopes

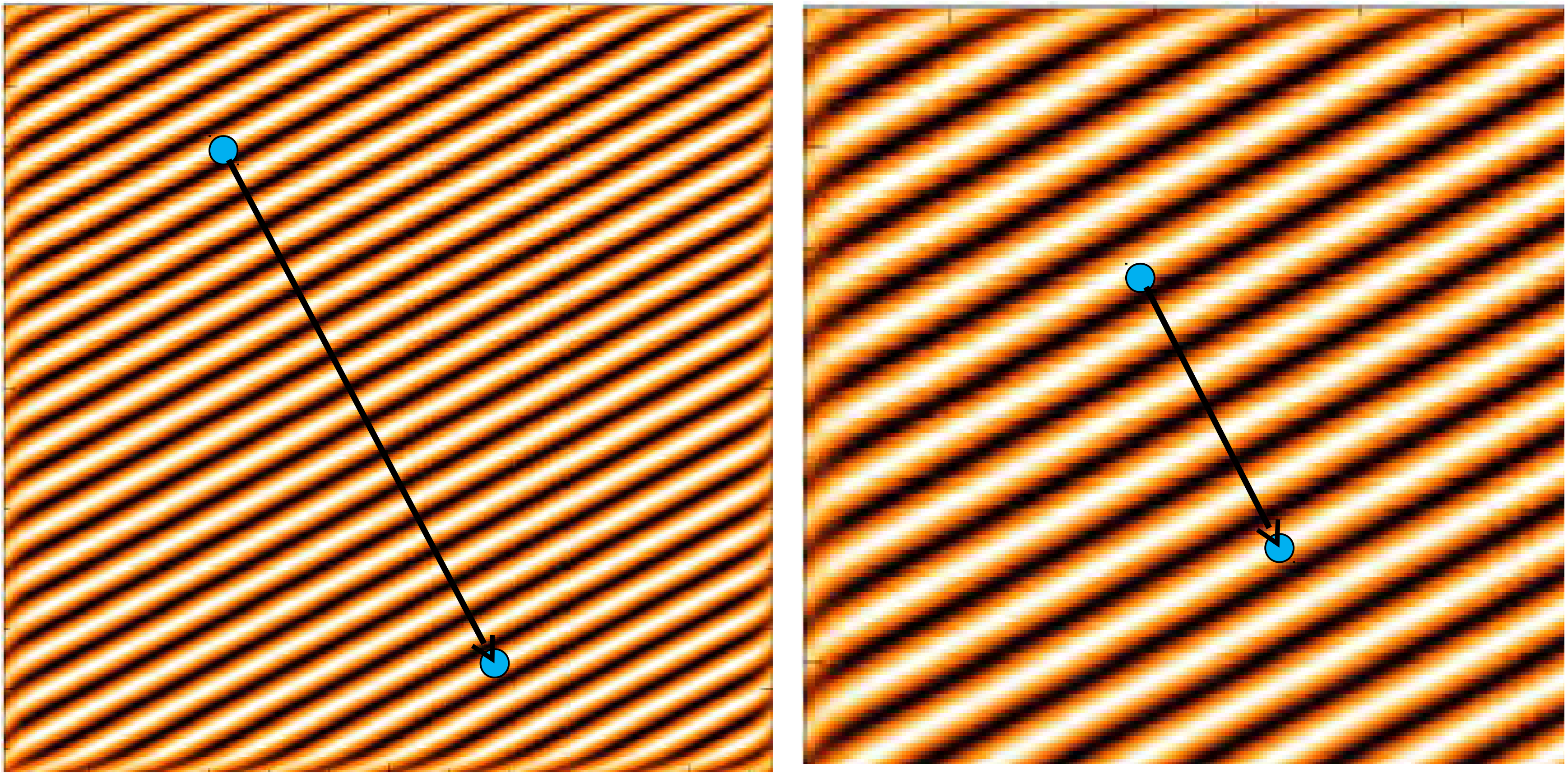


Phase comes from the (sum of) geometric path length, optical fiber, cables and electronic delays

Reciprocity theorem

Screen/Detector → Correlator

Sky response of an individual baseline

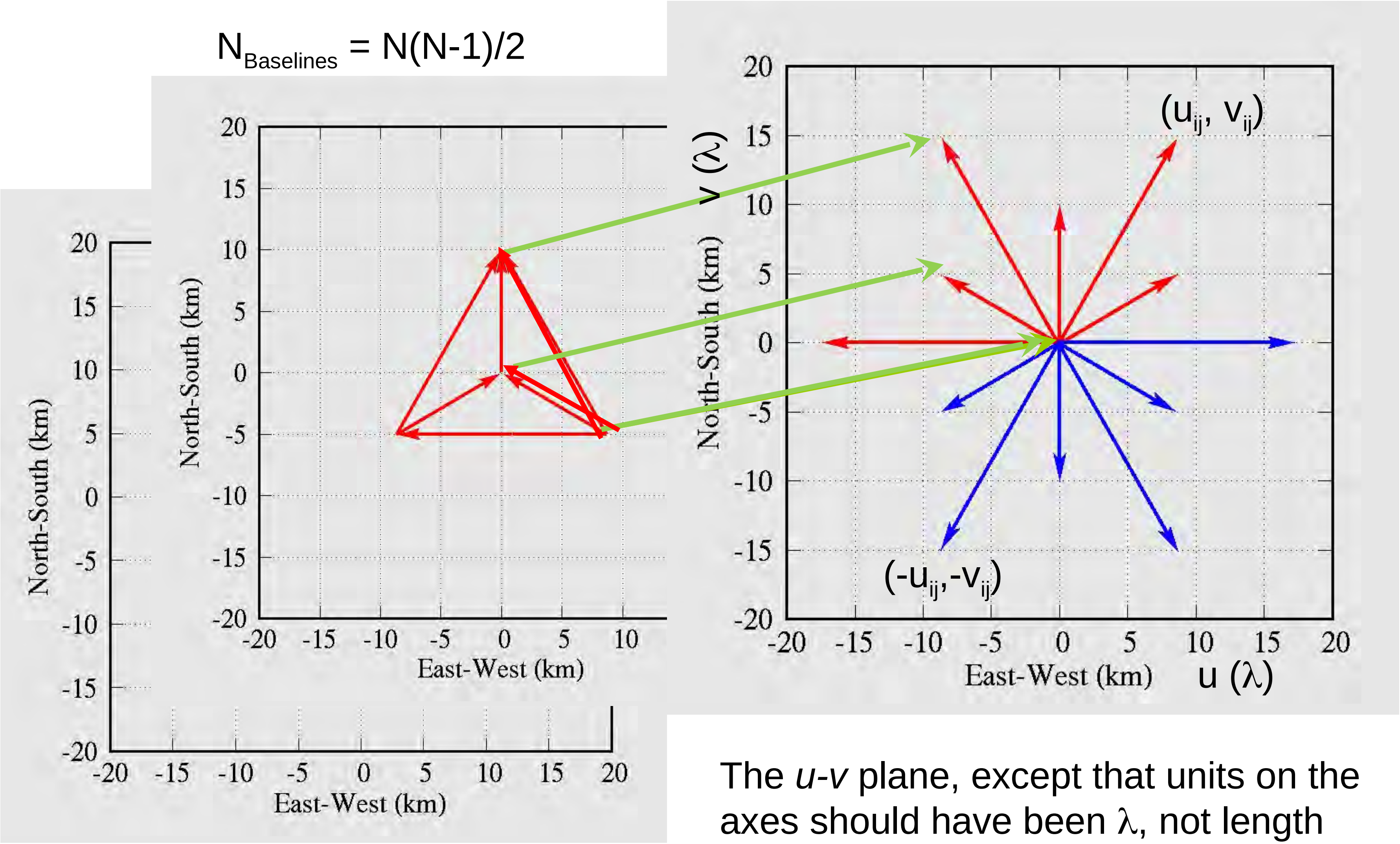


Cos $2\pi(ul + vm)$;

u,v – components of the baseline; l,m – coordinates in image plane

Baselines and *u-v plane*

$N_{Baselines} = N(N-1)/2$



The *u-v* plane, except that units on the axes should have been λ, not length

Visibility V(u,v)

- The fundamental Radio Astronomy measurable

  $V_{ij}(u,v,t,\Delta t,\nu_0,\Delta\nu) = <V_i(\ldots) \times V_j^*(\ldots,t+\tau,\ldots)>$

- van Cittert Zernike Theorem

  V(u,v) is 2D Fourier Transform of the sky Brightness distribution $B(\theta,\varphi)$

(T(x,y) in the following slides)

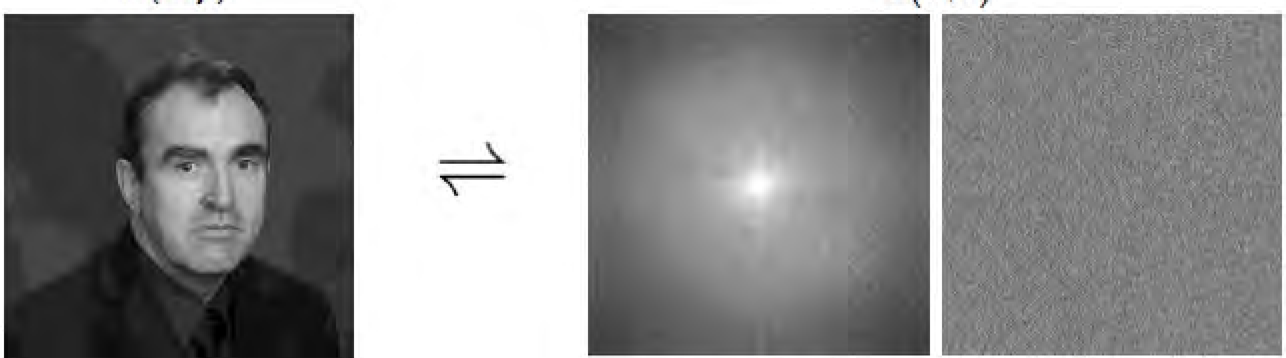
- Incoherent source,
- Small field of view
- Far-field

