Lecture 8 Notes: Stellar Evolution and Compact Objects

Introduction to Astronomy and Astrophysics I

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1 Nuclear Binding Energy

The binding energy per nucleon curve is fundamental to understanding stellar nucleosynthesis and energy generation in stars. The curve shows:

- Peak around Iron-56, making it the most stable nucleus
- Energy can be released through fusion of lighter elements or fission of heavier elements
- Explains why fusion in stars stops at iron

2 White Dwarfs

2.1 Introduction

White dwarfs are stellar remnants that represent the final evolutionary state for low and intermediate mass stars ($M < 8M_\odot$). Sirius B is a famous example of a white dwarf.

2.2 Binary Systems and Roche Lobes

- The Roche potential defines gravitational equipotential surfaces in binary systems
- Roche lobe: Critical equipotential surface determining mass transfer
- Eggleton's approximation for Roche lobe radius:

$$\frac{R_L}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}$$

  where $q = M_2/M_1$ and a is the binary separation

2.3 Type Ia Supernovae

- Caused by accretion onto a white dwarf from a companion star
- Occurs when white dwarf approaches Chandrasekhar limit
- Important as standard candles for cosmology
- Relatively uniform luminosity due to consistent mass at explosion

2.4 Astrophysical Importance

White dwarfs are important for:

- Cosmochronology (age dating stellar populations)
- Type Ia supernovae as standard candles
- Understanding binary evolution
- Studying planetary system evolution

3 Neutron Stars

3.1 Formation

- Result of core-collapse supernovae
- Progenitor mass range: $8M_\odot < M < 20M_\odot$
- Formation process involves:
  1. Iron core reaching Chandrasekhar limit
  2. Core collapse and neutronization
  3. Bounce and shockwave
  4. Neutrino emission
  5. Ejection of outer layers

3.2 Structure

- Radius: $\sim 10 - 14$ km
- Mass: $\sim 1.4 - 3M_\odot$
- Layers:
  - Outer crust: nuclei + electrons
  - Inner crust: neutron-rich nuclei + free neutrons
  - Outer core: neutron superfluid + protons + electrons
  - Inner core: possibly exotic matter

3.3 Properties

- Extreme density: $\rho \sim 10^{14} - 10^{15}$ g/cm^3
- Rapid rotation: periods from milliseconds to seconds
- Strong magnetic fields: $10^8 - 10^{15}$ Gauss
- Maximum mass: $\sim 2.5 - 3M_\odot$ (EoS dependent)

4 The TOV Equation

The Tolman-Oppenheimer-Volkoff (TOV) equation describes hydrostatic equilibrium in general relativity:

$$\frac{dP(r)}{dr} = -\frac{G\left[\rho(r) + \frac{P(r)}{c^2}\right]\left[M(r) + 4\pi r^3 \frac{P(r)}{c^2}\right]}{r\left[r - 2GM(r)/c^2\right]}$$

In the Newtonian limit ($c \to \infty$), this reduces to:

$$\frac{dp(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

5 Pulsars

5.1 Basic Properties

- Rotating neutron stars with strong magnetic fields
- Lighthouse model explains periodic pulses
- Types include radio pulsars, X-ray pulsars, and millisecond pulsars
- Spin periods range from milliseconds to seconds
- Magnetic field strength can be estimated from period and period derivative

5.2 Evolution and Characteristics

- Characteristic age: $\tau_c = P/(2\dot{P})$
- Evolution tracked on P-$\dot{P}$ diagram
- Magnetic field strength: $B \sim 3.2 \times 10^{19}\sqrt{P\dot{P}}$ Gauss