Lecture 13: Galaxy Luminosity Functions and Virial Theorem Applications

Course: Introduction to Astronomy and Astrophysics I

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1 Galaxy Luminosity Functions

1.1 Overview

The luminosity function specifies the relative number of galaxies at each luminosity. It represents a convolution of several physical processes:

- Primordial density fluctuations
- Galaxy creation and destruction processes
- Morphological transformation processes (mergers, stripping)
- Mass-to-light conversion mechanisms

1.2 Schechter Luminosity Function

The standard functional form (Schechter 1976):

$$\phi(L)dL = n_* \left(\frac{L}{L_*}\right)^{\alpha} \exp\left(-\frac{L}{L_*}\right) d\left(\frac{L}{L_*}\right)$$

Key parameters:

- L_* - characteristic luminosity ($\sim 10^{10} L_\odot$)
- α - faint-end slope (~ -0.8 to -1.3)
- n_* - normalization ($\sim 0.02 h^3$ Mpc^{-3})

1.3 Measuring Luminosity Functions

1.3.1 In Galaxy Clusters

Advantages:

- All galaxies at same distance

- Direct conversion from apparent to absolute magnitude
- Simpler statistical analysis

Method:

1. Bin galaxies by apparent magnitude
2. Convert to absolute magnitudes using cluster distance
3. Fit Schechter function via χ^2 minimization

1.3.2 In the Field

Challenges:

- Malmquist bias due to flux-limited samples
- Variable survey volume with luminosity
- Need for statistical corrections

Correction methods:

- V_{max} method
- Maximum likelihood estimators
- Corrections for incompleteness

2 Virial Theorem Applications

2.1 Basic Principles

For a system in equilibrium:

$$2T + U = 0$$

where:

- T = total kinetic energy
- U = gravitational potential energy

2.2 Applications in Astrophysics

The virial theorem can be applied to:

- Elliptical galaxies
- Globular clusters
- Galaxy clusters
- Relaxed stellar systems

2.3 Mass Estimation

For a spherical system:

$$M \simeq \frac{\sigma^2 R}{G}$$

where:

- M = total mass
- σ = velocity dispersion
- R = characteristic radius
- G = gravitational constant

2.4 Dark Matter Evidence

Applications of virial theorem reveal:

- Mass-to-light ratios much larger than stellar populations
- Evidence for dark matter in clusters (Zwicky 1937)
- Typical cluster M/L $\sim 200 - 500$ solar units

3 Further Reading

Key references:

- Schechter (1976) ApJ, 203, 297 - Original luminosity function paper
- Press & Schechter (1974) ApJ, 187, 425 - Theoretical basis
- Binney & Tremaine (2008) "Galactic Dynamics" - Detailed virial theorem applications
- Zwicky (1937) ApJ, 86, 217 - First dark matter evidence in clusters
- Blanton et al. (2003) ApJ, 592, 819 - Modern luminosity function measurements

Online resources:

- SDSS luminosity function calculator: `https://www.sdss.org/dr16/spectro/galaxy_kcorrections/`
- Galaxy Mass Assembly (GAMA) survey: `http://www.gama-survey.org/`

4 Special Topics

4.1 Malmquist Bias

Important observational effect where:

- Flux-limited samples preferentially detect intrinsically bright objects
- Can significantly affect luminosity function measurements
- Requires careful statistical corrections

4.2 Environment Effects

Luminosity functions vary with:

- Galaxy morphology
- Local density
- Cluster vs. field environment
- Redshift/cosmic epoch