Visibilities

- each V(u,v) contains information on T(x,y) *everywhere*, not just at a given (x,y) coordinate or within a given subregion

- V(u,v) is a complex quantity
  - visibility expressed as (real, imaginary) or (amplitude, phase)

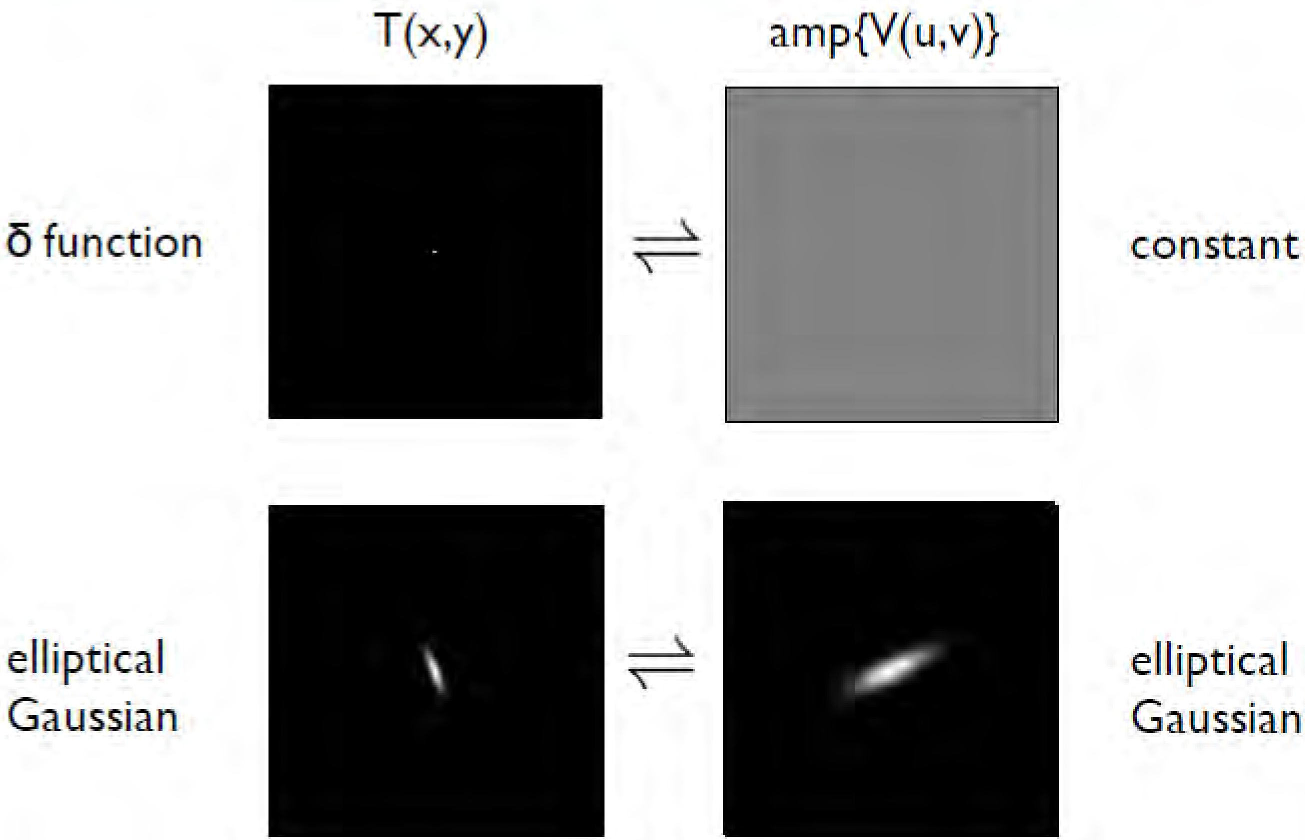
T(x,y) V(u,v)



amplitude phase

Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Example 2D Fourier Transform Pairs



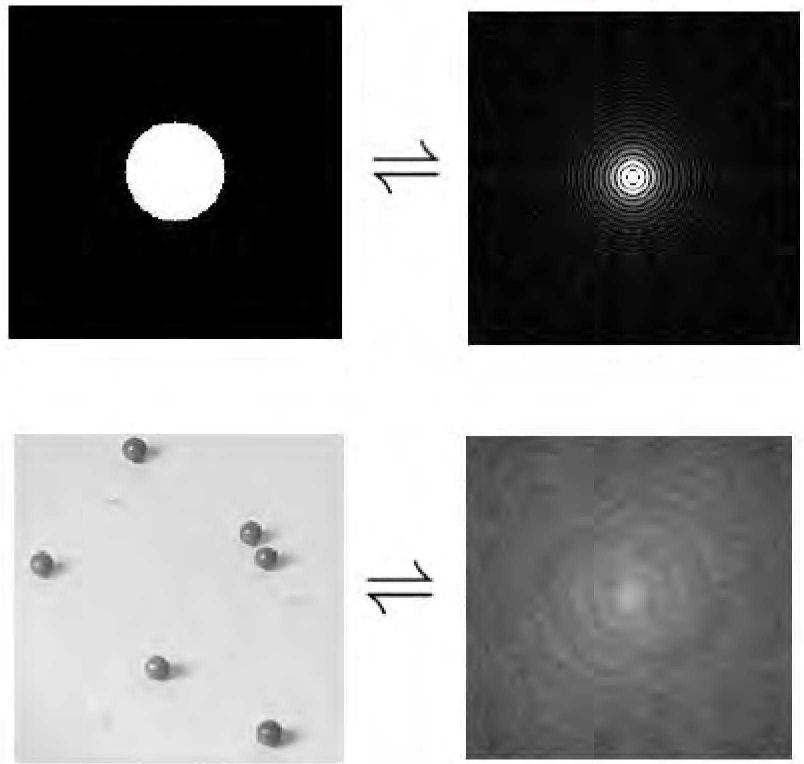
narrow features transform into wide features (and vice-versa)

Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Example 2D Fourier Transform Pairs

T(x,y) amp{V(u,v)}

disk

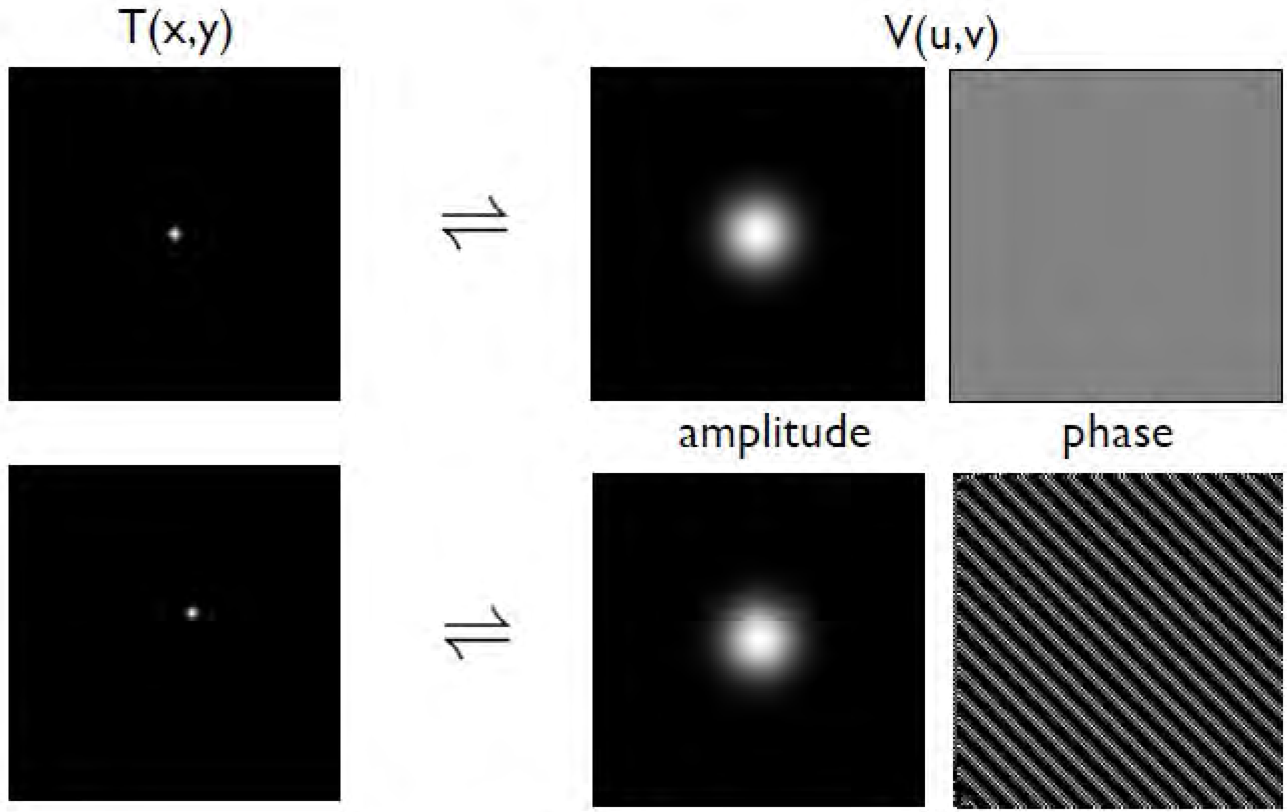


Bessel

sharp edges result in many high spatial frequencies

Amplitude and Phase

- amplitude tells "how much" of a certain spatial frequency
- phase tells "where" this component is located



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The Visibility Concept

$$V(u,v) = \int \int T(x,y) e^{2\pi i (ux+vy)} dxdy$$

- visibility as a function of baseline coordinates (u,v) is the Fourier transform of the sky brightness distribution as a function of the sky coordinates (x,y)

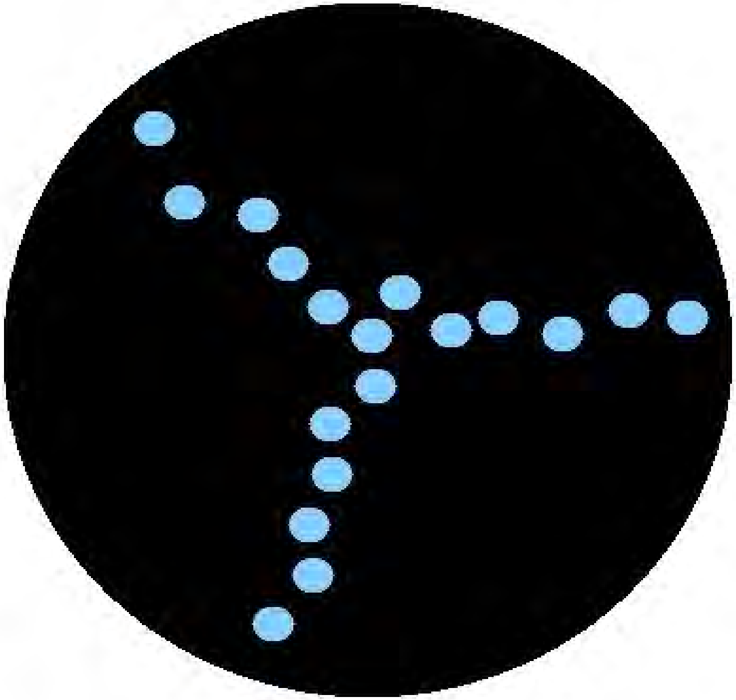
- V(u=0,v=0) is the integral of T(x,y)dxdy = total flux

- since T(x,y) is real, V(u,v) is Hermitian: V(-u,-v) = V*(u,v)
  - get two visibilities for one measurement

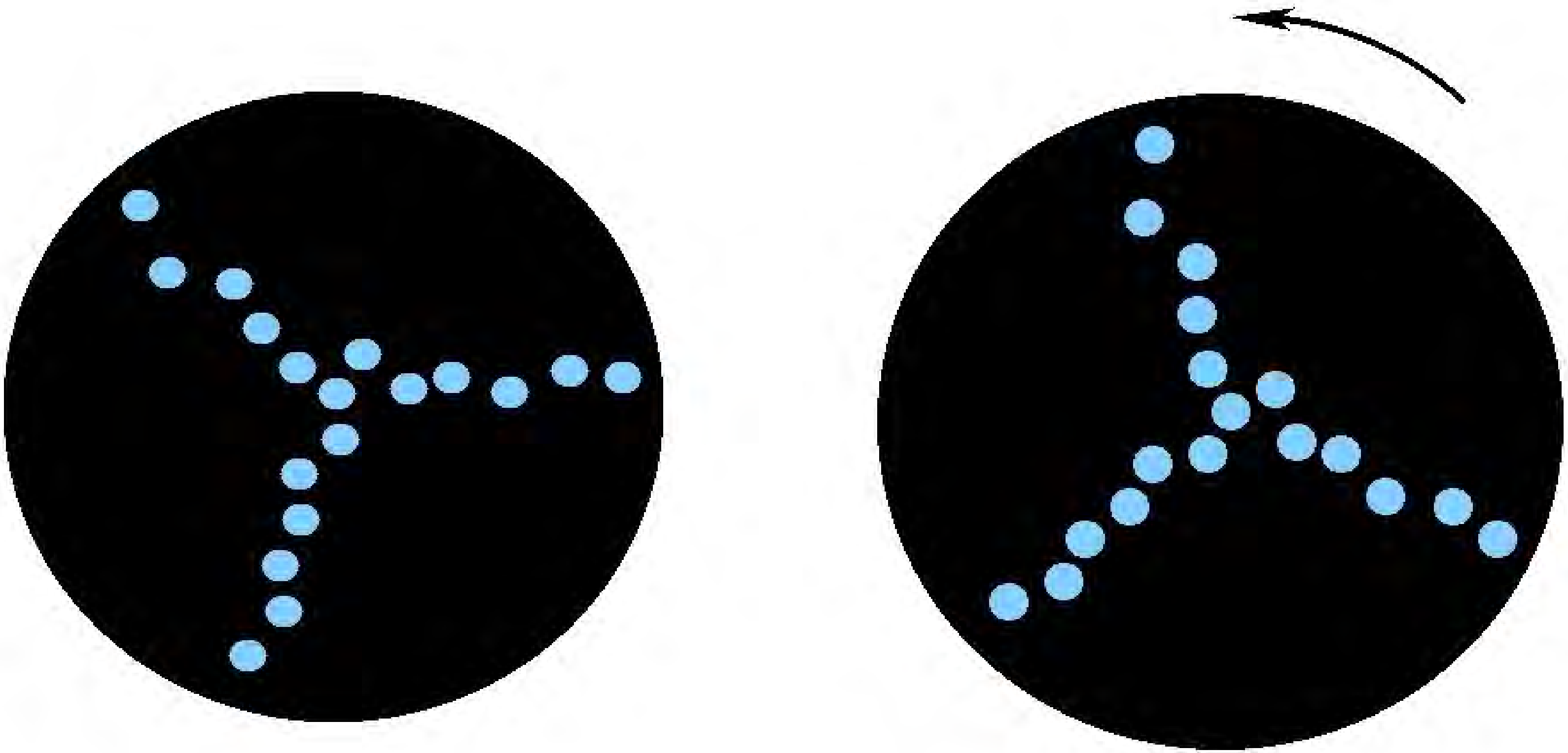
Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

An N element interferometer

- ‘Baselines’ from N elements – N(N-1)/2
- Each of these will lead to a ‘fringe’ with different orientation and spacing
- The final response of the interferometer will be the superposition of fringes from all the baselines



Synthesis imaging



VLA - 27 antennas ⇒ 351 baselines

GMRT - 30 antennas ⇒ 435 baselines

MWA – 128 elements ⇒ 8,128 baselines

The mathematical basis

- Brightness distribution in the sky is Fourier transform of the Visibilities

  $B(\theta,\varphi) \leftrightarrow V(u,v)$

  V(u,v) – The quantity measured by a baseline (amplitude, phase / real, imaginary)

- In the uv-plane, we measure visibilities only at a few places i.e. we have a sampling function

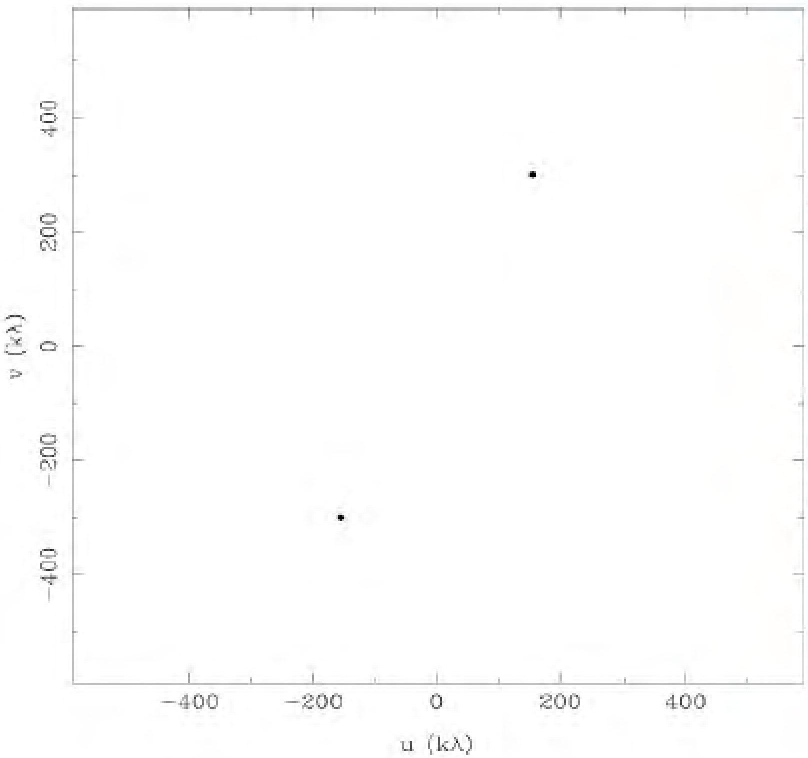
  $S(u,v) = \Sigma_k (u_k, v_k)$

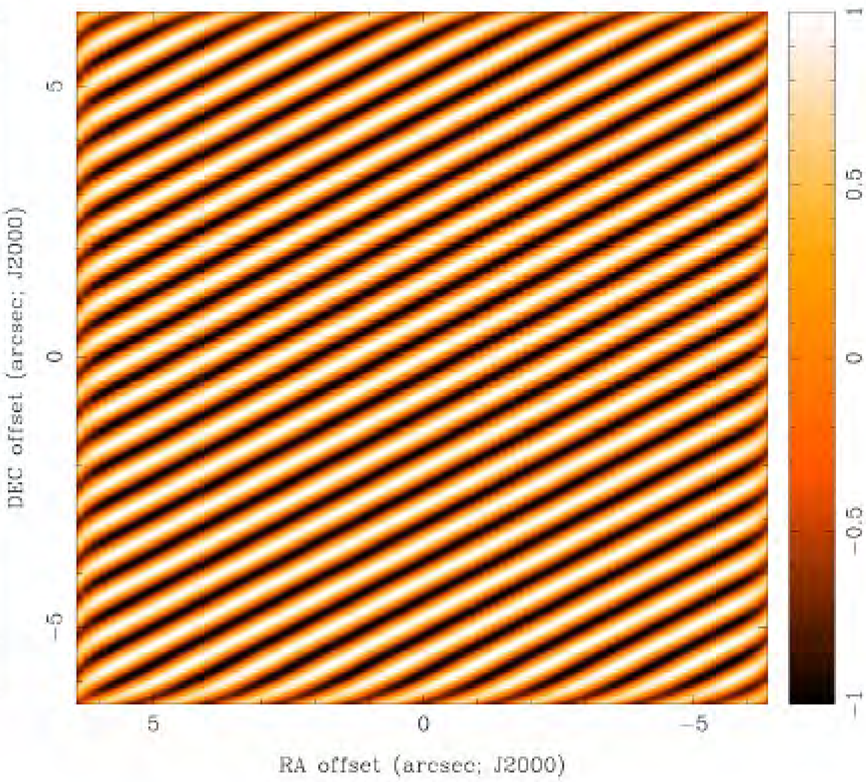
- Point source response of an interferometer (PSF) is Fourier transform of S(u,v)

  $P(\theta,\varphi) \leftrightarrow S(u,v)$

Dirty Beam Shape and N Antennas

2 Antennas





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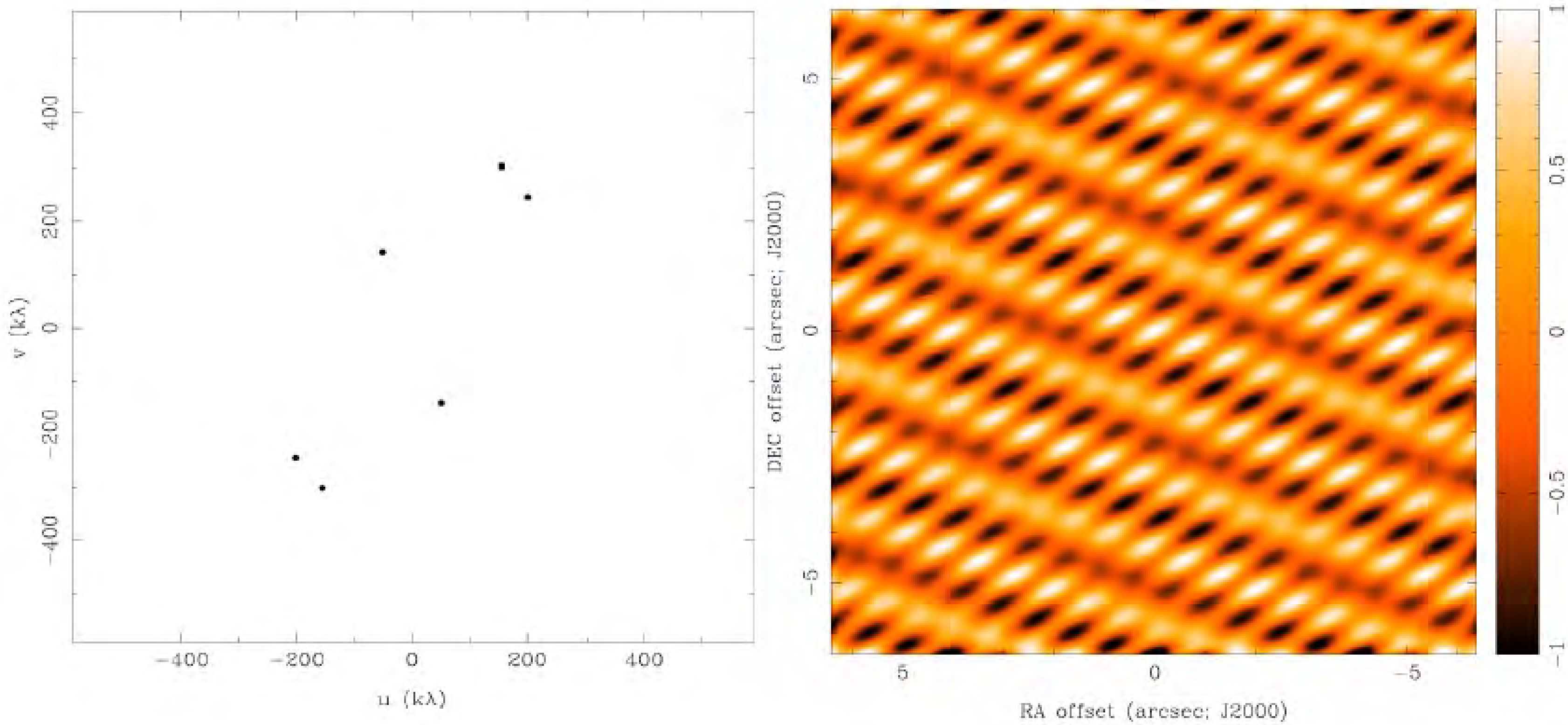
Dirty Beam Shape and N Antennas

3 Antennas



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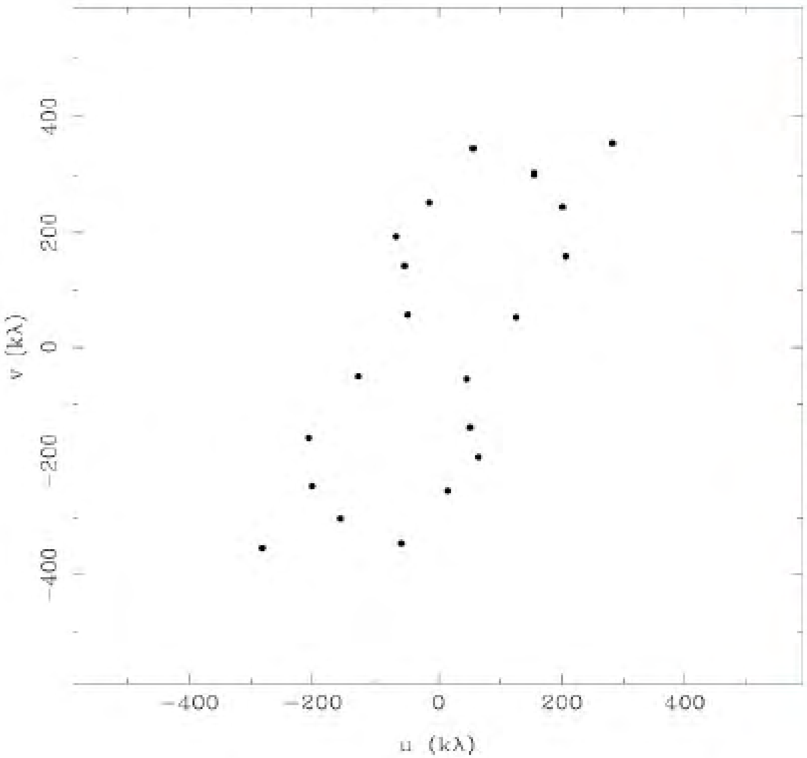
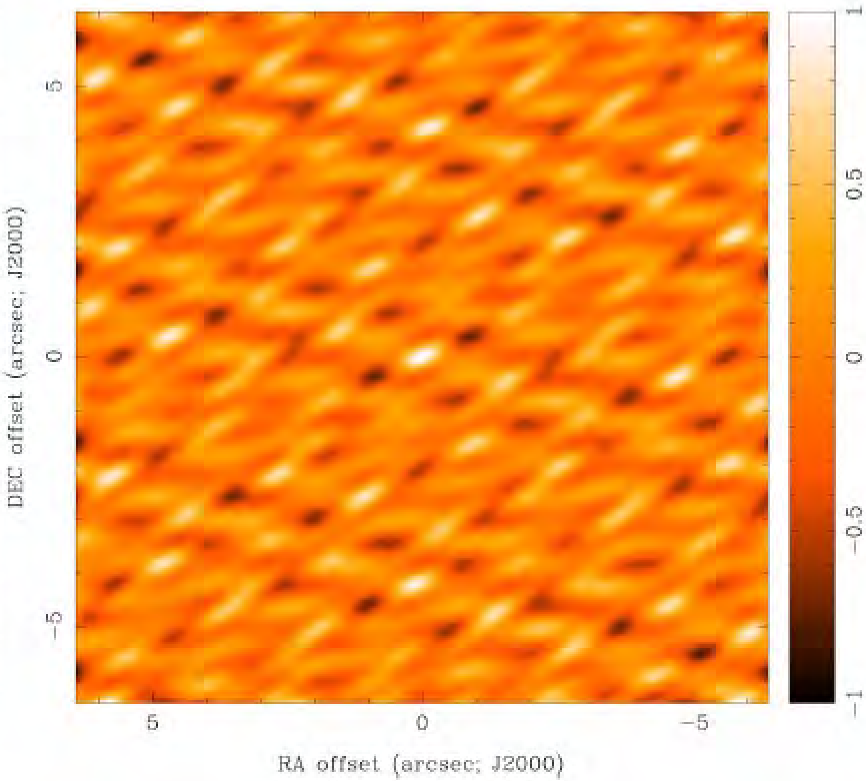
Dirty Beam Shape and N Antennas

5 Antennas





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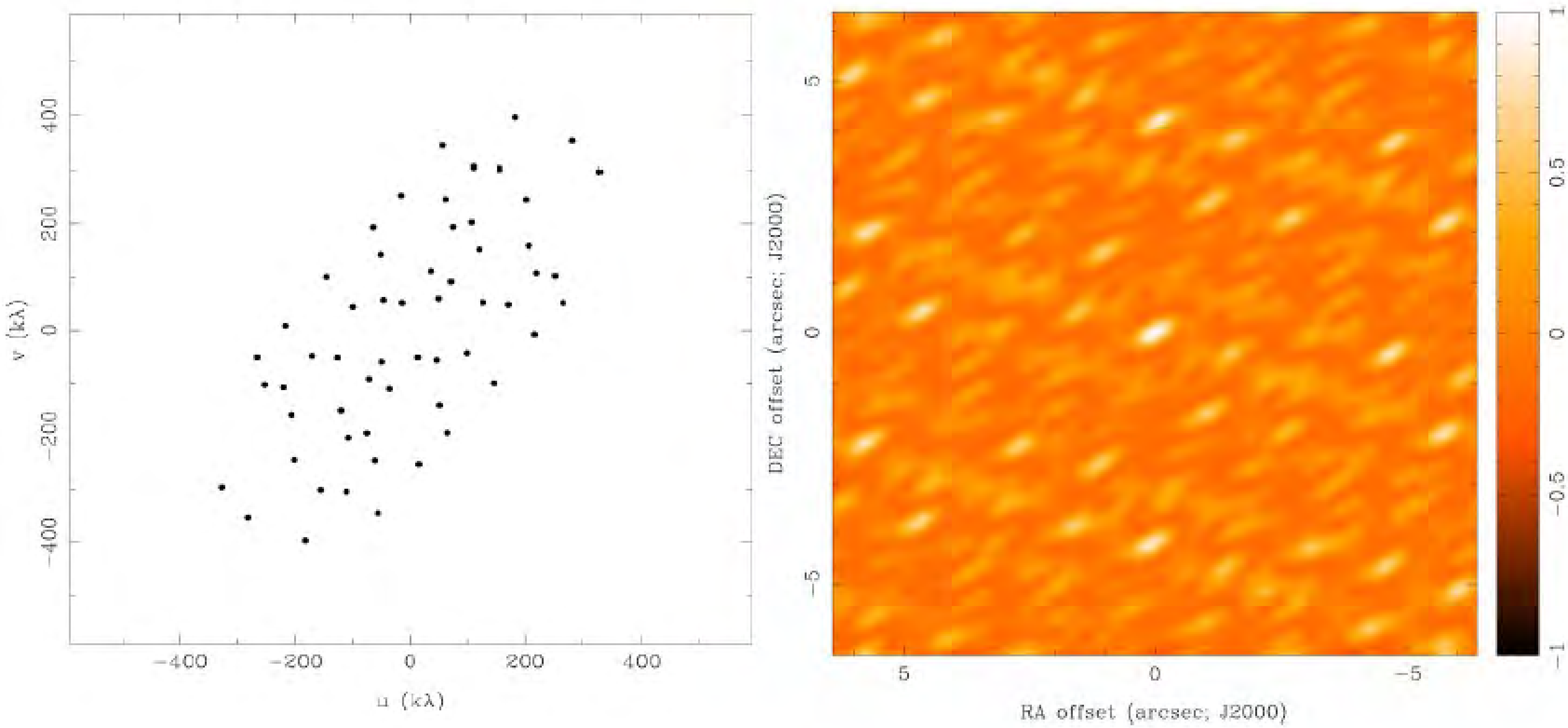
Dirty Beam Shape and N Antennas

8 Antennas



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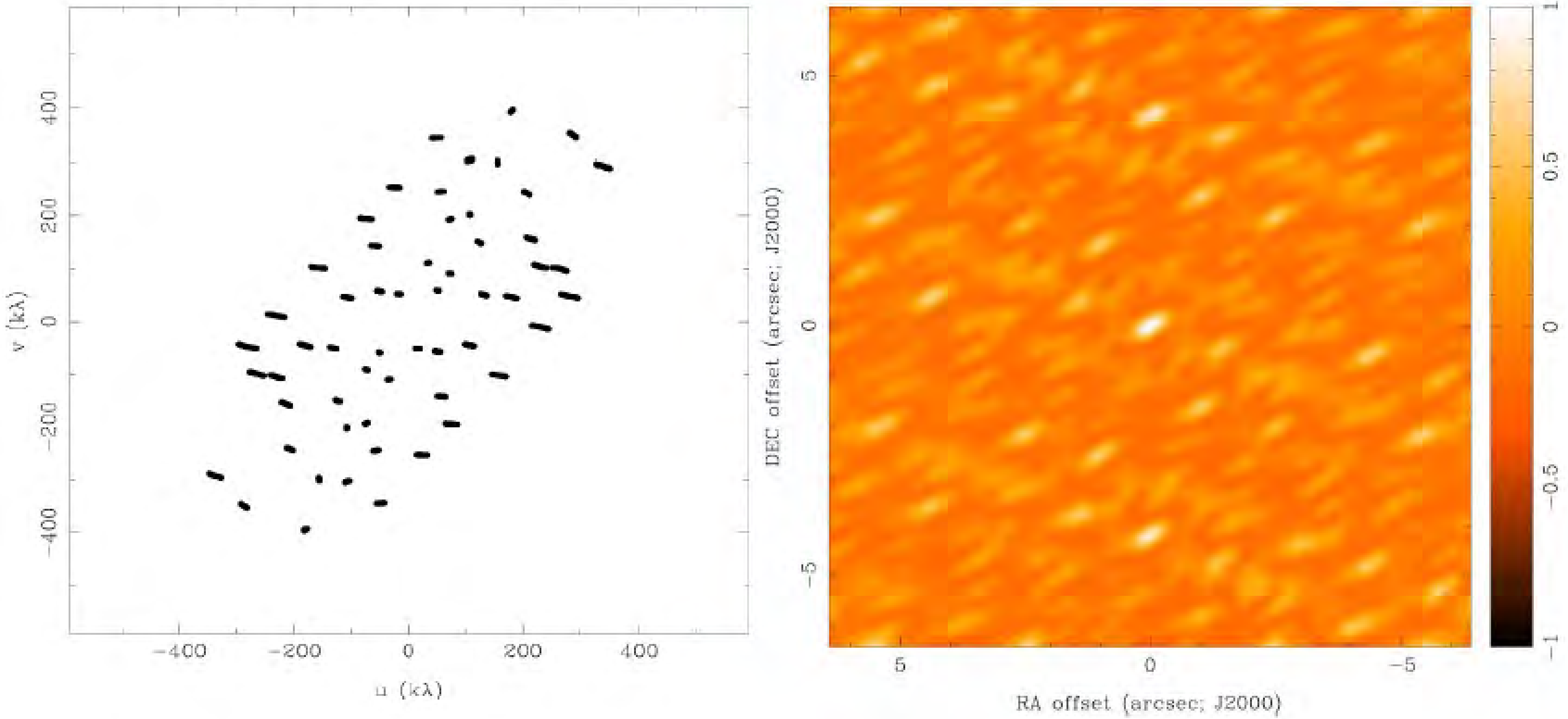
Dirty Beam Shape and N Antennas

8 Antennas x 30 samples



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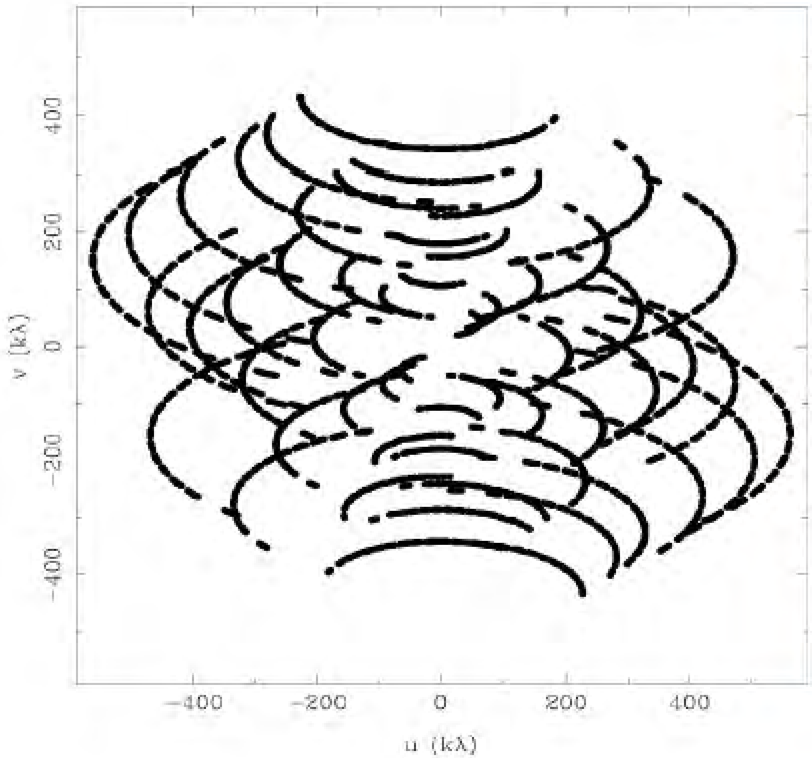
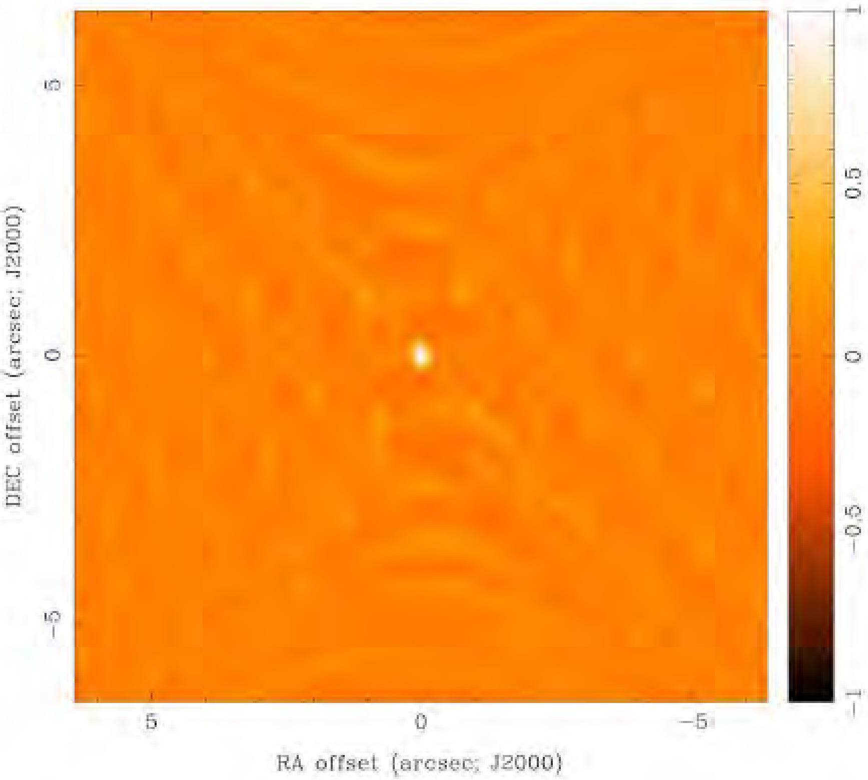
Dirty Beam Shape and N Antennas

8 Antennas x 480 samples





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So what do we finally have?

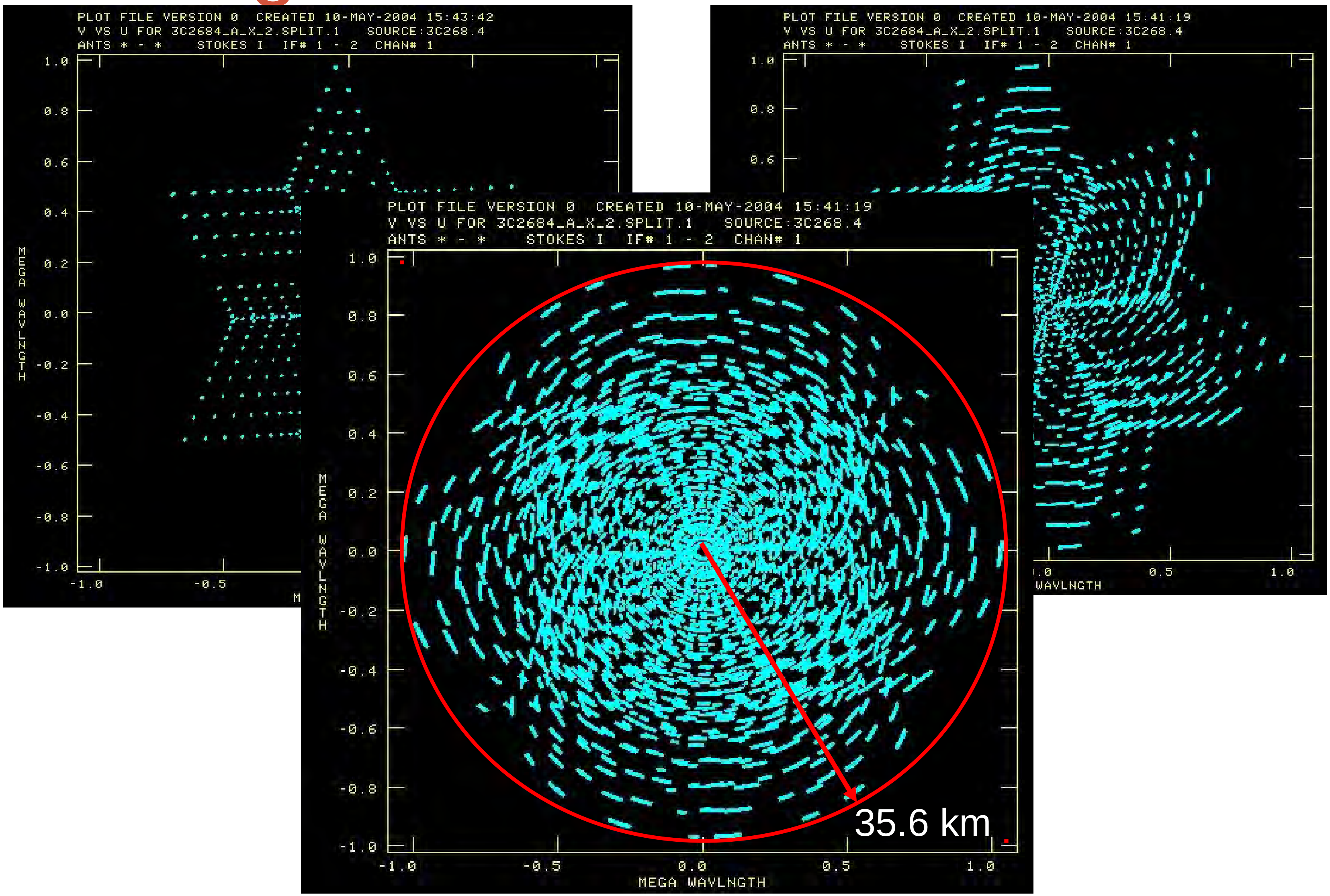
- Point Spread Function (PSF) – Dirty beam – FT of uv sampling
- Dirty image
  - – FT of the measured visibilities
  - – Convolution of the PSF and the true sky brightness distribution
- To get true sky brightness distribution, one needs to ‘deconvolve’ the PSF from the dirty image

A real life example

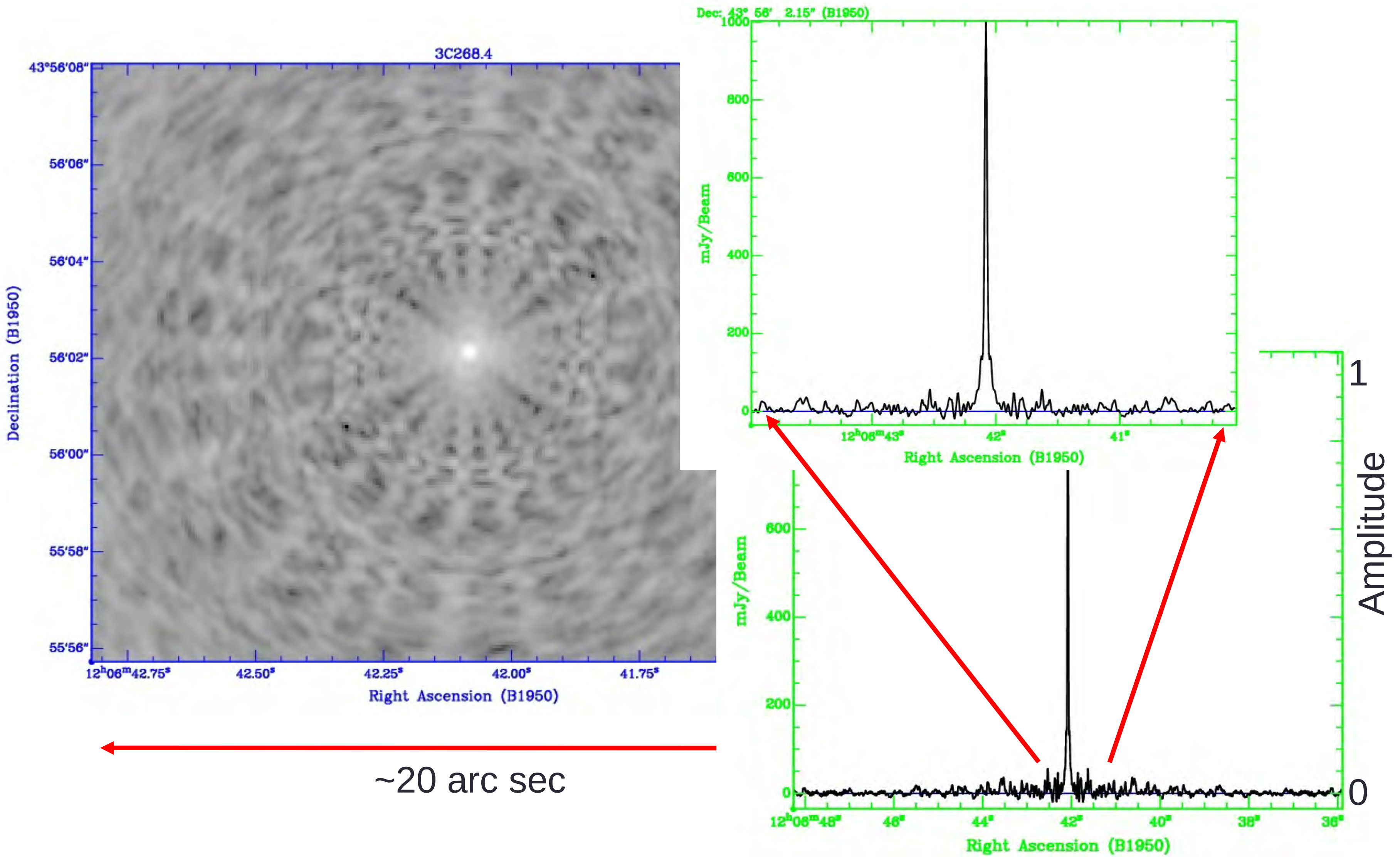
- The Very Large Array (VLA), NM
- 8.43 GHz (λ = 3.56cm)
- 3C268.4

- Data courtesy Colin Lonsdale, MIT Haystack Observatory

Array configuration and u-v coverage



The interferometer response function (Point Spread Function)

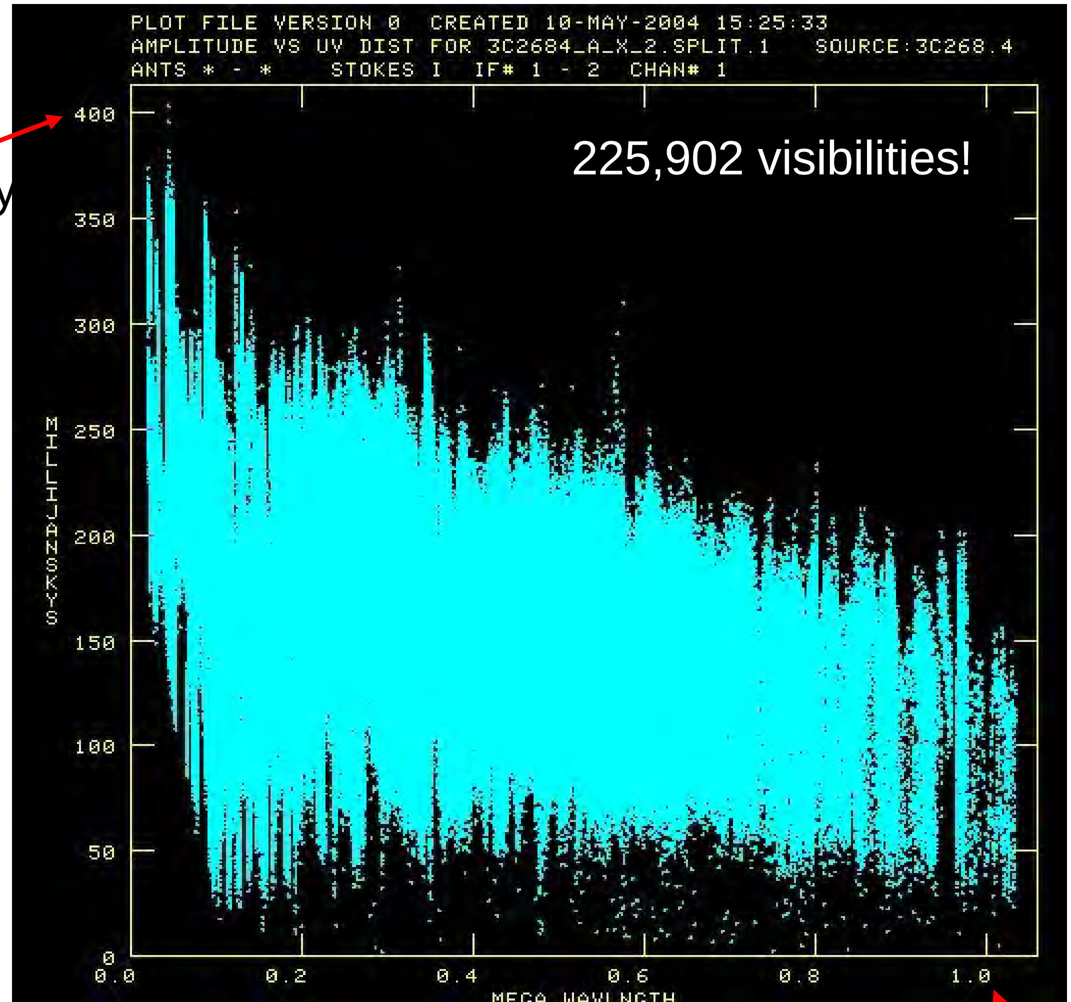


The measured cross-correlations

A typical FM radio station ~0.1 W Hz^{-1} placed at the distance of the Sun (1.5×10^8 km)
⇒ ~35 Jy at Earth

VLA sensitivity at 8 GHz ~45×10^{-6} Jy (10 min, 86 MHz)

In 10 min VLA can detect a source as strong as a typical FM station ~88 AU away!



400 mJy

Amplitude (mJy)

Sqrt($u^2 + v^2$) (λ)

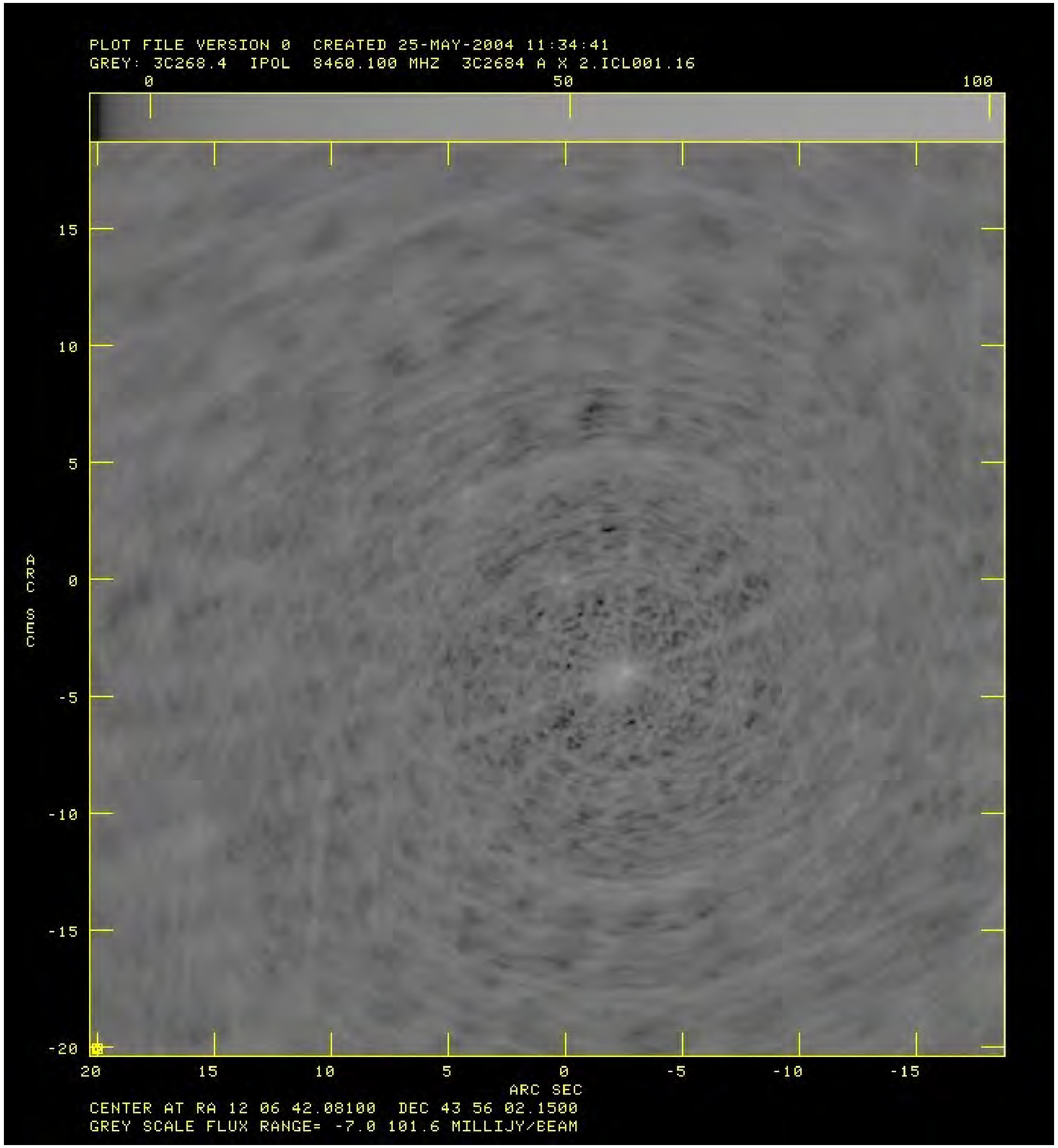
10^6 λ

1 Jy = 10^{-26} W m^{-2} Hz^{-1}

FT of gridded visibilities

The *dirty* map

Convolution of the PSF with the Brightness distribution



Log scale

The problem of deconvolution

- The measurements from any instrument are really the *convolution* of the *transfer function* of the instrument and the input signal.
- In order to figure out the true input signal, it is necessary to *deconvolve* the *transfer function* from the measurements
- Radio Astronomy solutions
  - CLEAN algorithm(s)
  - Maximum Entropy Method(s)

The *CLEANed* map

Actually, CLEANed and *Self-calibrated* map

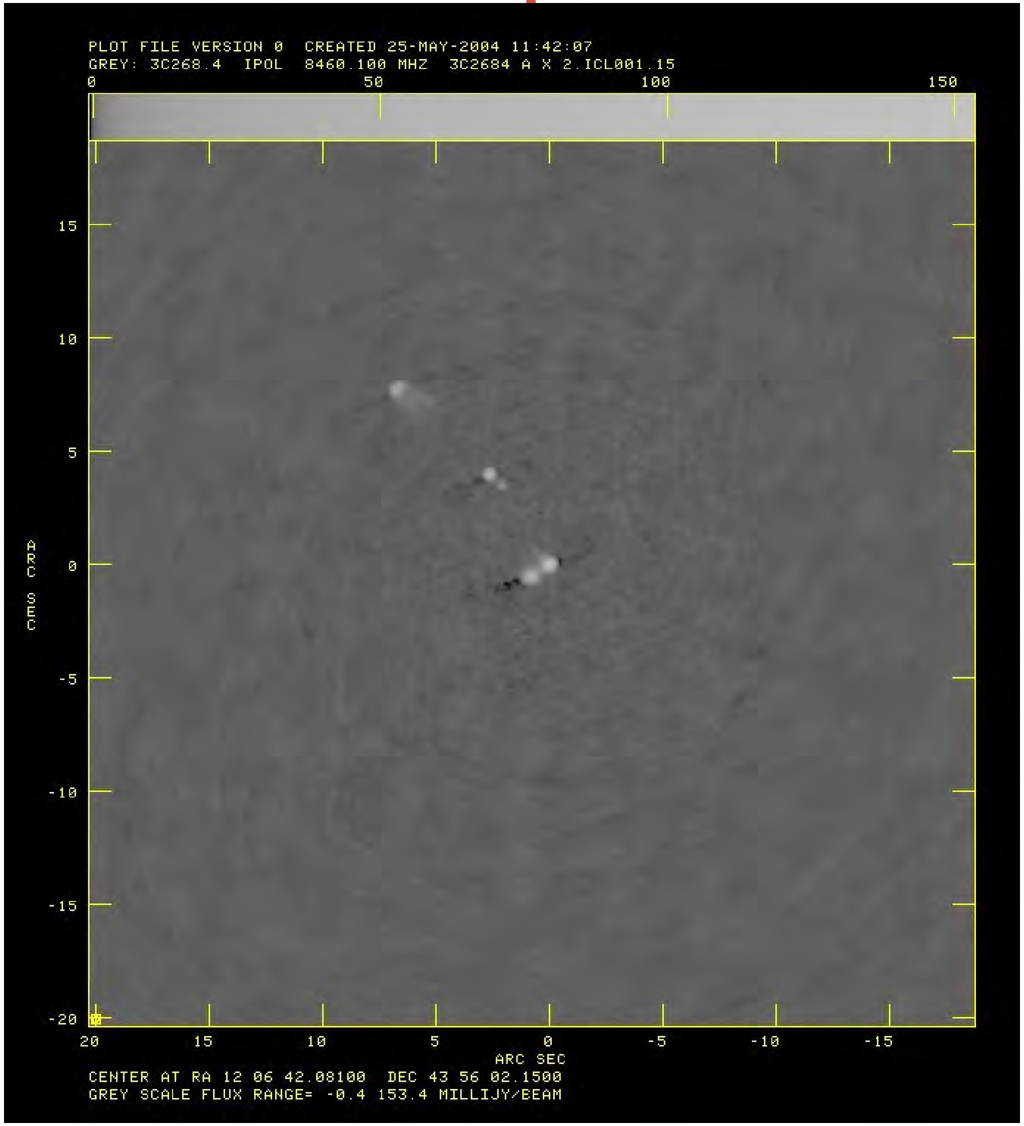
~50,000 Clean iterations

~4000 Clean components

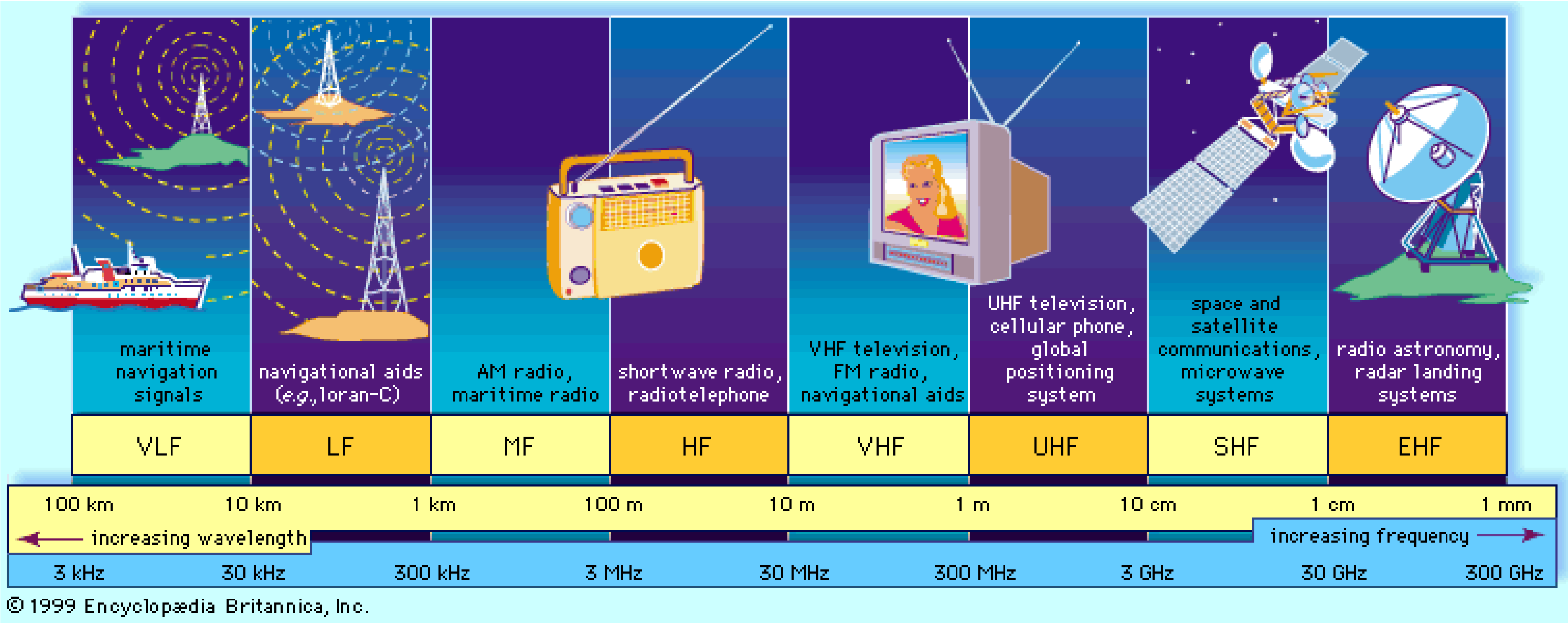
Dynamic range ~5000

Noise ~30 μJy/beam

Log scale



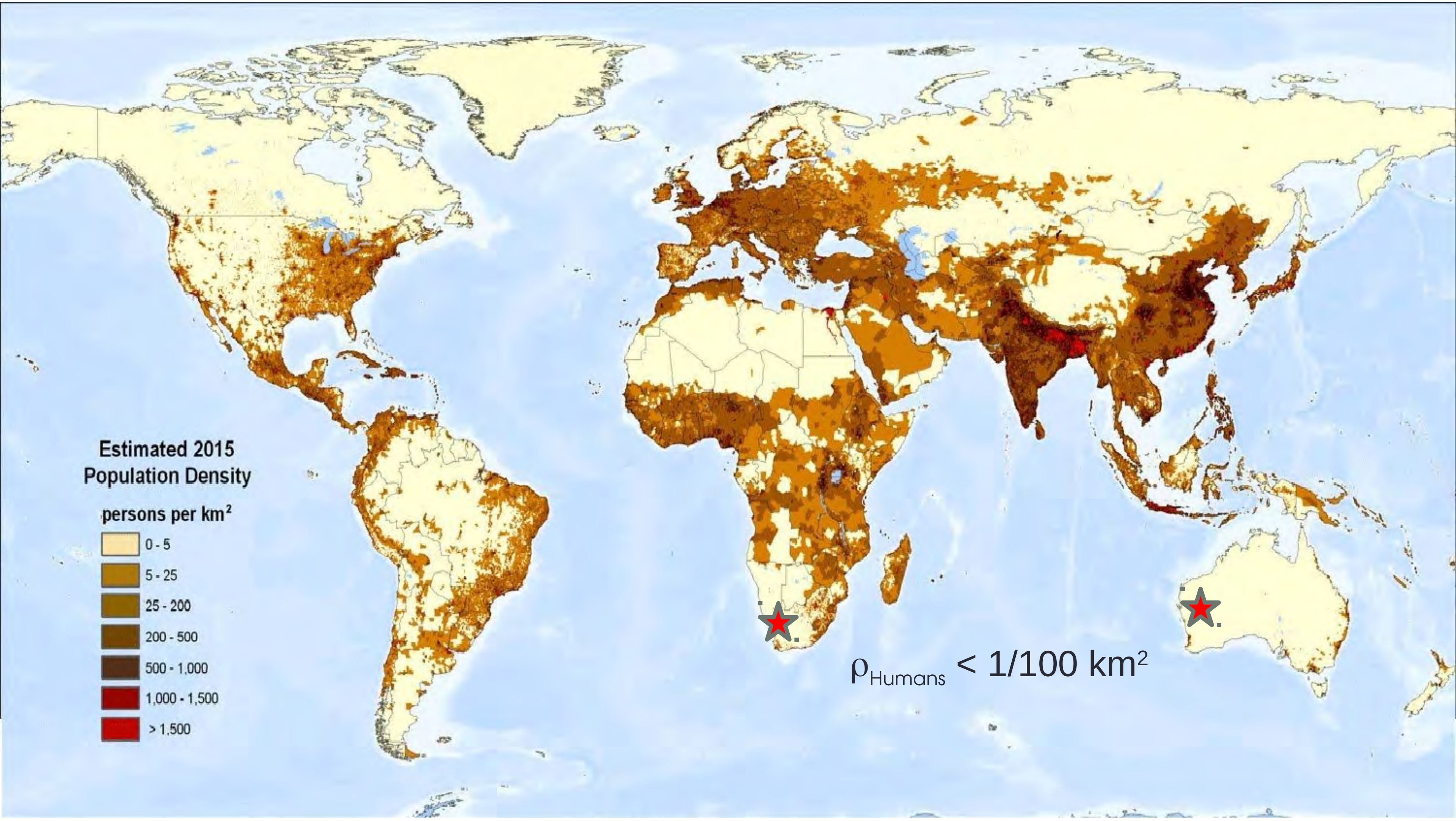
Radio analog of dark-sky problem



Human presence = radio pollution

Cell phones, chord less phones, garage door openers, keyless entry systems, computers, florescent lights, petrol vehicles, mircowave ovens, bluetooth devices,

Population density of the World



Looking Ahead

- Radio astronomy : reaping the benefits of astounding leaps in technology
  - Computational capacity and storage
  - Algorithms for data processing
  - Digital signal processing hardware
  - Analog electronics
  - Antenna design and modelling

New Telescopes on the horizon

- Jansky Very Large Array (US)
- Atacma Large Millimetre/Submillimetre Array (Chile)
- Low Frequency Array (The Netherlands, Europe)
- Murchison Widefield Array (Australia)

- Upgraded GMRT (India)
- Australian SKA Pathfinder (Australia)
- MEERKAT (South Africa)
- Square Kilometre Array (Australia, South Africa)

The challenges (and opportunities)

- Characteristics of new instruments
  - 1-2 orders of magnitude improvements in sensitivity and imaging fidelity
  - Gather more detailed and higher SN information about the sky
    - Ability to solve more challenging astrophysics problems
  - Assumptions/approximations made in most of the present analysis no longer valid
    - Require more sophisticated analysis algorithms
  - Higher sensitivity => need to reduce ‘systematics’ in the analysis
    - Requires deeper understanding of the instrument, the sky and the analysis procedures
- Not only will they enable new and exciting science, the process of getting to the new science requires really exciting research in its own right

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